CLASS IX: MATHS

Chapter 11: Surface Areas and Volumes

Questions and Solutions | Exercise 11.1 - NCERT Books

Q1. Diameter of the base of a cone is 10.5 cm and its slant height is 10 cm. Find its curved surface area.

Sol. $\because$ Diameter of the base = 10.5 cm

$\therefore$ Radius of the base (r) = $\frac{10.5}{2}$ cm = 5.25 cm

Slant height (ℓ) = 10 cm

$\therefore$ Curved surface area of the cone

$= \pi r\ell = \frac{22}{7} \times 5.25 \times 10 = 165 \text{ cm}^2.$

Q2. Find the total surface area of a cone, if its slant height is 21 m and diameter of its base is 24 m.

Sol. ℓ = 21m, r = 12 m

Total surface area $= \pi r (r + \ell) = \frac{22}{7} \times 12 \times 33 \text{ m}^2$

$= 1244.57 \text{ m}^2$

Q3. Curved surface area of a cone is 308 cm^2 and its slant height is 14 cm. Find
(i) radius of the base and (ii) total surface area of the cone.

Sol. (i) Slant height (ℓ) = 14 cm

Curved surface area = 308 cm^2

$\Rightarrow \pi r\ell = 308 \qquad \Rightarrow \frac{22}{7} \times r \times 14 = 308$

$\Rightarrow r = \frac{308 \times 7}{22 \times 14} \qquad \Rightarrow r = 7 \text{ cm}$

Hence, the radius of the base is 7 cm.

(ii) Total surface area of the cone = $\pi r(\ell + r) = \frac{22}{7} \times 7 \times (14 + 7) = \frac{22}{7} \times 7 \times 21 = 462 \text{ cm}^2$

Hence, the total surface area of the cone is 462 cm^2.

Q4. A conical tent is 10 m high and the radius of its base is 24 m. Find

(i) Slant height of the tent.

(ii) cost of the canvas required to make the tent, if the cost of 1 m^2 canvas is Rs. 70.

Sol. Height of the tent (h) = 10 m

Radius of the base (r) = 24 m

(i) The slant height, $\ell = \sqrt{h^2 + r^2}$

$\ell = \sqrt{(24)^2 + (10)^2}$ m $= \sqrt{576 + 100}$ m

$\ell = 26$ m

Thus, the required slant height of the tent is 26 m.

(ii) Curved surface area of the cone = $\pi r \ell$

$\therefore$ Area of the canvas required = $\frac{13728}{7} \text{ m}^2$

$\therefore$ Cost of $\frac{13728}{7} \text{ m}^2$ canvas

$= \text{Rs } 70 \times \frac{13728}{7} = \text{Rs } 137280$

Q5. What length of tarpaulin 3 m wide will be required to make conical tent of height 8 m and base radius 6 m ? Assume that the extra length of material that will be required for stitching margins and wastage in cutting is approximately 20 cm (Use $\pi = 3.14$)

Sol. Area of Tarpaulin required = Curved surface of the conical tent

$1 = \sqrt{8^2 + 6^2} = 10\text{m}$

Area of tarpaulin = $3.14 \times 6 \times 10$

Acc. to quest = 188.4

3m × length = 188.4

length = 62.8m

wastage = 20cm = 0.2 m

Total length required = 62.8 + 0.2 = 63m

i.e., $\ell \times b = \pi r \ell$]

Ans. 63 m.

Q6. The slant height and base diameter of a conical tomb are 25 m and 14 m respectively. Find the cost of white washing its curved surface at the rate of Rs. 210 per 100 m^2.

Sol. $\ell = 25$ m, r = 7 m

Curved surface $= \frac{22}{7} \times 7 \times 25 m^2 = 550\ m^2$

Cost of white washing $=$ Rs. $\frac{210}{100} \times 550 =$ Rs. 1155

Q7. A joker's cap is in the form of a right circular cone of base radius 7 cm and height 24 cm. Find the area of the sheet required to make 10 such caps.

Sol. r = 7 cm, h = 24 cm $\ell^2 = h^2 + r^2$

$= 576 + 49 = 625 \Rightarrow \ell = 25$ cm

Sheet required for one cap

$= \frac{22}{7} \times 7 \times 25\ cm^2 = 550\ cm^2$

Sheet required for 10 caps $= 10 \times 550\ cm^2 = 5500\ cm^2$

Q8. A bus stop is barricaded from the remaining part of the road, by using 50 hollow cones made of recycled cardboard. Each cone has a base diameter of 40 cm and height 1 m. If the outer side of each of the cones is to be painted and the cost of painting is Rs 12 per m^2, what will be the cost of painting all these cones? (Use $\pi = 3.14$ and take $\sqrt{1.04} = 1.02$)

Sol. Radius (r) $= \frac{40}{2}$ cm $= \frac{20}{100}$ m $= 0.2$ m

Height (h) = 1 m

Slant height (ℓ) $= \sqrt{r^2 + h^2} = \sqrt{(0.2)^2 + (1)^2}$

$= 1.02$ m.

Now, curved surface area $= \pi r \ell$

$\therefore$ Curved surface area of 1 cone

$= 3.14 \times 0.2 \times 1.02\ m^2$

$= \frac{314}{100} \times \frac{2}{10} \times \frac{102}{100} m^2$

Curved surface area of 50 cones

$$= 50 \times \left[\frac{314}{100} \times \frac{2}{10} \times \frac{102}{100}\right] m^2$$

$$= \frac{314 \times 102}{10 \times 100} m^2$$

Cost of painting per m^2 = Rs 12

$\therefore$ Cost of painting $\left[\frac{314 \times 102}{1000}\right] m^2$

$$= \frac{12 \times 314 \times 102}{1000} = \text{Rs } 384.34 \text{ (approx)}$$

Questions and Solutions | Exercise 11.2 - NCERT Books

Q1. Find the surface area of a sphere of radius :

(i) 10.5cm (ii) 5.6 cm (iii) 14 cm

Sol. (i) Surface area $= 4 \times \frac{22}{7} \times (10.5)^2 \text{ cm}^2$

$= 1386 \text{ cm}^2$

(ii) Surface area $= 4 \times \frac{22}{7} \times 5.6 \times 5.6 \text{ cm}^2$

$= 394.24 \text{ cm}^2$

(iii) Surface area $= 4 \times \frac{22}{7} \times 14 \times 14 \text{ cm}^2$

$= 2464 \text{ cm}^2$

Q2. Find the surface area of a sphere of diameter

(i) 14 cm. (ii) 21 cm (iii) 3.5 m

Sol. (i) Diameter = 14 cm

$\therefore$ Radius (r) $= \frac{14}{2}$ cm = 7 cm

$\therefore$ Surface area $= 4\pi r^2 = 4 \times \frac{22}{7} \times (7)^2 = 616 \text{ cm}^2$.

(ii) Diameter = 21 cm

$\therefore$ Radius (r) = $\frac{21}{2}$ cm = 10.5 cm

$\therefore$ Surface area = $4\pi r^2 = 4 \times \frac{22}{7} \times (10.5)^2 = 1385.4\ cm^2$.

(ii) Diameter = 3.5 m

$\therefore$ Radius (r) = $\frac{3.5}{2}$ cm = 1.75 cm

$\therefore$ Surface area = $4\pi r^2 = 4 \times \frac{22}{7} \times (1.75)^2 = 38.4\ m^2$

Q3. Find the total surface area of a hemisphere of radius 10 cm. (Use $\pi = 3.14$)

Sol. r = 10 cm.

$\therefore$ Total surface area of the hemisphere $= 3\pi r^2 = 3 \times 3.14 \times (10)^2 = 942\ cm^2$.

Q4. The radius of a spherical balloon increases from 7 cm to 14 cm as air is being pumped into it. Find the ratio of surface areas of the balloon in the two cases.

Sol. r_1 = 7 cm & r_2 =14cm and let S_1 and S_2 be the surface areas of respective spheres.

$$\frac{S_1}{S_2} = \frac{4\pi r_1^2}{4\pi r_2^2} = \frac{r_1^2}{r_2^2} = \left(\frac{r_1}{r_2}\right)^2$$

Ans. 1 : 4

Q5. A hemispherical bowl made of brass has inner diameter 10.5 cm. Find the cost of tin-plating it on the inside at the rate of Rs 16 per 100 cm^2.

Sol. Inner diameter = 10.5 cm, Radius = $\frac{105}{20}$ cm

Curved surface area of a hemisphere = $2\pi r^2$

$\therefore$ Inner curved surface area of hemispherical bowl

$$= 2 \times \frac{22}{7} \times \frac{105}{20} \times \frac{105}{20}\ cm^2 = \frac{17325}{100}\ cm^2$$

Cost of tinplating for 100 cm^2 = Rs 16

$\therefore$ Cost of tinplating for $\frac{17325}{100}$ cm^2

$= \text{Rs } \frac{16}{100} \times \frac{17325}{100}$

$= \text{Rs } \frac{277200}{100 \times 100} = \text{Rs } 27.72$

Q6. Find the radius of a sphere whose surface area is 154 cm^2

Sol. $4\pi r^2 = 154 \Rightarrow 4 \times \frac{22}{7} \times r^2 = 154$

$\Rightarrow r^2 = \frac{7 \times 7}{4} \Rightarrow r = \frac{7}{2}$ cm, i.e., r = 3.5 cm

Q7. The diameter of the moon is approximately one fourth of the diameter of the earth. Find the ratio of their surface areas.

Sol. Let d_1 and d_2 be the diameters of the moon and the earth respectively and S_1 and S_2 be their respective surface areas.

$d_1 = \frac{1}{4} d_2 \Rightarrow \frac{d_1}{d_2} = \frac{1}{4} \Rightarrow \frac{2r_1}{2r_2} = \frac{1}{4} \Rightarrow \frac{r_1}{r_2} = \frac{1}{4}$]

Ans. 1 : 16.

Q8. A hemispherical bowl is made of steel, 0.25 cm thick. The inner radius of the bowl is 5 cm. find the outer curved surface area of the bowl.

Sol. r = 5 cm, thickness of steel sheet = 0.25 cm

$\Rightarrow$ R = 5 cm + 0.25 cm = 5.25 cm

Outer curved surface area of the bowl = $2\pi R^2$

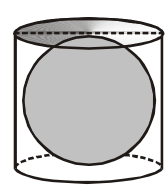
$= 2 \times \frac{22}{7} \times \frac{525}{100} \times \frac{525}{100}$ cm^2

= 173.25 cm^2

Q9. A right circular cylinder just encloses a sphere of radius r. Find

(i) Surface area of the sphere,

(ii) Curved surface area of the cylinder,



(iii) Ratio of the areas obtained in (i) and (ii).

Sol. Radius of cylinder = radius of sphere = r

Height of cylinder = 2 × radius of sphere = 2r]

Ans. (i) $4\pi r^2$ (ii) $4\pi r^2$ (iii)1 : 1.

Questions and Solutions | Exercise 11.3 - NCERT Books

Q1. Find the volume of the right circular cone with

(i) radius 6 cm, height 7 cm

(ii) radius 3.5 cm, height 12 cm

Sol. (i) $r = 6$ cm, $h = 7$ cm

$$\text{Volume} = \frac{1}{3}\times\frac{22}{7}\times(6)^2\times 7\text{ cm}^3 = 264\text{ cm}^3$$

(ii) $r = \frac{7}{2}$ cm, $h = 12$ cm

$$\text{Volume} = \frac{1}{3}\times\frac{22}{7}\times\left(\frac{7}{2}\right)^2\times 12\text{ cm}^3 = 154\text{ cm}^3$$

Q2. Find the capacity in litres of a conical vessel with

(i) radius 7 cm, slant height 25 cm.

(ii) height 12 cm, slant height 13 cm.

Sol. (i) $r = 7$ cm, $\ell = 25$ cm

$r^2 + h^2 = \ell^2$

$\Rightarrow (7)^2 + h^2 = (25)^2 \Rightarrow h^2 = (25)^2 - (7)^2$

$\Rightarrow h^2 = 625 - 49 \Rightarrow h^2 = 576$

$\Rightarrow h = \sqrt{576} \Rightarrow h = 24$ cm

$$\therefore \text{Capacity} = \frac{1}{3}\pi r^2 h = \frac{1}{3}\times\frac{22}{7}\times(7)^2\times 24$$

$= 1232 \text{ cm}^3 = 1.232\ \ell.$

(ii) $h = 12$ cm, $\ell = 13$ cm

$r^2 + h^2 = \ell^2$

$\Rightarrow r^2 + (12)^2 = (13)^2 \Rightarrow r^2 + 144 = 169$

$\Rightarrow r^2 = 169 - 144 \quad \Rightarrow r^2 = 25$

$\Rightarrow r = \sqrt{25} \quad \Rightarrow r = 5$ cm

$\therefore$ Capacity $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (5)^2 \times 12$

$= \frac{2200}{7} \text{ cm}^3 = \frac{2200}{7000}\ \ell = \frac{11}{35}\ell.$

Q3. The height of a cone is 15 cm. If its volume is 1570 cm^3, find the radius of the base. (Use $\pi = 3.14$)

Sol. $h = 15$ cm, volume $= 1570 \text{ cm}^3$

$\Rightarrow \frac{1}{3} \times 3.14 \times r^2 \times 15 = 1570$

$\Rightarrow r^2 = \frac{1570}{15.70} = 100$

$\Rightarrow r = 10$ cm

Q4. If the volume of a right circular cone of height 9 cm is $48\ \pi \text{ cm}^3$, find the diameter of its base.

Sol. $h = 9$ cm, volume $= 48\ \pi \text{cm}^3$

$\frac{1}{3}\pi r^2 \times h = 48\ \pi$

$\Rightarrow \frac{1}{3} r^2 \times 9 = 48$

$\Rightarrow r^2 = 16 \Rightarrow r = 4$ cm

Q5. A conical pit of top diameter 3.5 m is 12 m deep. What is its capacity in kilolitres?

Sol. For conical pit

Diameter = 3.5 m

$\therefore$ Radius (r) = $\frac{3.5}{2}$ m = 1.75 m

Depth (h) = 12 m

$\therefore$ Capacity of the conical pit

$= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (1.75)^2 \times 12 \text{ m}^3$

$= 38.5 \text{ m}^3 = 38.5 \times 1000\ \ell = 38.5$ kl.

Q6. The volume of a right circular cone is 9856 cm³. If the diameter of the base is 28 cm, find (i) height of the cone (ii) slant height of the cone (iii) curved surface area of the cone

Sol. (i) Volume = 9856 cm³, r = 14 cm

$\Rightarrow \frac{1}{3} \times \frac{22}{7} \times (14)^2 \times 4 = 9856$

$\Rightarrow h = \frac{9856 \times 3}{22 \times 28}$ cm $\Rightarrow h = 48$ cm

(ii) $\ell^2 = h^2 + r^2 = (48)^2 + (14)^2 = 2500$

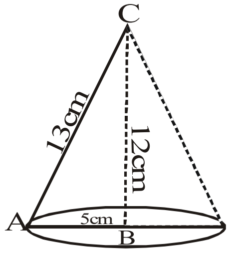
$\Rightarrow \ell = 50$ cm

(iii) Curved surface area $= \frac{22}{7} \times 14 \times 50 \text{ cm}^2$

$= 2200 \text{ cm}^2$

Q7. A right triangle ABC with sides 5 cm, 12 cm and 13 cm is revolved about the side 12 cm. Find the volume of the solid so obtained.

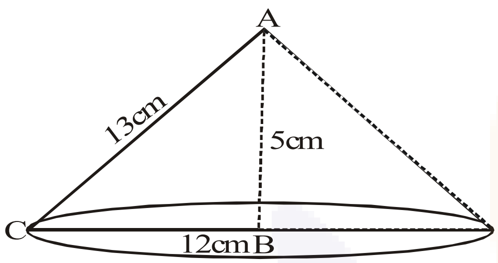
Sol.



Radius, r = 5 cm ; height, h = 12 cm & slant height, ℓ = 13 cm] $= \frac{1}{2}\pi 5^2 \times 12 = 100\pi$

Q8. If the triangle ABC in the question 7 above is revolved about the side 5 cm, then find the volume of the solid so obtained. Find also the ratio of the volumes of the two solids obtained in Question 7 and 8.

Sol.



Radius, r = 12 cm ; height, h = 5 cm & slant height, ℓ = 13 cm]

Vol. $= \frac{1}{3}\pi\, 12^2 \times 5 = 240\pi$

Ans. 240 π cm^3 ; 5 : 12.

Q9. A heap of wheat is in the form of a cone whose diameter is 1 0.5 m and height is 3 m. Find its volume. The heap is to be covered by canvas to protect it from rain. Find the area of the canvas required.

Sol. Diameter = 10.5 m

$\therefore$ Base Radius (r) = $\frac{10.5}{2}$ m = $\frac{105}{20}$ m

Height (h) = 3m

$\therefore$ Volume of the heap = $\frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times \left(\frac{105}{20}\right)^2 \times 3$$

$= 86.625\ m^3$

$\therefore$ Area of the canvas = $\pi r \ell$

where, $\ell = \sqrt{r^2 + h^2}$

$$= \sqrt{\left(\frac{10.5}{2}\right)^2 + (3)^2} = \sqrt{\frac{110.25}{4} + 9}$$

$$= \sqrt{\frac{146.25}{4}} = 6.046 \text{ m (approx)}$$

Now, $\pi r\ell = \frac{22}{7} \times \frac{10.5}{2} \times 6.05 \text{ m}^2$

$= 11 \times 1.5 \times 6.05 \text{ m}^2$

$= 99.825 \text{ m}^2$

Thus, the required area of the canvas is 99.825 m^2

Questions and Solutions | Exercise 11.4 - NCERT Books

Q1. Find the volume of a sphere whose radius is
(i) 7 cm (ii) 0.63 m

Sol. (i) r = 7 cm

Volume $= \frac{4}{3} \times \frac{22}{7} \times (7)^3 \text{ cm}^3 = 1437\frac{1}{3} \text{ cm}^3$

(ii) r = 0.63 m

Volume $= \frac{4}{3} \times \frac{22}{7} \times (0.63)^3 \text{ m}^3 = 1.047816 \text{ m}^3 = 1.05 \text{ m}^3$ (approx)

Q2. (i) Find the amount of water displaced by a solid spherical ball of diameter 28 cm.

Sol. Diameter = 28 cm

$\therefore$ Radius (r) $= \frac{28}{2}$ cm = 14 cm

$\therefore$ Amount of water displaced

$= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times (14)^3 = \frac{34496}{3} \text{ cm}^3$

$= 11498\frac{2}{3} \text{ cm}^3$.

Q3. The diameter of a metallic ball is 4.2 cm. What is the mass of the ball, if the density of the metal is 8.9 g per cm^3 ?

Sol. Density $= \frac{\text{mass}}{\text{volume}}$

Volume of metallic ball = $\frac{4}{3}\pi r^3$

$= \frac{4}{3}\times\frac{22}{7}\times(4.2)^3 = 310.46 cm^3$

mass = density × Volume

$8.9 g/cm^3 \times 310.46 cm^3$

$= 2763.12 gm = 2.7 kg$

Q4. The diameter of the moon is approximately one-fourth the diameter of the earth. What fraction of the volume of the earth is the volume of the moon?

Sol. Let d_1 and d_2 be the diameters of the moon and the earth respectively. Then, $d_1 = \frac{1}{4}d_2$

$\Rightarrow \frac{r_1}{r_2} = \frac{1}{4}$; $\frac{\text{Volume of moon}}{\text{Volume of earth}} = \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \left(\frac{r_1}{r_2}\right)^3$] Ans. $\frac{1}{64}$

Q5. How many litres of milk can a hemispherical bowl of diameter 10.5 cm hold?

Sol. $r = \frac{10.5}{2} = \frac{21}{4}$ cm

Capacity of the bowl $= \frac{2}{3}\pi r^3$

$= \frac{2}{3}\times\frac{22}{7}\times\frac{21}{4}\times\frac{21}{4}\times\frac{21}{4} cm^3 = \frac{4851}{16} cm^3$

$= 303.2$ cm^3 (approx.)

$= \frac{303.2}{1000}$ lit. $= 0.303$ lit. (approx.)

Q6. A hemispherical tank is made up of an iron sheet 1 cm thick. If the inner radius is 1 m, then find the volume of the iron used to make the tank.

Sol. Inner radius (r) = 1 m

Thickness of iron sheet = 1 cm = 0.01 m

∴ Outer radius (R) = Inner radius (r) + Thickness of iron sheet = 1 m + 0.01 m = 1.01 m

∴ Volume of the iron used to make the tank

$= \frac{2}{3}\pi(R^3 - r^3) = \frac{2}{3}\times\frac{22}{7}\times\{(1.01)^3 - 1^3\}$

$= 0.06348\ m^3$ (Approx).

Q7. Find the volume of a sphere whose surface area is 154 cm^2.

Sol. $4\pi r^2 = 154 \Rightarrow 4 \times \frac{22}{7} \times r^2 = 154$

$\Rightarrow\ r^2 = \frac{49}{4} \Rightarrow r = \frac{7}{2}\ cm$

Volume of the sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2} cm^3 = \frac{539}{3}\ cm^3$

$= 179\frac{2}{3}\ cm^3$

Q8. A dome of a building is in the form of a hemisphere. From inside, it was white washed at the cost of Rs. 498.96. if the cost of white washing is Rs. 2.00 per square metre, find the
(i) Inside surface area of the dome,
(ii) Volume of the air inside the dome.

Sol. (i) Total cost of white washing = Rs 498.96
Cost of 1 m^2 of white washing = Rs 2
$\therefore$ Inside surface Area = 498.96 = 249.48 m^2
$\therefore$ Inside surface area = $2\pi r^2$
$\Rightarrow 2\pi r^2 = 249.48$

$\Rightarrow 2 \times \frac{22}{7} \times r^2 = \frac{24948}{100}$; $r^2 = \frac{3969}{100}$

$\Rightarrow r = \left(\frac{63}{10}\right)^2 m \Rightarrow r = \frac{63}{10} = 6.3\ m$

(ii) The volume of air in the dome

Volume $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times (6.3)^3\ m^3$

$= \frac{523908}{1000} m^3 = 523.9\ m^3$ (approx)

Q9. Twenty seven solid iron spheres, each of radius r and surface area S are melted to form a sphere with surface area S'. Find the (i) radius r' of the new sphere, (ii) ratio of S and S'.

Sol. Volume of 27 solid iron sphere each of radius r = volume of new sphere of radius R.

$$27 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$$

$\Rightarrow$ R = 3r

S = $4\pi r^2$

S' = $4\pi(3r)^2]$

Ans. 3r ; 1 : 9

Q10. A capsule of medicine is in the shape of a sphere of diameter 3.5 mm. How much medicine (in mm^3) is needed to fill this capsule?

Sol. $r = \frac{3.5}{2} mm$

Capacity of the capsule $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2} \times \frac{3.5}{2} mm^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} \times \frac{7}{4} mm^3 = \frac{11}{24} \times 49\ mm^3$$

$$= \frac{539}{24} mm^3 = 22.346\ mm^3$$