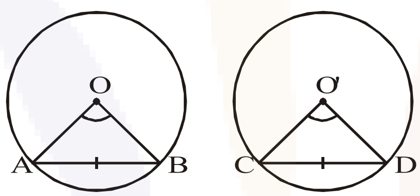
CLASS IX: MATHS
Chapter 9: Circles

Questions and Solutions | Exercise 9.1 - NCERT Books

Q1. Recall that two circles are congruent if they have the same radii. Prove that equal chords of congruent circles subtend equal angles at their centres.

Sol. **Given :** Two congruent circles C(O, r) and C(O', r) which have chords AB and CD respectively such that AB = CD.



To prove : ∠AOB = ∠CO'D

Proof : From ΔAOB and ΔCO'D, we have

AB = CD [Given]

OA = O'C [Each equal to r]

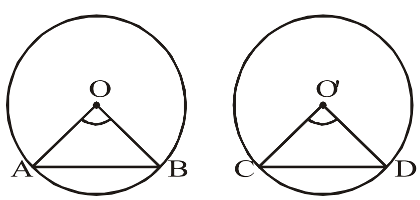
OB = O'D [Each equal to r]

∴ AOB ≅ ΔCO'D [By SSS-congruence]

⇒ ∠AOB = ∠CO'D [C.P.C.T.]

Q2. Prove that if chords of congruent circles subtend equal angles at their centres, then the chords are equal.

Sol. **Given :** Two congruent circle C(O, r) and C(O', r) which have chords AB and CD respectively, such that ∠AOB = ∠CO'D



To prove : AB = CD

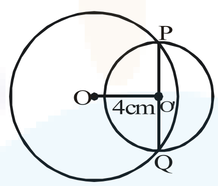
Proof : In Δ AOB and Δ CO'D, we have :

$OA = O'C$	[each equal to r]
$OB = O'D$	[each equal to r]
$\angle AOB = \angle CO'D$	[given]
$\therefore$ $\Delta AOB \cong \Delta CO'D$	[by SAS - criterion]
Hence, $AB = CD$	[C.P.C.T.]

Questions and Solutions | Exercise 9.2 - NCERT Books

Q1. Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of common chord.

Sol. We know that if two circles intersect each other at two points, then the line joining their centres is the perpendicular bisector of their common chord.



$\therefore$ Length of the common chord

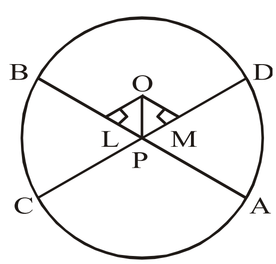
$\Rightarrow$ $PQ = 2O'P = 2 \times 3 = 6$ cm

Q2. If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.

Sol. O is the centre of the circle. Chords AB and CD of the circle are equal. P is the point of intersection of AB and CD. Join OP, draw $OL \perp AB$ and $OM \perp OD$.

Here, we find $OL = OM$ $(\because AB = CD)$ ...(1)

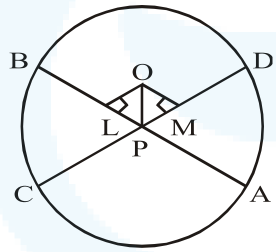
In ΔOLP and ΔOMP,



OL = OM (By 1)
OP = OP (Common hypotenuse)
∠OLP = ∠OMP (Each = 90°)
Then we have ΔOLP ≅ ΔOMP (RHS congruence)
By CPCT, or PL = PM ...(2)
Now, AL = BL = ½ AB; CM = DM = ½ CD
⇒ AL = CM (∵ AB = CD) ...(3)
and BL = DM ...(4)
Subtracting (1) from (3),
AL – PL = CM – PM ⇒ AP = CP
Adding (2) from (4),
PL + BL = PM + DM ⇒ PB = PD

Q3. If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chords.

Sol. O is the centre of the circle. Chords AB and CD of the circle are equal. P is the point of intersection of AB and CD. Join OP, draw OL ⊥ AB and OM ⊥ OD.



Here, we find OL = OM (∵ AB = CD) ...(1)
In ΔOLP and ΔOMP,
OL = OM (By 1)
OP = OP (Common hypotenuse)
∠OLP = ∠OMP (Each = 90°)
Then we have ΔOLP ≅ ΔOMP (RHS congruence)
By CPCT, or PL = PM ...(2)
Now, AL = BL = ½ AB;
CM = DM = ½ CD

$\Rightarrow$ AL = CM ($\because$ AB = CD) ...(3)

and BL = DM ...(4)

Subtracting (1) from (3),

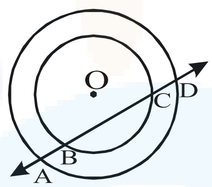
AL – PL = CM – PM

$\Rightarrow$ AP = CP

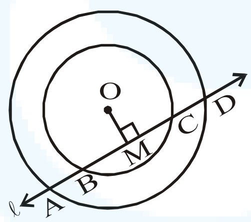
Adding (2) from (4),

PL + BL = PM + DM $\Rightarrow$ PB = PD

Q4. If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that AB = CD (see fig).



Sol. **Given :** Two circles with the common centre O. A line "ℓ" intersects the outer circle at A and D and the inner circle at B and C.



To prove : AB = CD

Construction : Draw OM $\perp \ell$.

Proof : OM $\perp \ell$ [Construction]

For the outer circle,

$\therefore$ AM = MD [Perpendicular from the centre bisects the chord] .... (1)

For the inner circle,

OM $\perp \ell$ [Construction]

$\therefore$ BM = MC [Perpendicular from the centre to the chord bisects the chord] ...(2)

Subtracting (2) from (1), we have

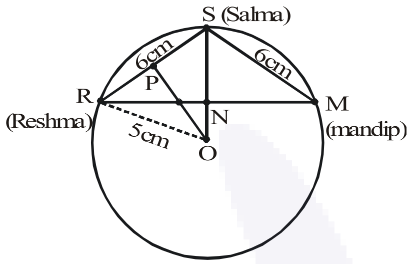
$\Rightarrow$ AM – BM = MD – MC

$\Rightarrow$ AB = CD

Q5. Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5 m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6 m each, what is the distance between Reshma and Mandip?

Sol. We draw SN ⊥ RS.

Now, SN bisects RM and also SN (produced) passes through the centre O.



Put RN = x

The ar(ΔORS) = $\frac{1}{2}$ × OS × RN

$= \frac{1}{2} \times 5 \times x$ (∵ OS = OR = 5 m)

i.e., ar(ΔORS) = $\frac{5}{2}x$ ...(1)

Now, draw OP ⊥ RS, P is mid-point of RS.

$\Rightarrow$ PR = PS = 3 m $\Rightarrow OP^2 = (5)^2 - (3)^2 = 16 \Rightarrow OP = 4$ m

Here, ar(ΔORS) = $\frac{1}{2}$ × RS × OP = $\frac{1}{2} \times 6 \times 4$

i.e., ar(ΔORS) = 12 m^2

From (1) and (2),

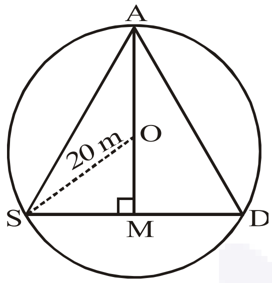
$\frac{5}{2}x = 12 \Rightarrow x = 4.8 \text{ m} \Rightarrow RM = 2x = 2 \times 4.8 \text{ m} \Rightarrow RM = 9.6 \text{ m}$

Thus, distance between Reshma and Mandip is 9.6 m.

Q6. A circular park of radius 20 m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.

Sol. Let Ankur, Syed and David are sitting at A, S and D respectively such that AS = SD = AD i.e., ΔASD is an equilateral triangle.

Let the length of each side of the equilateral triangle is 2x metres.



Draw $AM \perp SD$.

Since, ΔASD is an equilateral triangle,

$\therefore$ AM passes through O.

$\Rightarrow$ $SM = \frac{1}{2} SD = \frac{1}{2}(2x) = x$

Now, in ΔASM, we have $AM^2 + SM^2 = AS^2$

$\Rightarrow$ $AM^2 = AS^2 - SM^2 = (2x)^2 - x^2 = 4x^2 - x^2 = 3x^2$

$\Rightarrow$ $AM = \sqrt{3}x$.

Now, $OM = AM - OA = (\sqrt{3}x - 20)$ m

$\Rightarrow$ (OS = OA = 20 cm)

$\Rightarrow$ $(20)^2 = x^2 + (\sqrt{3}x - 20)^2$

$\Rightarrow$ $400 = x^2 + 3x^2 - 40\sqrt{3}x + 400$

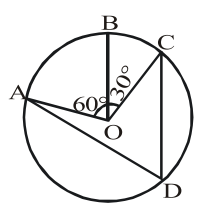
$\Rightarrow$ $4x^2 = 40\sqrt{3}x \Rightarrow 4x = 40\sqrt{3} \Rightarrow x = 10\sqrt{3}$ m

Now, $SD = 2x = 2 \times 10\sqrt{3}$ m $= 20\sqrt{3}$ m

Thus, the length of the string of each phone $= 20\sqrt{3}$ m

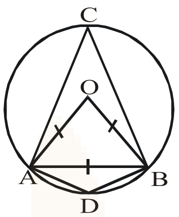
Questions and Solutions | Exercise 9.3 - NCERT Books

Q1. In Fig. A, B and C are three points on a circle with centre O such that $\angle BOC = 30°$ and $\angle AOB = 60°$. If D is a point on the circle other than the arc $\angle ABC$, find $\angle ADC$.



Sol. $\angle ADC = \frac{1}{2}\angle AOC = \frac{1}{2}(60° + 30°) = 45°$

Q2. A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.



Sol. $\because$ OA = OB = AB [Given]

$\therefore$ ΔOAB is equilateral

$\therefore$ $\angle AOB = 60°$ $\angle ACB = \frac{1}{2}\angle AOB$

[The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle]

$= \frac{1}{2} \times 60 = 30°$

$\because$ ADBC is a cyclic quadrilateral.

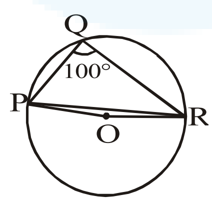
$\therefore$ $\angle ADB + \angle ACB = 180°$

[The sum of either pair of opposite angles of a cyclic quadrilateral is 180°]

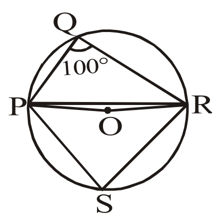
$\Rightarrow$ $\angle ADB + 30° = 180° \Rightarrow \angle ADB = 180° – 30°$

$\Rightarrow$ $\angle ADB = 50°$

Q3. In figure, $\angle PQR = 100°$, where P, Q and R are points on a circle with centre O. Find $\angle OPR$.



Sol. Take a point S in the major arc. Join PS and RS.



$\because$ PQRS is a cyclic quadrilateral.

$\therefore$ $\angle PQR + \angle PSR = 180°$

[The sum of either pair of opposite angles of a cyclic quadrilateral is 180°]

$\Rightarrow$ $100° + \angle PSR = 180° \Rightarrow \angle PSR = 180° - 100°$

$\Rightarrow$ $\angle PSR = 80°$ ...(i)

Now $\angle PSR = 2\angle PSR$

[The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle]

$= 2 \times 80° = 160°$ ... (2) [Using (i)]

In ΔOPR,

$\because$ OP = OR [radii of a circle]

$\therefore$ $\angle OPR = \angle ORP$ ... (3) [Angles opposite to equal sides of a triangle is 180°]

In ΔOPR,

$\angle OPR + \angle ORP + \angle POR = 180°$ [Sum of all the angles of a triangle is 180°]

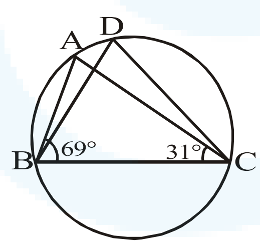
$\Rightarrow$ $\angle OPR + \angle OPR + 160° = 180°$ [Using (2) and (1)]

$\Rightarrow$ $2\angle OPR + 160° = 180°$

$\Rightarrow$ $2\angle OPR = 180° - 160° = 20°$

$\Rightarrow$ $\angle OPR = 10°$

Q4. In fig. $\angle ABC = 69°$, $\angle ACB = 31°$, find $\angle BDC$.



Sol. $\angle ABC + \angle ACB + \angle BAC = 180°$ (By angle sum property)

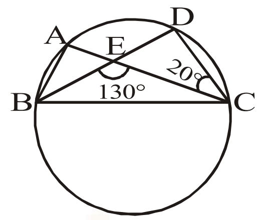
$\Rightarrow$ $69° + 31° + \angle BAC = 180°$

$\Rightarrow$ $\angle BAC = 180° - 100° = 80°$

Since, angles in the same segment are equal

$\angle BDC = \angle BAC$, $\angle BDC = 80°$.

Q5. In figure, A, B, C and D are four points on a circle. AC and BD intersect at a point E such that ∠BEC = 130° and ∠ECD = 20°. Find ∠BAC.



Sol. ∠CED + ∠BEC = 180° [Linear Pair]

⇒ ∠CED + 130° = 180°

⇒ ∠CED + 180° – 130° = 50° ...(i)

∠ECD = 20° ...(ii)

In ΔCED, ∠CED + ∠ECD + ∠CDE = 180°

[Sum of all the angles of a triangle is 180°]

⇒ 50° + 20° + ∠CDE = 180° [Using (i) and (ii)]

⇒ 70° + ∠CDE = 180°

⇒ ∠CDE = 180° – 70°

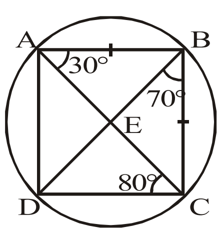
⇒ ∠CDE = 110° ...(iii)

Now ∠BAC = ∠CDE = 110°

[Angle in the same segment of a circle are equal]

Q6. ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. If ∠DBC = 70°, ∠BAC = is 30°, find ∠BCD. Further, if AB = BC, find ∠ECD.

Sol.



Since angles in the same segment of a circle are equal

$\therefore$ $\angle BAC = BDC$

$\Rightarrow$ $BDC = 30°$

Also $\angle DBC = 70°$ (Given)

$\therefore$ In $\angle BCD$, we have

$\Rightarrow$ $\angle BCD + \angle DBC + \angle CDB = 180°$ [sum of angles of a triangle is 180°]

$\Rightarrow$ $\angle BCD + 70° + 30° = 180°$

$\Rightarrow$ $\angle BCD = 80°$

Now, in ΔABC, $AB = BC$ (given)

$\therefore$ $\angle BCA = \angle BAC$

(angles opp. to equal sides of a triangle are equal)

$\Rightarrow$ $\angle BCA = 30°$ [$\angle BAC = 30°$]

Now, $\angle BCA + \angle ECD = \angle BCD$

$\Rightarrow$ $30° + \angle ECD = 80°$

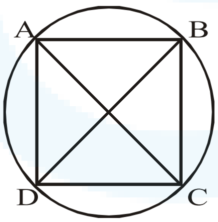
$\Rightarrow$ $\angle ECD = 50°$

Q7. If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.

Sol. Since, AC and BD are diameters.

$\Rightarrow$ $AC = BD$ [all diameters of a circle are equal]

Also, $\angle BAD = 90°$



[angle formed in a semicircle is 90°]

Similarly, $\angle ABC = 90°$, $\angle BCD = 90°$

and $\angle CDA = 90°$.

Now, in right ΔABC and ΔBAD, we have

$AC = BD$ (from (1))

$AB = BA$ (common)

$\angle ABC = \angle BAD$ (each equal to 90°)

$\therefore$ $\Delta ABC \cong \Delta BAD$ (By RHS congruence)

$\Rightarrow$ $BC = AD$ (CPCT)

Similarly, $AB = DC$

Thus, ABCD is a rectangle.

Q8. If the non-parallel sides of a trapezium are equal, prove that it is cyclic.

Sol. **Given :** ABCD is a trapezium whose two non-parallel sides AD and BC are equal.

To Prove : Trapezium ABCD is a cyclic.

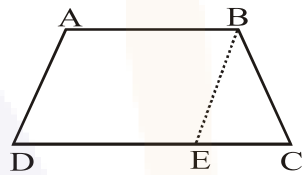
Construction : Draw BE||AD

Proof :

∵ AB||DE [Given]

AD||BE [By construction]

∴ Quadrilateral ABCD is a parallelogram.



∴ ∠BAD = ∠BED ...(i) [Opp. ∠s of a || gm]

and AD = BE ...(ii) [Opp. sides of a || gm]

But AD = BC ...(iii) [Given]

From (ii) and (iii)

BE = BC

∴ ∠BEC = ∠BCE ...(iv) [Angle opposite to equal sides]

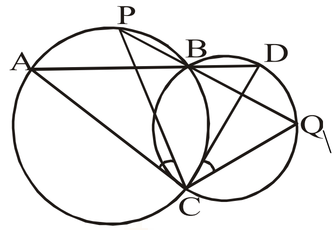
∠BEC + ∠BED = 180° [Linear pair]

⇒ ∠BCE + ∠BAD = 180° [From (iv) and (i)]

⇒ Trapezium ABCD is cyclic.

[∵ If a pair of opposite angles of a quadrilateral is 180°, then the quadrilateral is cyclic]

Q9. Two circles intersect at two points B and C. Through B, two line segment ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively. Prove that $\angle ACP = \angle QCD$.



Sol. Given : Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively.

To Prove : $\angle ACP = \angle QCD$

Proof : $\angle ACP = \angle ABP$ ...(i) [Angles in the same segment of a circle are equal]

$\angle QCD = \angle QBD$ ...(ii) [Angles in the same segment of a circle are equal]

$\angle ABP = \angle QBD$ ...(iii) [Vertically Opposite Angles]

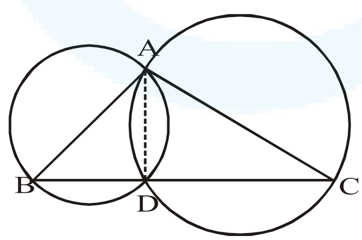
From (i), (ii) and (iii),

$\angle ACP = \angle QCD$.

Q10. If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side.

Sol. We have ΔABC, and two circles described with diameter as AB and AC respectively. They intersect at a point D, other than A.

Let us join A and D.



AB is a diameter

$\therefore$ $\angle ADB$ is an angle formed in a semicircle.

$\Rightarrow$ $\angle ADB = 90°$ ......(1)

Similarly, $\angle ADC = 90°$ ......(2)

adding (1) and (2) $\angle ADB + \angle ADC = 180°$

i.e., B, D and C are collinear points BC is a straight line. Thus, D lies on BC.

Q11. ABC and ADC are two right triangles with common hypotenuse AC. Prove that $\angle CAD = \angle CBD$.

Sol. AC is a hypotenuse

$\angle ADC = 90° = \angle ABC$

$\therefore$ Both the triangles are in the same semicircle.

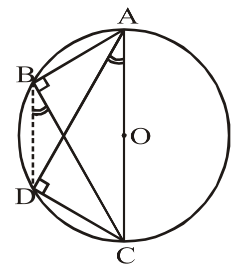
$\Rightarrow$ A, B, C and D are concyclic.

Join BD

DC is chord

$\therefore$ $\angle CAD$ and $\angle CBD$ are formed on the same segment

$\therefore$ $\angle CAD = \angle CBD$



Q12. Prove that a cyclic parallelogram is a rectangle.

Sol. We have a cyclic parallelogram ABCD.

$\therefore$ Sum of its opposite angles is 180°

$\therefore$ $\angle A + \angle C = 180°$ ......(1)

But $\angle A = \angle C$ ......(2)

From (1) and (2), we have

$\angle A = \angle C = 90°$

Similarly, $\angle B = \angle D = 90°$

$\Rightarrow$ Each angle of the parallelogram ABCD is 90°

Thus, ABCD is a rectangle.