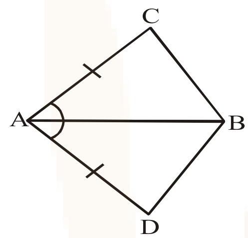
CLASS IX: MATHS
Chapter 7: Triangles

Questions and Solutions | Exercise 7.1 - NCERT Books

Q1. In quadrilateral ACBD, AC = AD and AB bisects ∠A. Show that ΔABC ≅ ΔABD. What can you say about BC and BD?



Sol. Given : In quadrilateral ACBD, AC = AD and AB bisect ∠A.

To prove : ΔABC ≅ ΔABD

Proof : In ΔABC and ΔABD

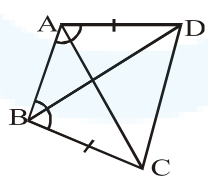
AC = AD	(Given)
AB = AB	(Common)
∠CAB = ∠DAB	(AB bisect ∠A)
∴ ΔABC ≅ ΔABD	(by SAS criteria)
BC = BD	(by CPCT)

Q2. ABCD is a quadrilateral in which AD = BC and ∠DAB = ∠CBA. Prove that

(i) ΔABD ≅ ΔBAC

(ii) BD = AC

(iii) ∠ABD = ∠BAC.



Sol. In ΔABD and ΔBAC,

AD = BC	(Given)
∠DAB = ∠CBA	(Given)
AB = AB	(Common side)

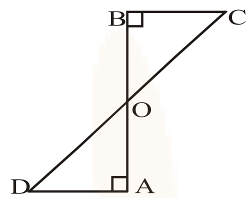
∴ By SAS congruence rule, we have

ΔABD ≅ ΔBAC

Also, by CPCT, we have

BD = AC and ∠ABD = ∠BAC

Q3. AD and BC are equal perpendiculars to a line segment AB. Show that CD bisects AB.



Sol. Given : AD and BC are equal perpendiculars to line AB.

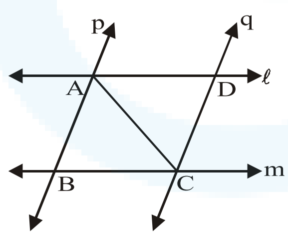
To prove : CD bisect AB

Proof : In ΔOAD and ΔOBC

AD = BC	(Given)
$\angle$OAD = $\angle$OBC	(Each 90°)
$\angle$AOD = $\angle$BOC	(Vertically opposite angles)
ΔOAD $\cong$ ΔOBC	(AAS rule)
OA = OB	(by CPCT)

$\therefore$ CD bisect AB.

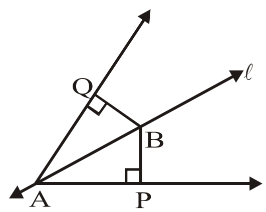
Q4. ℓ and m are two parallel lines intersected by another pair of parallel lines p and q. Show that ΔABC $\cong$ ΔCDA.



Sol. In ΔABC and ΔCDA

$\angle$CAB = $\angle$ACD	(Pair of alternate angle)
$\angle$BCA = $\angle$DAC	(Pair of alternate angle)
AC = AC	(Common side)
$\therefore$ ΔABC $\cong$ ΔCDA	(ASA criteria)

Q5. Line ℓ is the bisector of an angle $\angle$A and B is any point on ℓ. BP and BQ are perpendiculars from B to the arms of $\angle$A. Show that :



(i) $\Delta APB \cong \Delta AQB$

(ii) $BP = BQ$ or B is equidistant from the arms of $\angle A$.

Sol. Given : line ℓ is bisector of angle A and B is any point on ℓ. BP and BQ are perpendicular from B to arms of $\angle A$.

To prove : (i) $\Delta APB \cong \Delta AQB$ (ii) $BP = BQ$.

Proof :

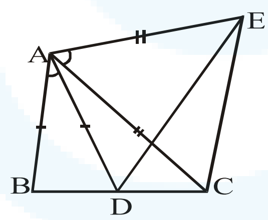
(i) In ΔAPB and ΔAQB

$\angle BAP = \angle BAQ$	(ℓ is bisector)
$AB = AB$	(common)
$\angle BPA = \angle BQA$	(Each 90°)
$\therefore \Delta APB \cong \Delta AQB$	(AAS rule)

(ii) $\Delta APB \cong \Delta AQB$

$BP = BQ$	(By CPCT)

Q6. In figure, $AC = AE$, $AB = AD$ and $\angle BAD = \angle EAC$. Show that $BC = DE$.



Sol. Given : $AC = AE$

$AB = AD$,

$\angle BAD = \angle EAC$

To prove : $BC = DE$

Proof : In ΔABC and ΔADE

$AB = AD$	(Given)
$AC = AE$	(Given)

$\angle BAD = \angle EAC$

Add $\angle DAC$ to both

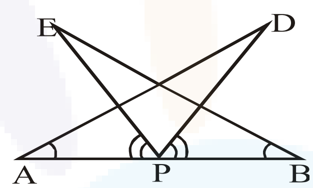
$\Rightarrow \angle BAD + \angle DAC = \angle DAC + \angle EAC$

$\angle BAC = \angle DAE$

$\Delta\ ABC \cong \Delta\ ADE$ (SAS rule)

$BC = DE$ (By CPCT)

Q7. AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that
$\angle BAD = \angle ABE$ and $\angle EPA = \angle DPB$. Show that
(i) $\Delta DAP \cong \Delta EBP$ (ii) $AD = BE$



Sol. $\angle EPA = \angle DPB$ (Given)

$\Rightarrow \angle EPA + \angle DPE = \angle DPB + \angle DPE$

$\Rightarrow \angle APD = \angle BPE$ ...(1)

Now, in ΔDAP and ΔEBP, we have

$AP = PB$ ($\because$ P is mid point of AB)

$\angle PAD = \angle PBE$

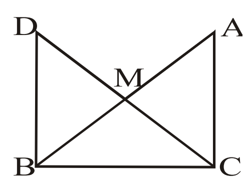
$\left\{\begin{array}{l}\because \angle PAD = \angle BAD,\ \angle PBE = \angle ABE \\ \text{and we are given that } \angle BAD = \angle ABE\end{array}\right\}$

Also, $\angle APD = \angle BPE$ (By 1)

$\therefore\ \Delta DAP \cong \Delta EBP$ (By ASA congruence)

$\Rightarrow AD = BE$ (By CPCT)

Q8. In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B. Show that :



(i) $\Delta AMC \cong \Delta BMD$

(ii) $\angle DBC$ is a right angle.

(iii) $\Delta DBC \cong \Delta ACB$

(iv) $CM = \frac{1}{2} AB$

Sol. (i) In $\Delta AMC \cong \Delta BMD$,

$AM = BM$	($\because$ M is mid point of AB)
$\angle AMC = \angle BMD$	(Vertically opposite angles)
$CM = DM$	(Given)
$\therefore \Delta AMC \cong \Delta BMD$	(By SAS congruence)

(ii) $\angle AMC = \angle BMD$,

$\Rightarrow \angle ACM = \angle BDM$ (By CPCT)

$\Rightarrow CA \parallel BD$

$\Rightarrow \angle BCA + \angle DBC = 180°$

$\Rightarrow \angle DBC = 90°$ ($\because \angle BCA = 90°$)

(iii) In ΔDBC and ΔACB,

$DB = AC$	($\because \Delta BMD \cong \Delta AMC$)
$\angle DBC = \angle ACB$	(Each = 90°)
$BC = BC$	(Common side)

$\therefore \Delta DBC \cong \Delta ACB$ (By SAS congruence)

(iv) In $\Delta DBC \cong \Delta ACB \Rightarrow CD = AB$ ...(1)

Also, $\Delta AMC \cong \Delta BMD$

$\Rightarrow CM = DM$

$\Rightarrow CM = DM = \frac{1}{2} CD$

$\Rightarrow CD = 2\ CM$ ...(2)

From (1) and (2),

$2\ CM = AB$

$\Rightarrow CM = \frac{1}{2} AB$