Class XI : Physics
Chapter 5 : Systems of Particles And Rotational Motion

Questions and Solutions | Exercises - NCERT Books

Question 1:

Give the location of the centre of mass of a (i) sphere, (ii) cylinder, (iii) ring, and (iv) cube, each of uniform mass density. Does the centre of mass of a body necessarily lie inside the body?

Answer

Geometric centre; No

The centre of mass (C.M.) is a point where the mass of a body is supposed to be concentrated. For the given geometric shapes having a uniform mass density, the C.M. lies at their respective geometric centres.

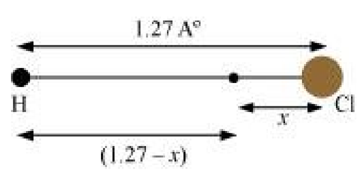
The centre of mass of a body need not necessarily lie within it. For example, the C.M. of bodies such as a ring, a hollow sphere, etc., lies outside the body.

Question 2:

In the HCl molecule, the separation between the nuclei of the two atoms is about 1.27 Å (1 Å = 10^{-10} m). Find the approximate location of the CM of the molecule, given that a chlorine atom is about 35.5 times as massive as a hydrogen atom and nearly all the mass of an atom is concentrated in its nucleus.

Answer

The given situation can be shown as:



Distance between H and Cl atoms = 1.27Å

Mass of H atom = m

Mass of Cl atom = $35.5m$

Let the centre of mass of the system lie at a distance x from the Cl atom.

Distance of the centre of mass from the H atom = $(1.27 - x)$

Let us assume that the centre of mass of the given molecule lies at the origin. Therefore, we can have:

$$\frac{m(1.27 - x) + 35.5mx}{m + 35.5m} = 0$$

$$m(1.27 - x) + 35.5mx = 0$$
$$1.27 - x = -35.5x$$

$$\therefore x = \frac{-1.27}{(35.5 - 1)} = -0.037\,\text{Å}$$

Here, the negative sign indicates that the centre of mass lies at the left of the molecule. Hence, the centre of mass of the HCl molecule lies 0.037Å from the Cl atom.

Question 3:

A child sits stationary at one end of a long trolley moving uniformly with a speed V on a smooth horizontal floor. If the child gets up and runs about on the trolley in any manner, what is the speed of the CM of the (trolley + child) system?

Answer

No change

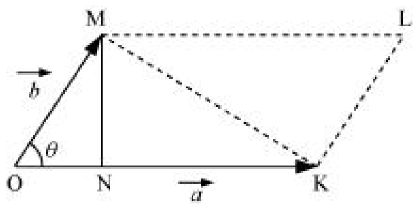
The child is running arbitrarily on a trolley moving with velocity v. However, the running of the child will produce no effect on the velocity of the centre of mass of the trolley. This is because the force due to the boy's motion is purely internal. Internal forces produce no effect on the motion of the bodies on which they act. Since no external force is involved in the boy–trolley system, the boy's motion will produce no change in the velocity of the centre of mass of the trolley.

Question 4:

Show that the area of the triangle contained between the vectors **a** and **b** is one half of the magnitude of **a** × **b**.

Answer

Consider two vectors $\overline{OK} = |\vec{a}|$ and $\overline{OM} = |\vec{b}|$, inclined at an angle θ, as shown in the following figure.



In ΔOMN, we can write the relation:

$$\sin\theta = \frac{MN}{OM} = \frac{MN}{|\vec{b}|}$$

$$MN = |\vec{b}|\sin\theta$$

$$|\vec{a} \times \vec{a}| = |\vec{a}|\,|\vec{b}|\sin\theta$$

$$= OK \cdot MN \times \frac{2}{2}$$

= 2 × Area of ΔOMK

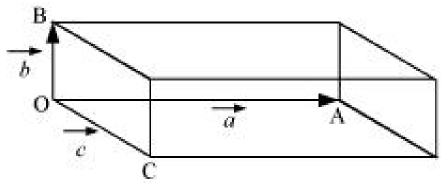
∴Area of ΔOMK $= \frac{1}{2}|\vec{a} \times \vec{b}|$

Question 5:

Show that **a**. (**b** × **c**) is equal in magnitude to the volume of the parallelepiped formed on the three vectors, **a**, **b** and **c**.

Answer

A parallelepiped with origin O and sides *a*, *b*, and *c* is shown in the following figure.



Volume of the given parallelepiped = *abc*

$\overrightarrow{OC} = \vec{a}$

$\overrightarrow{OB} = \vec{b}$

$\overrightarrow{OC} = \vec{c}$

Let $\hat{\mathbf{n}}$ be a unit vector perpendicular to both *b* and *c*. Hence, $\hat{\mathbf{n}}$ and *a* have the same direction.

$\therefore \vec{b} \times \vec{c} = bc \sin\theta \, \hat{\mathbf{n}}$

$= bc \sin 90° \, \hat{\mathbf{n}}$

$= bc \, \hat{n}$

$\vec{a} \cdot \left(\vec{b} \times \vec{c}\right)$

$= a \cdot \left(bc \, \hat{\mathbf{n}}\right)$

$= abc \cos\theta \; \hat{\mathbf{n}}$

$= abc \cos 0°$

$= abc$

= Volume of the parallelepiped

Question 6:

Find the components along the *x, y, z* axes of the angular momentum **l** of a particle, whose position vector is **r** with components x, y, z and momentum is **p** with components p_x, p_y and p_z. Show that if the particle moves only in the x-y plane the angular momentum has only a z-component.

Answer

$l_x = yp_z - zp_y$

$l_y = zp_x - xp_z$

$l_z = xp_y - yp_x$

Linear momentum of the particle, $\vec{p} = p_x\,\hat{\mathbf{i}} + p_y\,\hat{\mathbf{j}} + p_z\,\hat{\mathbf{k}}$

Position vector of the particle, $\vec{r} = x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}} + z\,\hat{\mathbf{k}}$

Angular momentum, $\vec{l} = \vec{r} \times \vec{p}$

$$= \left(x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}} + z\,\hat{\mathbf{k}}\right) \times \left(p_x\,\hat{\mathbf{i}} + p_y\,\hat{\mathbf{j}} + p_z\,\hat{\mathbf{k}}\right)$$

$$= \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ x & y & z \\ p_x & p_y & p_z \end{vmatrix}$$

$$l_x\,\hat{\mathbf{i}} + l_y\,\hat{\mathbf{j}} + l_z\,\hat{\mathbf{k}} = \hat{\mathbf{i}}\left(yp_z - zp_y\right) - \hat{\mathbf{j}}\left(xp_z - zp_x\right) + \hat{\mathbf{k}}\left(xp_y - zp_x\right)$$

Comparing the coefficients of $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$, we get:

$$\left.\begin{aligned} l_x &= yp_z - zp_y \\ l_y &= xp_z - zp_x \\ l_z &= xp_y - yp_x \end{aligned}\right\} \quad \ldots (i)$$

The particle moves in the x-y plane. Hence, the z-component of the position vector and linear momentum vector becomes zero, i.e.,

$z = p_z = 0$

Thus, equation (*i*) reduces to:

$$\left.\begin{aligned} l_x &= 0 \\ l_y &= 0 \\ l_z &= xp_y - yp_x \end{aligned}\right\}$$

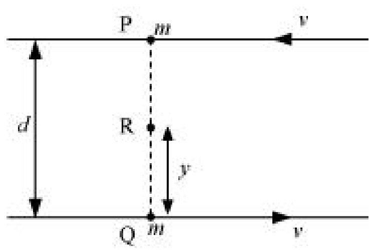
Therefore, when the particle is confined to move in the *x*-*y* plane, the direction of angular momentum is along the *z*-direction.

Question 7:

Two particles, each of mass *m* and speed *v*, travel in opposite directions along parallel lines separated by a distance *d*. Show that the vector angular momentum of the two particle system is the same whatever be the point about which the angular momentum is taken.

Answer

Let at a certain instant two particles be at points P and Q, as shown in the following figure.



Angular momentum of the system about point P:

$$\vec{L}_P = mv \times 0 + mv \times d$$
$$= mvd \qquad \ldots (i)$$

Angular momentum of the system about point Q:

$$\vec{L}_Q = mv \times d + mv \times 0$$
$$= mvd \qquad \ldots (ii)$$

Consider a point R, which is at a distance y from point Q, i.e.,

QR = y

$\therefore$PR = $d - y$

Angular momentum of the system about point R:

$$\begin{aligned}\vec{L}_R &= mv \times (d - y) + mv \times y \\ &= mvd - mvy + mvy \\ &= mvd \qquad \ldots (iii)\end{aligned}$$

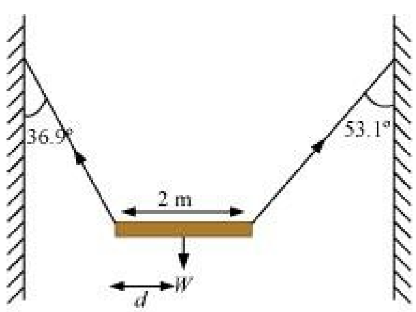
Comparing equations (*i*), (*ii*), and (*iii*), we get:

$$\vec{L}_P = \vec{L}_Q = \vec{L}_R \qquad \ldots (iv)$$

We infer from equation (*iv*) that the angular momentum of a system does not depend on the point about which it is taken.

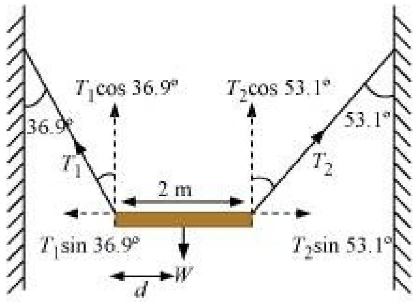
Question 8:

A non-uniform bar of weight W is suspended at rest by two strings of negligible weight as shown in Fig.7.39. The angles made by the strings with the vertical are 36.9° and 53.1° respectively. The bar is 2 m long. Calculate the distance d of the centre of gravity of the bar from its left end.



Answer

The free body diagram of the bar is shown in the following figure.



Length of the bar, $l = 2$ m

T_1 and T_2 are the tensions produced in the left and right strings respectively.

At translational equilibrium, we have:

$$T_1 \sin 36.9° = T_2 \sin 53.1$$

$$\frac{T_1}{T_2} = \frac{\sin 53.1°}{\sin 36.9}$$

$$= \frac{0.800}{0.600} = \frac{4}{3}$$

$$\Rightarrow T_1 = \frac{4}{3} T_2$$

For rotational equilibrium, on taking the torque about the centre of gravity, we have:

$$T_1 \cos 36.9 \times d = T_2 \cos 53.1 (2 - d)$$

$$T_1 \times 0.800 d = T_2 \; 0.600 (2 - d)$$

$$\frac{4}{3} \times T_2 \times 0.800 d = T_2 [0.600 \times 2 - 0.600 d]$$

$$1.067 d + 0.6 d = 1.2$$

$$\therefore d = \frac{1.2}{1.67}$$

$$= 0.72 \text{ m}$$

Hence, the C.G. (centre of gravity) of the given bar lies 0.72 m from its left end.

Question 9:

A car weighs 1800 kg. The distance between its front and back axles is 1.8 m. Its centre of gravity is 1.05 m behind the front axle. Determine the force exerted by the level ground on each front wheel and each back wheel.

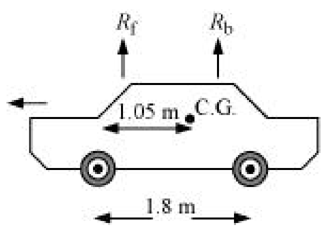
Answer

Mass of the car, m = 1800 kg

Distance between the front and back axles, d = 1.8 m

Distance between the C.G. (centre of gravity) and the back axle = 1.05 m

The various forces acting on the car are shown in the following figure.



R_f and R_b are the forces exerted by the level ground on the front and back wheels respectively.

At translational equilibrium:

$R_f + R_b = mg$

= 1800 × 9.8

= 17640 N … (*i*)

For rotational equilibrium, on taking the torque about the C.G., we have:

$$R_f(1.05) = R_b(1.8 - 1.05)$$
$$R_f \times 1.05 = R_b \times 0.75$$

$$\frac{R_f}{R_b} = \frac{0.75}{1.05} = \frac{5}{7}$$

$$\frac{R_b}{R_f} = \frac{7}{5}$$

$$R_b = 1.4\ R_f \qquad \ldots (ii)$$

Solving equations (*i*) and (*ii*), we get:

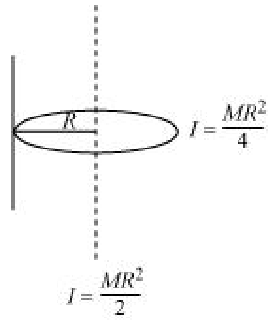
$$1.4R_f + R_f = 17640$$

$$R_f = \frac{17640}{2.4} = 7350\ \text{N}$$

$\therefore R_b$ = 17640 – 7350 = 10290 N

Therefore, the force exerted on each front wheel $= \frac{7350}{2} = 3675\ \text{N}$, and

The force exerted on each back wheel $= \frac{10290}{2} = 5145\ \text{N}$



Applying the theorem of parallel axes:

The moment of inertia about an axis normal to the disc and passing through a point on its edge $= \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$

Question 10:

Torques of equal magnitude are applied to a hollow cylinder and a solid sphere, both having the same mass and radius. The cylinder is free to rotate about its standard axis of symmetry, and the sphere is free to rotate about an axis passing through its centre. Which of the two will acquire a greater angular speed after a given time?

Answer

Let *m* and *r* be the respective masses of the hollow cylinder and the solid sphere.

The moment of inertia of the hollow cylinder about its standard axis, $I_{\text{I}} = mr^2$

The moment of inertia of the solid sphere about an axis passing through its centre,

$I_{\text{II}} = \frac{2}{5}mr^2$

We have the relation:

$\tau = I\alpha$

Where,

α = Angular acceleration

τ = Torque

I = Moment of inertia

For the hollow cylinder, $\tau_{I} = I_{I}\alpha_{I}$

For the solid sphere, $\tau_{II} = I_{II}\alpha_{II}$

As an equal torque is applied to both the bodies, $\tau_1 = \tau_2$

$$\therefore \frac{\alpha_{II}}{\alpha_{I}} = \frac{I_{I}}{I_{II}} = \frac{mr^2}{\frac{2}{5}mr^2} = \frac{2}{5}$$

$\alpha_{II} > \alpha_{I}$ ... (i)

Now, using the relation:

$\omega = \omega_0 + \alpha t$

Where,

ω_0 = Initial angular velocity

t = Time of rotation

ω = Final angular velocity

For equal ω_0 and t, we have:

$\omega \propto \alpha$ … (ii)

From equations (i) and (ii), we can write:

$\omega_{II} > \omega_{I}$

Hence, the angular velocity of the solid sphere will be greater than that of the hollow cylinder.

Question 11:

A solid cylinder of mass 20 kg rotates about its axis with angular speed 100 rad s^{-1}. The radius of the cylinder is 0.25 m. What is the kinetic energy associated with the rotation of the cylinder? What is the magnitude of angular momentum of the cylinder about its axis?

Answer

Mass of the cylinder, m = 20 kg

Angular speed, ω = 100 rad s^{-1}

Radius of the cylinder, r = 0.25 m

The moment of inertia of the solid cylinder:

$$I = \frac{mr^2}{2}$$

$$= \frac{1}{2} \times 20 \times (0.25)^2$$

$$= 0.625 \text{ kg m}^2$$

∴Kinetic energy $= \frac{1}{2} I\omega^2$

$$= \frac{1}{2} \times 6.25 \times (100)^2 = 3125 \text{ J}$$

∴Angular momentum, $L = I\omega$

= 6.25 × 100

= 62.5 Js

Question 12:

A child stands at the centre of a turntable with his two arms outstretched. The turntable is set rotating with an angular speed of 40 rev/min. How much is the angular speed of the child if he folds his hands back and thereby reduces his moment of inertia to 2/5 times the initial value? Assume that the turntable rotates without friction.

Show that the child's new kinetic energy of rotation is more than the initial kinetic energy of rotation. How do you account for this increase in kinetic energy?

Answer

100 rev/min

Initial angular velocity, ω_1= 40 rev/min

Final angular velocity = ω_2

The moment of inertia of the boy with stretched hands = I_1

The moment of inertia of the boy with folded hands = I_2

The two moments of inertia are related as:

$$I_2 = \frac{2}{5} I_1$$

Since no external force acts on the boy, the angular momentum L is a constant.

Hence, for the two situations, we can write:

$$I_2\omega_2 = I_1\omega_1$$

$$\omega_2 = \frac{I_1}{I_2}\omega_1$$

$$= \frac{I_1}{\frac{2}{5}I_1} \times 40 = \frac{5}{2} \times 40$$

$$= 100 \text{ rev/min}$$

(b) Final K.E. = 2.5 Initial K.E.

Final kinetic rotation, E_F $= \frac{1}{2} I_2 \omega_2^2$

Initial kinetic rotation, E_I $= \frac{1}{2} I_1 \omega_1^2$

$$\frac{E_F}{E_1}=\frac{\frac{1}{2}I_2\omega_2^2}{\frac{1}{2}I_1\omega_1^2}$$

$$=\frac{2}{5}\frac{I_1}{I_1}\frac{(100)^2}{(40)^2}$$

$$=\frac{2}{5}\times\frac{100\times100}{40\times40}$$

$$=\frac{5}{2}=2.5$$

$$\therefore E_F=2.5E_1$$

The increase in the rotational kinetic energy is attributed to the internal energy of the boy.

Question 13:

A rope of negligible mass is wound round a hollow cylinder of mass 3 kg and radius 40 cm. What is the angular acceleration of the cylinder if the rope is pulled with a force of 30 N? What is the linear acceleration of the rope? Assume that there is no slipping.

Answer

Mass of the hollow cylinder, m = 3 kg

Radius of the hollow cylinder, r = 40 cm = 0.4 m

Applied force, F = 30 N

The moment of inertia of the hollow cylinder about its geometric axis:

$I = mr^2$

$= 3 \times (0.4)^2 = 0.48$ kg m^2

Torque, $\tau = F \times r$

= 30 × 0.4 = 12 Nm

For angular acceleration α , torque is also given by the relation:

$$\tau = I\alpha$$

$$\alpha = \frac{\tau}{I} = \frac{12}{0.48}$$

$$= 25 \text{ rad s}^{-2}$$

Linear acceleration = $r\alpha$ = 0.4 × 25 = 10 m s^{-2}

Question 14:

To maintain a rotor at a uniform angular speed of 200 rad s^{-1}, an engine needs to transmit a torque of 180 Nm. What is the power required by the engine?

(Note: uniform angular velocity in the absence of friction implies zero torque. In practice, applied torque is needed to counter frictional torque). Assume that the engine is 100 % efficient.

Answer

Angular speed of the rotor, ω = 200 rad/s

Torque required, τ = 180 Nm

The power of the rotor (P) is related to torque and angular speed by the relation:

$P = \tau\omega$

= 180 × 200 = 36 × 10^3

= 36 kW

Hence, the power required by the engine is 36 kW.

Question 15:

From a uniform disk of radius R, a circular hole of radius $R/2$ is cut out. The centre of the hole is at $R/2$ from the centre of the original disc. Locate the centre of gravity of the resulting flat body.

Answer

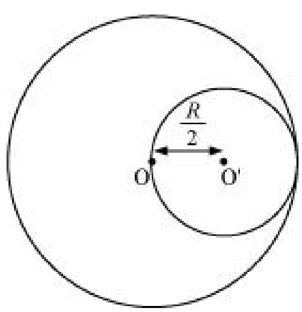
$R/6$; from the original centre of the body and opposite to the centre of the cut portion.

Mass per unit area of the original disc = σ

Radius of the original disc = R

Mass of the original disc, $M = \pi R^2\sigma$

The disc with the cut portion is shown in the following figure:



Radius of the smaller disc = $\frac{R}{2}$

Mass of the smaller disc, $M' = \pi\left(\frac{R}{2}\right)^2\sigma = \frac{1}{4}\pi R^2\sigma = \frac{M}{4}$

Let O and O′ be the respective centres of the original disc and the disc cut off from the original. As per the definition of the centre of mass, the centre of mass of the original disc is supposed to be concentrated at O, while that of the smaller disc is supposed to be concentrated at O′.

It is given that:

OO′= $\frac{R}{2}$

After the smaller disc has been cut from the original, the remaining portion is considered to be a system of two masses. The two masses are:

M (concentrated at O), and

$-M'\left(=\frac{M}{4}\right)$ concentrated at O′

(The negative sign indicates that this portion has been removed from the original disc.)

Let *x* be the distance through which the centre of mass of the remaining portion shifts from point O.

The relation between the centres of masses of two masses is given as:

$$x=\frac{m_1r_1+m_2r_2}{m_1+m_2}$$

For the given system, we can write:

$$x=\frac{M\times 0-M'\times\left(\frac{R}{2}\right)}{M+(-M')}$$

$$=\frac{\frac{-M}{4}\times\frac{R}{2}}{M-\frac{M}{4}}=\frac{-MR}{8}\times\frac{4}{3M}=\frac{-R}{6}$$

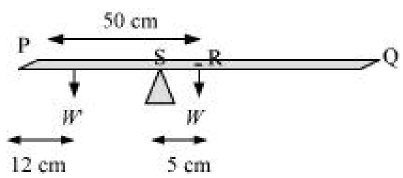
(The negative sign indicates that the centre of mass gets shifted toward the left of point O.)

Question 16:

A metre stick is balanced on a knife edge at its centre. When two coins, each of mass 5 g are put one on top of the other at the 12.0 cm mark, the stick is found to be balanced at 45.0 cm. What is the mass of the metre stick?

Answer

Let *W* and *W'* be the respective weights of the metre stick and the coin.



The mass of the metre stick is concentrated at its mid-point, i.e., at the 50 cm mark.

Mass of the meter stick = m'

Mass of each coin, m = 5 g

When the coins are placed 12 cm away from the end P, the centre of mass gets shifted by 5 cm from point R toward the end P. The centre of mass is located at a distance of 45 cm from point P.

The net torque will be conserved for rotational equilibrium about point R.

$$10 \times g(45 - 12) - m'g(50 - 45) = 0$$

$$\therefore m' = \frac{10 \times 33}{5} = 66 \text{ g}$$

Hence, the mass of the metre stick is 66 g.

Question 17:

The oxygen molecule has a mass of 5.30×10^{-26} kg and a moment of inertia of 1.94×10^{-46} kg m^2 about an axis through its centre perpendicular to the lines joining the two atoms. Suppose the mean speed of such a molecule in a gas is 500 m/s and that its kinetic energy of rotation is two thirds of its kinetic energy of translation. Find the average angular velocity of the molecule.

Answer

Mass of an oxygen molecule, $m = 5.30 \times 10^{-26}$ kg

Moment of inertia, $I = 1.94 \times 10^{-46}$ kg m^2

Velocity of the oxygen molecule, $v = 500$ m/s

The separation between the two atoms of the oxygen molecule = $2r$

Mass of each oxygen atom = $\frac{m}{2}$

Hence, moment of inertia I, is calculated as:

$$\left(\frac{m}{2}\right)r^2 + \left(\frac{m}{2}\right)r^2 = mr^2$$

$$r = \sqrt{\frac{I}{m}}$$

$$\sqrt{\frac{1.94 \times 10^{-46}}{5.36 \times 10^{-26}}} = 0.60 \times 10^{-10} \text{ m}$$

It is given that:

$$KE_{rot} = \frac{2}{3} KE_{trans}$$

$$\frac{1}{2} I\omega^2 = \frac{2}{3} \times \frac{1}{2} \times mv^2$$

$$mr^2\omega^2 = \frac{2}{3} mv^2$$

$$\omega = \sqrt{\frac{2}{3}} \frac{v}{r}$$

$$= \sqrt{\frac{2}{3}} \times \frac{500}{0.6 \times 10^{-10}}$$

$$= 6.80 \times 10^{12} \text{ rad/s}$$