CLASS X: MATHS

Chapter 12: Surface Areas and Volumes

Questions and Solutions | Exercise 12.1 - NCERT Books

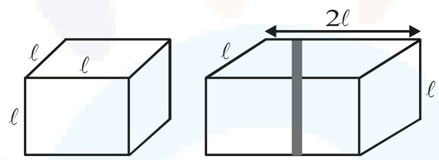
NOTE: Unless stated otherwise, take $\pi = \frac{22}{7}$

Q1. 2 cubes each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboid.

Sol. Let ℓ cm be the length of an edge of the cube having volume = 64 cm^3.

Then, $\ell^3 = 64 = (4)^3 \Rightarrow \ell = 4$ cm

Now, the dimensions of the resulting cuboid made by joining two cubes (see figure) are 8 cm × 4 cm × 4 cm (i.e., length = 8 cm, breadth = 4 cm and height = 4 cm)



Surface area of cuboid = $2(\ell b + bh + h\ell)$

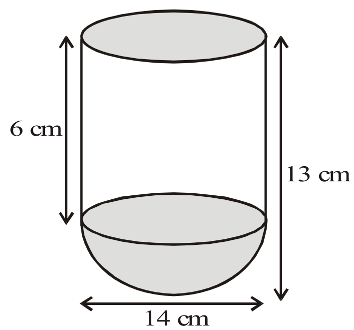
$= 2(8 \times 4 + 4 \times 4 + 4 \times 8)$

$= 2(32 + 16 + 32) = 2 \times 80 = 160 \text{ cm}^2$

Q2. A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel.

Sol. For hemispherical part, radius (r) = $\frac{14}{2} = 7$ cm

$\therefore$ Curved surface area = $2\pi r^2$



$= 2 \times \frac{22}{7} \times 7 \times 7 \text{ cm}^2 = 308 \text{ cm}^2$

Total height of vessel = 13 cm

$\therefore$ Height of cylinder = (13 – 7)cm = 6 cm and radius (r) = 7 cm

$\therefore$ Curved surface area of cylinder = $2\pi rh$

$= 2 \times \frac{22}{7} \times 7 \times 6 \text{ cm}^2 = 264 \text{ cm}^2$

$\therefore$ Inner surface area of vessel = Curved surface area of hemispherical part + Curved surface area of cylinder

$= (308 + 264) \text{ cm}^2 = 572 \text{ cm}^2$

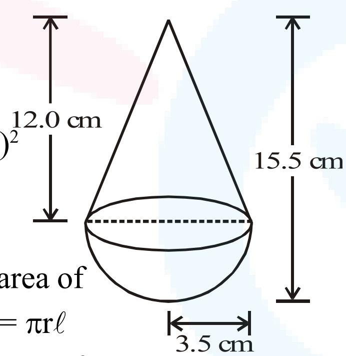
Q3. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. find the total surface area of the toy.

Sol. Let r and h be the radius of cone, hemisphere and height of cone

$\therefore h = (15.5 - 3.5)$ cm

$= 12.0$ cm

Also $\ell^2 = h^2 + r^2$



$= 12^2 + (3.5)^2$

$= 156.25$

$\therefore \ell = 12.5$ cm

Curved surface area of the conical part = $\pi r\ell$

Curved surface area of the hemispherical part = $2\pi r^2$

Total surface area of the toy = $\pi r\ell + 2\pi r^2$

$= \pi r(\ell + 2r)$

$= \frac{22}{7} \times \frac{35}{10} (12.5 + 2 \times 3.5) \text{ cm}^2$

$= 11 \times (12.5 + 7) \text{ cm}^2 = 11 \times 19.5 \text{ cm}^2$

$= 214.5 \text{ cm}^2$

Q4. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid.

Sol. On 7 cm × 7 cm base of the cubical block, we can mount hemisphere having greatest diameter equal to 7 cm.

Here, the radius of the hemisphere = 3.5 cm.

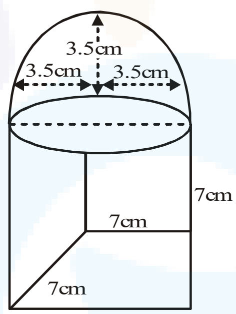
Now, the surface area of the solid made in figure.

= The surface area of the cube + The curved surface area of the hemisphere – The area of the base of the hemisphere.

$= \{6 \times (7)^2 + 2\pi \times (3.5)^2 - \pi \times (3.5)^2\}\ cm^2$

($\because$ the part of the top of the cubical part which is covered by the hemisphere is not visible outside)

$= \left\{6 \times 49 + \frac{22}{7} \times \frac{35}{10} \times \frac{35}{10}\right\} cm^2$



$= \left\{294 + 11 \times \frac{35}{10}\right\} cm^2 = 332.5\ cm^2$

Q5. A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter ℓ of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

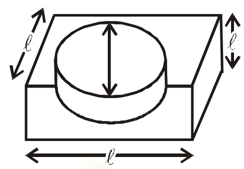
Sol. Let ℓ be the side of the cube.

$\therefore$ The greatest diameter of the hemisphere = ℓ

$\Rightarrow$ Radius of the hemisphere = $\frac{\ell}{2}$

$\therefore$ Surface area of hemisphere = $2\pi r^2$

$= 2 \times \pi \times \frac{\ell}{2} \times \frac{\ell}{2} = \frac{\pi\ell^2}{2}$



Base area of the hemisphere $= \pi\left(\frac{\ell}{2}\right)^2 = \frac{\pi\ell^2}{4}$

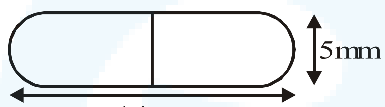
Surface area of the cube $= 6 \times \ell^2 = 6\ell^2$

$\therefore$ Surface area of the remaining solid

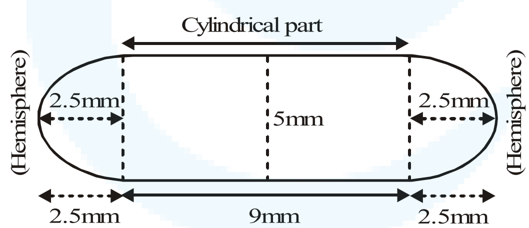
$= 6\ell^2 + \frac{\pi\ell^2}{2} - \frac{\pi\ell^2}{4} = \frac{24\ell^2 + 2\pi\ell^2 - \pi\ell^2}{4} = \frac{24\ell^2 + \pi\ell^2}{4}$

$= \frac{\ell^2}{4}(24 + \pi)$ sq. units.

Q6. A medicine capsule is in the shape of a cylinder with two hemispheres struck to each of its ends (see figure). The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area.



Sol. Surface area of the cylindrical part $= 2\pi \times r \times h$



$= 2\pi \times \left(\frac{5}{2}\right) \times 9 \text{ mm}^2 = 45\,\pi \text{ mm}^2$

Sum of the curved surface areas of two hemispherical parts.

$= 2\left\{2\pi \times \left(\frac{5}{2}\right)^2\right\}\text{mm}^2 = 25\,\pi \text{ mm}^2$

Total surface area of the capsule

$= 45\pi + 25\pi \text{ mm}^2 = 70\pi \text{ mm}^2$

$= 70 \times \frac{22}{7} \text{ mm}^2 = 220 \text{ mm}^2$

7. A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2·1 m and 4 m respectively, and the slant height of the top is 2·8 m, find the area of the canvas used for making the tent. Also find the cost of the canvas of the tent at the rate of Rs.500 per m^2 (Note that the base of the tent will not be covered with canvas).

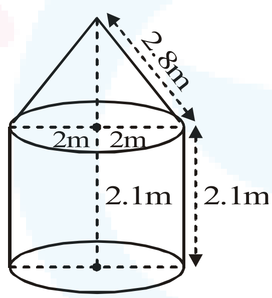
Sol. Radius of the cylindrical base = 2m and

height = 2.1 m. The curved surface area of the cylindrical part

$= 2\pi \times (2) \times (2.1) \text{ m}^2$ (i.e., $2\pi rh$)

$= 4 \times \frac{22}{7} \times 2.1 \text{ m}^2$

$= 26.4 \text{ m}^2$



Now, for the conical part,

we have r = 2m and

ℓ (slant height) = 2.8 m

The curved surface area of

the conical part = $\pi r \ell$

$= \frac{22}{7} \times 2 \times 2.8 \text{ m}^2$

$= 17.6 \text{ m}^2$

Then the area of the canvas

$= 26.4 \text{ m}^2 + 17.6 \text{ m}^2 = 44 \text{ m}^2$

Total cost of the canvas at the rate of Rs. 500 per m^2

= Rs. 500 × 44 = Rs. 22000

8. From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2.

Sol. For cylinder part :

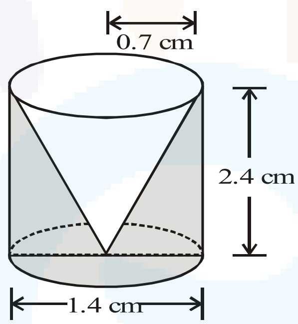
Height = 2.4 cm and diameter = 1.4 cm

$\Rightarrow$ Radius (r) = 0.7 cm

$\therefore$ Total surface area of the cylindrical part

$= 2 \times \frac{22}{7} \times \frac{7}{10}[2.4 + 0.7]\ cm^2$

$= \frac{44}{10} \times 3.1\ cm^2 = \frac{44 \times 31}{100} = \frac{1364}{100} cm^2$



For conical part :

Base radius (r) = 0.7 cm and height (h) = 2.4 cm

$\therefore$ Slant height (ℓ) = $\sqrt{r^2 + h^2} = \sqrt{(0.7)^2 + (2.4)^2}$

$= \sqrt{0.49 + 5.76} = \sqrt{6.25} = 2.5$ cm

$\therefore$ Curved surface area of the conical part

$= \pi r\ell = \frac{22}{7} \times 0.7 \times\ 2.5\ cm^2 = \frac{550}{100} cm^2$

Base area of the conical part

$\pi r^2 = \frac{22}{7} \times \left(\frac{7}{10}\right)^2 cm^2 = \frac{22 \times 7}{100} cm^2 = \frac{154}{100} cm^2$

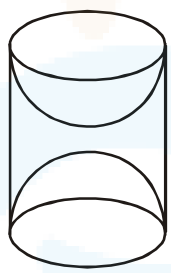
Total surface area of the remaining solid

= [(Total surface area of cylinderical part) + (Curved surface area of conical part) – (Base area of the conical part)] $= \left[\frac{1364}{100} + \frac{550}{100} - \frac{154}{100}\right] cm^2$

$= \frac{1760}{100} cm^2 = 17.6\ cm^2$.

Hence, total surface area to the nearest cm^2 is 18 cm^2.

9. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in fig. If the height of the cylinder is 10 cm, and its base is of radius 3·5 cm, find the total surface area of the article.



Sol. Radius of the cylinder (r) = 3.5 cm

Height of the cylinder (h) = 10 cm

$\therefore$ Curved surface area of cylinder = $2\pi rh$

$= 2 \times \frac{22}{7} \times \frac{35}{10} \times 10\ cm^2 = 220\ cm^2$

Curved surface area of a hemisphere = $2\pi r^2$

$\therefore$ Curved surface area of both hemispheres

$= 2 \times 2\pi r^2 = 4\pi r^2 = 4 \times \frac{22}{7} \times \frac{35}{10} \times \frac{35}{10} cm^2 = 154\ cm^2$

Total surface area of the remaining solid

$= (220 + 154)\ cm^2 = 374\ cm^2$.

Questions and Solutions | Exercise 12.2 - NCERT Books

NOTE: Unless stated otherwise, take $\pi = \frac{22}{7}$

Q1. A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius.
Find the volume of the solid in terms of π.

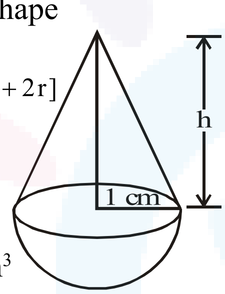
Sol. Here, r = 1 cm and h = 1 cm.

Volume of the conical part = $\frac{1}{3}\pi r^2 h$

and volume of the hemispherical part $\frac{2}{3}\pi r^3$

$\therefore$ Volume of the solid shape

$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3 = \frac{1}{3}\pi r^2 [h + 2r]$



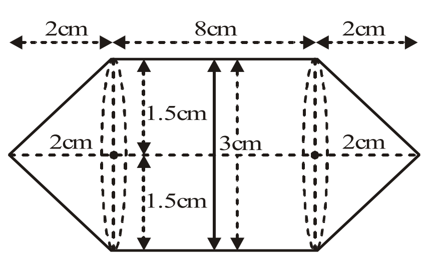
$= \frac{1}{3}\pi(1)^2[1 + 2(1)]\ cm^3$

$= \frac{1}{3}\pi \times 1 \times 3\ cm^3 = \pi cm^3$

Q2. Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly the same).

Sol. Volume of the cylindrical part

$= \pi \times (1.5)^2 \times 8\ cm^3 = 18\ \pi\ cm^3$



Volume of each conical part

$= \frac{1}{3}\pi \times (1.5)^2 \times 2\ cm^3 = \frac{3}{2}\pi\ cm^3$

Therefore, the volume of the air

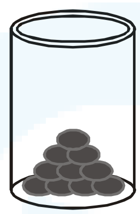
= The volume of cylindrical part

+ The volumes of two conical parts

$= 18\ \pi + 2 \times \frac{3}{2}\pi\ cm^3 = 21\ \pi\ cm^3$

$= 21 \times \frac{22}{7}\ cm^3 = 66\ cm^3$

3. A gulab jamun, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm. (see fig.)



Sol. Since, a gulab jamun is like a cylinder with hemispherical ends.

Total height of the

gulab jamun = 5 cm.

Diameter = 2.8 cm

⇒ Radius = 1.4 cm

∴ Length (height) of the cylindrical part

= 5cm – (1.4 + 1.4) cm = 5 cm – 2.8 cm = 2.2 cm

Now, volume of the cylindrical part = $\pi r^2 h$ and volume of both the hemispherical ends

$= 2\left(\frac{2}{3}\pi r^3\right) = \frac{4}{3}\pi r^3$

∴ Volume of a gulab jamun

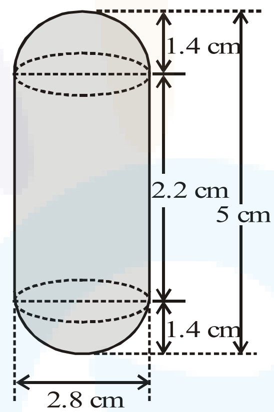
$= \pi r^2 h + \frac{4}{3}\pi r^3 = \pi r^2\left[h + \frac{4}{3}r\right]$

$$= \frac{22}{7} \times (1.4)^2 \left[2.2 + \frac{4}{3}(1.4)\right] \text{cm}^3$$

$$= \frac{22}{7} \times \frac{14}{10} \times \frac{14}{10}\left[\frac{22}{10} + \frac{56}{30}\right] \text{cm}^3$$

$$= \frac{22 \times 2 \times 14}{10 \times 10}\left[\frac{66 + 56}{30}\right] \text{cm}^2$$

$$= \frac{44 \times 14}{100} \times \frac{122}{30} \text{cm}^3$$



Volume of 45 gulab jamuns

$$= 45 \times \left[\frac{44 \times 14}{100} \times \frac{122}{30}\right] \text{cm}^3$$

$$= \frac{15 \times 44 \times 14 \times 122}{1000} \text{cm}^3$$

Since, the quantity of syrup in gulab jamuns

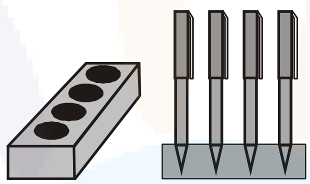
= 30% of [volume]

$$= 30\% \text{ of } \left[\frac{15 \times 44 \times 14 \times 122}{1000}\right] \text{cm}^3$$

$= \frac{30}{100} \times \frac{15 \times 44 \times 14 \times 122}{1000}$ cm^3 = 338.184 cm^3

= 338 cm^2 (approx)

Q4. A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm. Find the volume of wood in the entire stand (see fig.).



Sol. Radius of conical cavity = 0.5 cm and depth (i.e., vertical height) = 1.4 cm

Volume of wood taken out to make one cavity

$= \frac{1}{3}\pi r^2 \times h = \frac{1}{3} \times \frac{22}{7} \times (0.5)^2 \times (1.4)$ cm^3

$= \frac{1}{3} \times \frac{22}{7} \times \frac{1}{4} \times \frac{14}{10}$ cm^3 $= \frac{11}{30}$ cm^3

Volume of wood taken out to make four cavities

$= 4 \times \frac{11}{30}$ cm^3 $= \frac{44}{30}$ cm^3

volume of the wood in the pen stand

$= (15 \times 10 \times 3.5) - \frac{44}{30}$ cm^3 = (525 – 1.47) cm^3 (approx.) = 523.53 cm^3 (approx.)

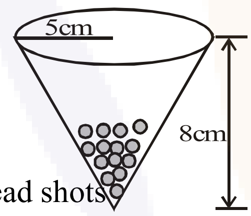
Q5. A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water upto the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one fourth of the water flows out. Find the number of lead shots dropped in the vessel.

Sol. Height of the conical vessel (h) = 8 cm

Base radius (r) = 5 cm

Volume of water in conical vessel $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times (5)^2 \times 8 \text{ cm}^3$



$= \frac{4400}{21} \text{ cm}^3$

Now, Total volume of lead shots

$= \frac{1}{4} \times \frac{4400}{21} \text{ cm}^3 = \frac{1100}{21} \text{ cm}^3$

Since, radius of spherical lead shot (r) = 0.5 cm

$\therefore$ Volume of 1 lead shot $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times \frac{5}{10} \times \frac{5}{10} \times \frac{5}{10} \text{ cm}^3$

$\therefore$ Number of lead shots

$$= \frac{\text{Total volume of lead shots}}{\text{Volume of 1 lead shot}} = \frac{\left[\frac{1100}{21}\right]}{\left[\frac{4 \times 22 \times 5 \times 5 \times 5}{3 \times 7 \times 1000}\right]}$$

= 100

Thus, the required number of lead shots = 100.

Q6. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm^3 of iron has approximately 8 g mass. (Use $\pi = 3.14$)

Sol. First cylindrical part has height 220 cm and radius 12 cm.

Its volume $= \pi \times (12)^2 \times 220\ cm^3$.

Second cylindrical part has height 60 cm and radius 8 cm.

Its volume $= \pi \times (8)^2 \times 60\ cm^3$

Total volume $= \{144 \times 220 + 64 \times 60\}\ \pi\ cm^3$

$= 35520\ \pi\ cm^3 = 35520 \times 3.14\ cm^3$

$= 111532.8\ cm^3$

Total weight (at the rate of 8 gm per 1 cm^3)

$= \frac{111532.8 \times 8}{1000}$ kg $= 111.5328 \times 8$ kg $= 892.2624$ kg

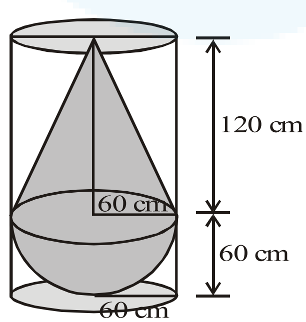
Q7. A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius is 60 cm and its height is 180 cm.

Sol. Height of the conical part = 120 cm

Base radius of the conical part = 60 cm

$\therefore$ Volume of the conical part $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 60^2 \times 120\ cm^3$



Radius of the hemispherical part = 60 cm.

$\therefore$ Volume of the hemispherical part $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times (60)^3$ cm^3

$\therefore$ Volume of the solid = [Volume of conical part] + [Volume of hemispherical part]

$= \left[\frac{1}{3} \times \frac{22}{7} \times 60^2 \times 120\right] + \left[\frac{2}{3} \times \frac{22}{7} \times 60^3\right]$ cm^3

$= \frac{2}{3} \times \frac{22}{7} \times 60^2\ [60 + 60]$ cm^3

$= \frac{2}{3} \times \frac{22}{7} \times 60 \times 60 \times 120$ cm^3 $= \frac{6336000}{7}$ cm^3

$\Rightarrow$ Volume of water in the cylinder = $\pi r^2 h$

$= \frac{22}{7} \times 60 \times 60 \times 180$

$= \frac{14256000}{7}$ cm^3

$\therefore$ Volume of water left in the cylinder

$= \left[\frac{14256000}{7} - \frac{6336000}{7}\right]$ cm^3

$= \frac{7920000}{7}$ cm^3

$= 1131428.57142$ cm^3 $= \frac{1131428.57142}{1000000}$ m^3

$= 1.13142857142$ m^3 $= 1.131$ m^3 (approx).

8. A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the diamter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm^3. Check whether she is correct, taking the above as the inside measurements, and π = 3.14.

Sol. The cylinder neck has length = 8 cm

and radius = 1 cm

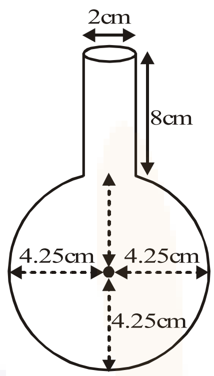
Volume of the cylinder part

$= \pi\,(1)^2 \times 8\ cm^3$

$= 8\,\pi\ cm^3$

The radius of the spherical part

$= \frac{8.5}{2} cm = 4.25\ cm$



Volume of the spherical part

$= \frac{4}{3}\pi \times (4.25)^3$

Total volume of water

$= 8\pi + \frac{4}{3} \times (4.25)^3\ \pi\ cm^3$

$= 8 \times 3.14 + \frac{4}{3} \times 3.14 \times (4.25)^3\ cm^3$

$= 25.12 + 321.38$ (approx.)

$= 346.5\ cm^3$ (approx.),

So, $345\ cm^3$ is not correct.

$= \frac{22}{7} \times H \times 7 \times 4 m^3$

Since, Volume of the embankment = Volume of the cylindrical well

$\therefore \left[\frac{22}{7} \times H \times 7 \times 4\right] = 99$

$\Rightarrow H = 99 \times \frac{7}{22} \times \frac{1}{7} \times \frac{1}{4}\ m = \frac{9}{8}\ m = 1.125\ m$

$= \frac{9}{8}\ m = 1.125\ m$

Thus, the required height of the embankment

$= 1.125$ m