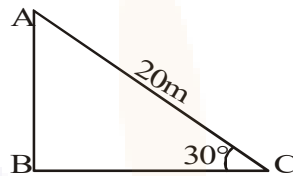


## CLASS X: MATHS

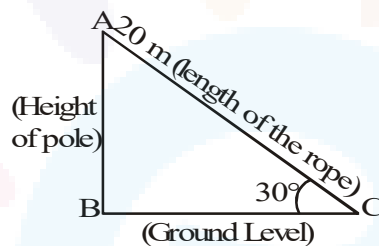
## Chapter 9: Some Applications of Trigonometry

## Questions and Solutions | Exercise 9.1 - NCERT Books

- Q1.** A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole if the angle made by the rope with the ground level is  $30^\circ$  (see fig.).



- Sol.** AC = 20 m is the length of the rope.  
 Let AB = h metres be the height of the pole  
 $\angle ACB = 30^\circ$  (Given)



$$\text{Now, } \frac{AB}{AC} = \sin 30^\circ = \frac{1}{2} \Rightarrow \frac{h}{20} = \frac{1}{2} \Rightarrow h = 10 \text{ m}$$

- Q2.** A tree breaks due to storm and the broken part bends so that the top of the trees touches the ground making an angle of  $30^\circ$  with the ground. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

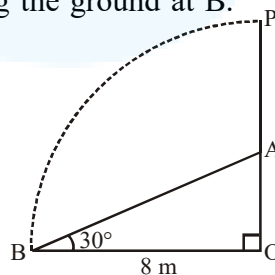
- Sol.** Let tree is broken at A and its top is touching the ground at B.

Now, in right  $\triangle AOB$ , we have

$$\frac{AO}{OB} = \tan 30^\circ$$

$$\Rightarrow \frac{AO}{8} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{AO}{8} = \frac{1}{\sqrt{3}} \Rightarrow AO = \frac{8}{\sqrt{3}} \text{ m}$$





Also,  $\frac{AB}{OB} = \sec 30^\circ$

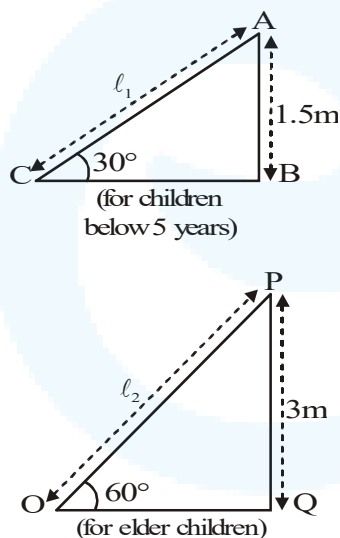
$$\Rightarrow \frac{AB}{8} = \frac{2}{\sqrt{3}} \Rightarrow AB = \frac{2 \times 8}{\sqrt{3}} = \frac{16}{\sqrt{3}} \text{ m}$$

Now, height of the tree  $OP = OA + AB$

$$= \frac{8}{\sqrt{3}} + \frac{16}{\sqrt{3}} = \frac{24}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \text{ m} = 8\sqrt{3} \text{ m}$$

**Q3.** A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose top is at a height of 1.5 m and is inclined at an angle of  $30^\circ$  to the ground, whereas for elder children, she wants to have a steep slide at a height of 3 m, and inclined at an angle of  $60^\circ$  to the ground. What should be the length of the slide in each case?

**Sol.** In figure,  $l_1$  is the length of the slide made for children below the age of 5 years and  $l_2$  is the length of the slide made for elder children.



In figure,  $AB = 1.5 \text{ m}$ ,  $AC = l_1 \text{ m}$  and  $\angle ACB = 30^\circ$ ;  $PQ = 3 \text{ m}$ ,  $OP = l_2 \text{ m}$  and  $\angle POQ = 60^\circ$

$$\frac{AB}{AC} = \sin 30^\circ \quad \text{and} \quad \frac{PQ}{OP} = \sin 60^\circ$$



$$\Rightarrow \frac{1.5}{l_1} = \frac{1}{2} \quad \text{and} \quad \frac{3}{l_2} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow l_1 = 2 \times 1.5 \text{ m and } l_2 = \frac{3 \times 2}{\sqrt{3}} \text{ m}$$

$$\Rightarrow l_1 = 3 \text{ m} \quad \text{and} \quad l_2 = 2\sqrt{3} \text{ m}$$

**Q4.** The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower, is  $30^\circ$ . Find the height of the tower.

**Sol.** In right ABC, AB = height of the tower and point C is 30m away from the foot of the tower,

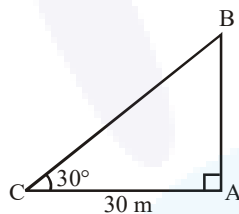
$$\therefore AC = 30 \text{ m}$$

$$\text{Now } \frac{AB}{AC} = \tan 30^\circ$$

$$\Rightarrow \frac{h}{30} = \frac{1}{\sqrt{3}}$$

$$[\because \tan 30^\circ = \frac{1}{\sqrt{3}}]$$

$$\Rightarrow h = \frac{30}{\sqrt{3}} = \frac{30}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 10\sqrt{3}$$



Thus, the required height of the tower is  $10\sqrt{3}$  m

**Q5.** A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is  $60^\circ$ . find the length of the string, assuming that there is no slack in the string.

**Sol.** P is the position of the kite. Its height from the point Q (on the ground) = PQ = 60 m

Let OP =  $l$  be the length of the string.

$$\angle POQ = 60^\circ \text{ (Given)}$$

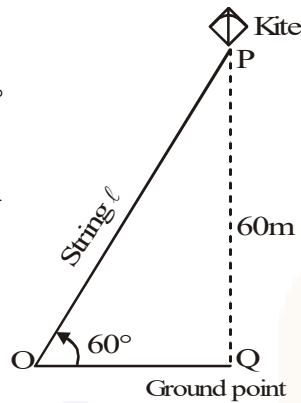


Now,  $\frac{PQ}{OP} = \sin 60^\circ$

$\Rightarrow \frac{60}{l} = \sin 60^\circ = \frac{\sqrt{3}}{2}$

$\Rightarrow \frac{60}{l} = \frac{\sqrt{3}}{2}$

$\Rightarrow l = 40\sqrt{3}\text{m}$



**Q6.** A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from  $30^\circ$  to  $60^\circ$  as he walks towards the building. Find the distance he walked towards the building.

**Sol.**  $PQ = 30\text{ m}$  is the height of the building.  $OA = 1.5\text{ m}$  is the height of the boy. Its first position is at  $OA$   $OR$  is horizontal line through the position of the eye at  $O$ .

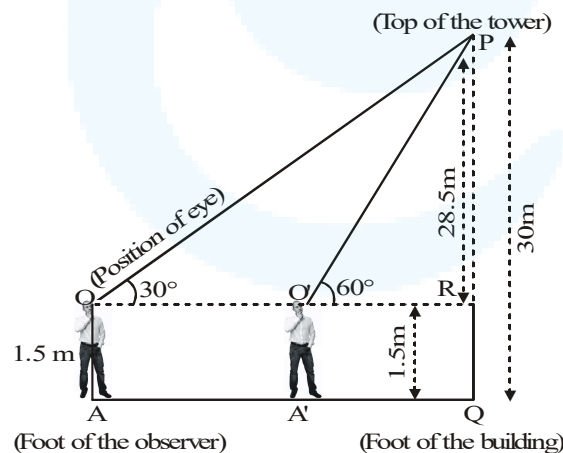
$\angle POR = 30^\circ$  (Given)

The second position of the boy is at  $O'A'$  and

$\angle PO'R = 60^\circ$ .

Here,  $RQ = OA = 1.5\text{ m}$

and  $PR = 30\text{ m} - 1.5\text{ m} = 28.5\text{ m}$



From  $\Delta POR$ ,

From  $\Delta PO'R$ ,



$$\frac{PR}{OR} = \tan 30^\circ \qquad \frac{PR}{O'R} = \tan 60^\circ$$

$$\Rightarrow \frac{28.5}{OR} = \frac{1}{\sqrt{3}} \qquad \frac{28.5}{O'R} = \sqrt{3}$$

$$\Rightarrow OR = 28.5 \times \sqrt{3} \text{ m} \dots(1) \Rightarrow O'R = \frac{28.5}{\sqrt{3}} \text{ m} \dots(2)$$

The distance walked by the boy towards the building.

$$= OO' = OR - O'R$$

$$= 28.5 \times \sqrt{3} \text{ m} - \frac{28.5}{\sqrt{3}} \text{ m} \quad [\text{From (1) and (2)}]$$

$$= 28.5 \times \left\{ \sqrt{3} - \frac{1}{\sqrt{3}} \right\} \text{ m}$$

$$= 28.5 \times \frac{(3-1)}{\sqrt{3}} \text{ m} = 28.5 \times \frac{2}{\sqrt{3}} \text{ m}$$

$$= \frac{57}{\sqrt{3}} \text{ m} = 19\sqrt{3} \text{ m}$$

**Q7.** From a point on the ground the angles of elevation of the bottom and top of a transmission tower fixed at the top of a 20 m high building are  $45^\circ$  and  $60^\circ$  respectively. Find the height of the tower.

**Sol.**  $PQ = 20$  m is the height of the building.

Let  $PR = h$  metres be the height of the transmission tower. P is the bottom and R is the top of the transmission tower.

$$\angle POQ = 45^\circ \text{ and } \angle ROQ = 60^\circ$$

From  $\triangle OPQ$ ,

$$\frac{PQ}{OQ} = \tan 45^\circ$$

$$\Rightarrow \frac{20}{OQ} = 1$$

$$\Rightarrow OQ = 20 \text{ m}$$

From  $\triangle ORQ$ ,

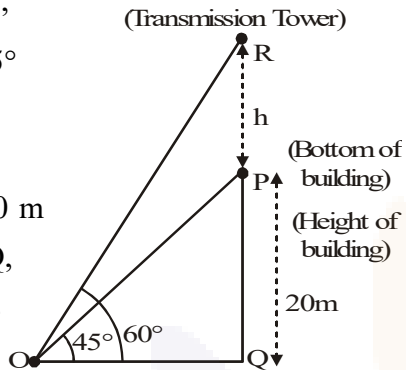
$$\frac{RQ}{OQ} = \tan 60^\circ$$

$$\Rightarrow \frac{20 + h}{20} = \sqrt{3}$$

$$(\because RQ = PQ + PR = 20 + h \text{ metres and } OQ = 20 \text{ metres})$$

$$\Rightarrow 1 + \frac{h}{20} = \sqrt{3} \Rightarrow \frac{h}{20} = (\sqrt{3} - 1)$$

$$\Rightarrow h = 20(\sqrt{3} - 1) \text{ m}$$



- Q8.** A statue, 1.6 m tall, stands on the top of pedestal. From a point on the ground, the angle of elevation of the top of the statue is  $60^\circ$  and from the same point the angle of elevation of the top of the pedestal is  $45^\circ$ . Find the height of the pedestal.

**Sol.** In the figure, DC represents the statue and BC represents the pedestal.

Now, in right  $\triangle ABC$ , we have

$$\frac{AB}{BC} = \cot 45^\circ = 1$$

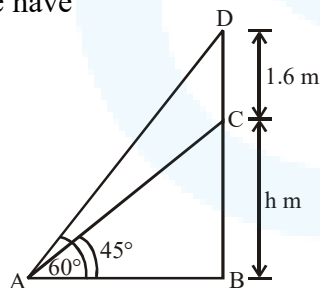
$$\Rightarrow \frac{AB}{h} = 1$$

$$\Rightarrow AB = h \text{ metres.}$$

Now in right  $\triangle ABD$ , we have

$$\frac{BD}{AB} = \tan 60^\circ = \sqrt{3}$$

$$\Rightarrow BD = \sqrt{3} \times AB = \sqrt{3} \times h$$



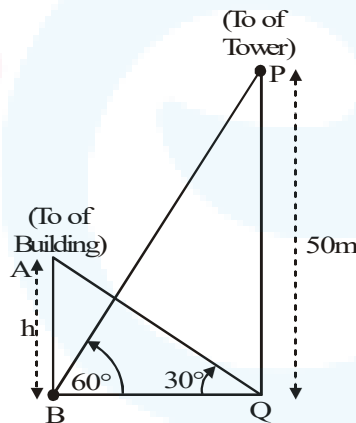


$$\begin{aligned} \Rightarrow h + 1.6 &= \sqrt{3} h \\ \Rightarrow h(\sqrt{3} - 1) &= 1.6 \\ \Rightarrow h &= \frac{1.6}{\sqrt{3} - 1} = \frac{1.6}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1} \\ \Rightarrow h &= \frac{1.6}{3 - 1} \times (\sqrt{3} + 1) = \frac{1.6}{2} \times (\sqrt{3} + 1) \\ &= 0.8(\sqrt{3} + 1)\text{m} \end{aligned}$$

Thus, the height of the pedestal is  $0.8(\sqrt{3} + 1)\text{m}$ .

**Q9.** The angle of elevation of the top of the building from the foot of the tower is  $30^\circ$  and the angle of elevation of the top of the tower from the foot of the building is  $60^\circ$ . If the tower is 50 m high, find the height of the building.

**Sol.** PQ = 50 metres is the height of the tower. Let AB = h metres be the height of the building. Angle of elevation of the top of the building from the foot of the tower =  $30^\circ$ , i.e.,  $\angle AQB = 30^\circ$ .



Angle of elevation of the top of the tower from the foot of the building

$$= 60, \text{i.e., } \angle PBQ = 60$$

From  $\triangle AQB$

$$\frac{h}{BQ} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\Rightarrow BQ = h\sqrt{3} \quad \dots(1)$$



From  $\triangle PBQ$

$$\frac{50}{BQ} = \tan 60^\circ = \sqrt{3}$$

$$\Rightarrow BQ = \frac{50}{\sqrt{3}} \quad \dots(2)$$

From (1) and (2), we have  $h\sqrt{3} = \frac{50}{\sqrt{3}}$

$$\Rightarrow h = \frac{50}{3} \text{ m, i.e., } h = 16\frac{2}{3} \text{ m}$$

**Q10.** Two poles of equal heights are standing opposite to each other on either side of the road which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are  $60^\circ$  and  $30^\circ$ , respectively. Find the height of the poles and the distances of the point from the poles.

**Sol.** Let AB and CD be the towers & P is the point between them.

AB = h metres

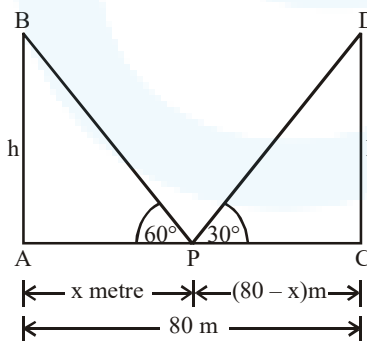
CD = h metres

AP = x m

CP = (80 - x) m

Now, in right  $\triangle APB$ , We have

$$\frac{AB}{AP} = \tan 60^\circ \Rightarrow \frac{h}{x} = \sqrt{3}$$



$$\Rightarrow h = x\sqrt{3} \quad \dots\dots(1)$$

Again in right  $\triangle CPD$ , we have





$$\frac{CD}{CP} = \tan 30^\circ$$

$$\Rightarrow \frac{h}{(80 - x)} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow h = \frac{80 - x}{\sqrt{3}} \quad \dots(2)$$

From (1) and (2), we get

$$\sqrt{3}x = \frac{80 - x}{\sqrt{3}}$$

$$\Rightarrow \sqrt{3} \times \sqrt{3} \times x = 80 - x \Rightarrow 3x = 80 - x$$

$$\Rightarrow 3x + x = 80 \Rightarrow 4x = 80 \Rightarrow x = \frac{80}{4} = 20$$

$$\therefore CP = 80 - x = 80 - 20 = 60 \text{ m}$$

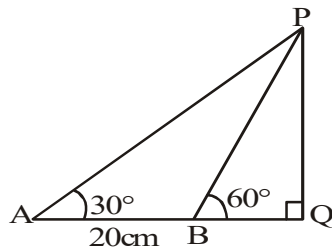
Now, from (1), we have

$$h = \sqrt{3} \times 20 = 1.732 \times 20 = 34.64$$

Thus, the required point is 20 m away from the first pole and 60 m away from the second pole.

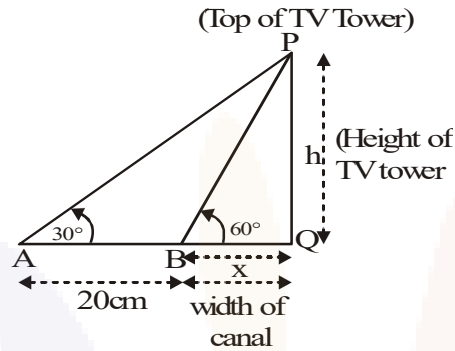
Height of each pole = 34.64 m.

- Q11.** A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is  $60^\circ$ . From another point 20 m away this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is  $30^\circ$  (see fig.). Find the height of the tower and the width of the canal.





**Sol.** Let  $PQ = h$  metres be the height of the tower and  $BQ = x$  metres be the width of the canal  
 $\angle PBQ = 60^\circ$



Now, the angle of elevation of the top of the tower from the point  $A = 30^\circ$ ,  
 i.e.,  $\angle PAQ = 30^\circ$  where  $AB = 20$  metres.

From  $\triangle PBQ$ ,

$$\frac{h}{x} = \tan 60^\circ$$

$$\Rightarrow \frac{h}{x} = \sqrt{3} \text{ or } h = x\sqrt{3} \dots(1)$$

From  $\triangle PAQ$ ,

$$\frac{h}{20+x} = \tan 30^\circ = \frac{1}{\sqrt{3}} \Rightarrow h = \frac{20+x}{\sqrt{3}} \dots(2)$$

$$\text{From (1) and (2), we have } x\sqrt{3} = \frac{20+x}{\sqrt{3}}$$

$$\Rightarrow 3x = 20 + x \text{ or } 2x = 20 \Rightarrow x = 10$$

$$\text{From (1), } h = 10\sqrt{3} \text{ m}$$

**Q12.** From the top of a 7 m high building, the angle of elevation of the top of a cable tower is  $60^\circ$  and the angle of depression of its foot is  $45^\circ$ . Determine the height of the tower.

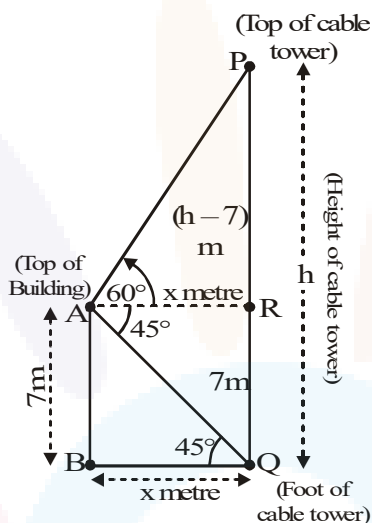
**Sol.** Let  $PQ = h$  metres be the height of the cable tower.



$AB = 7$  metres is the height of the building

$\angle PAR = 60^\circ$  is the angle of elevation of the top of the cable tower from the top of the building.

$\angle RAQ = 45^\circ$  is the angle of depression of the foot of the cable tower from the top of the building. Then  $\angle AQB = 45^\circ$ .



Now,  $BQ = AR = x$  metres (say)

$$\text{From } \triangle AQB, \frac{AB}{BQ} = \tan 45^\circ \Rightarrow \frac{7}{x} = 1 \Rightarrow x = 7 \text{ m}$$

$$\text{Now, from } \triangle PAR, \frac{PR}{AR} = \tan 60^\circ \Rightarrow \frac{PQ - QR}{x} = \sqrt{3}$$

$$\Rightarrow \frac{h-7}{x} = \sqrt{3} \Rightarrow \frac{h-7}{7} = \sqrt{3} \Rightarrow h = 7(\sqrt{3} + 1)$$

Hence, the height of the cable tower is  $7(\sqrt{3} + 1)$  metre.

**Q13.** As observed from the top of a 75m high lighthouse from the sea-level, the angles of depression of two ships are  $30^\circ$  and  $45^\circ$ . If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.

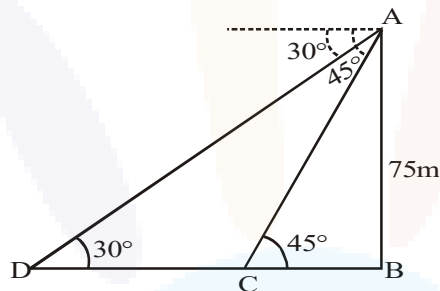
**Sol.** In the figure, let AB represent the light house.

$$\therefore AB = 75 \text{ m}$$

Let the two ships be C and D such that angle of depression from A are  $45^\circ$  and  $30^\circ$  respectively.

Now, in right  $\triangle ABC$ , we have

$$\frac{AB}{BC} = \tan 45^\circ$$



$$\Rightarrow \frac{75}{BC} = 1 \Rightarrow BC = 75$$

Again, in right  $\triangle ABD$ , We have

$$\frac{AB}{BD} = \tan 30^\circ \Rightarrow \frac{75}{BD} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow BD = 75\sqrt{3}$$

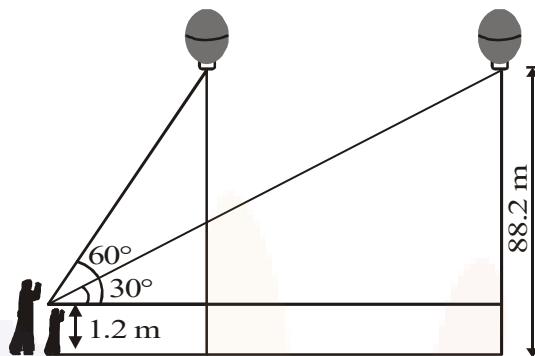
Since the distance between the two ships

$$= CD = BD - BC = 75\sqrt{3} - 75 = 75[\sqrt{3} - 1]$$

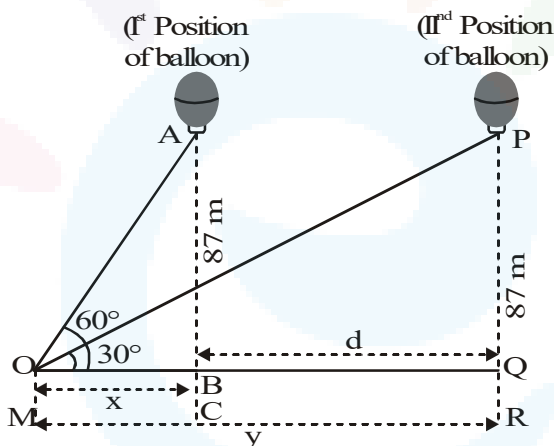
$$= 75[1.732 - 1] = 75 \times 0.732 = 54.9$$

Thus, the required distance between the ships is 54.9 m.

**Q14.** A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is  $60^\circ$ . After some time, the angle of elevation reduces to  $30^\circ$ . Find the distance travelled by the balloon during the interval.



**Sol.** From figure, we have  $\angle POQ = 30^\circ$  is the angle of elevation for the first position of the balloon. Let  $OQ = y$  m.



We are given that

$$AC = 88.2 \text{ m.}, AB = 88.2 - 1.2 = 87 \text{ m}$$

For the second position of the balloon, we have

$$\angle POQ = 30^\circ. \text{ Let } OB = x \text{ m.}$$

We have to find  $d = BQ = (y - x)$

$$\frac{AB}{OB} = \tan 60^\circ \text{ and } \frac{PQ}{OQ} = \tan 30^\circ$$

$$\Rightarrow \frac{87}{x} = \sqrt{3} \text{ and } \frac{87}{y} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow x = \frac{87}{\sqrt{3}} \text{ m and } y = 87\sqrt{3} \text{ m}$$

$$\text{Then } d = y - x = \left\{ 87\sqrt{3} - \frac{87}{\sqrt{3}} \right\} \text{ m}$$

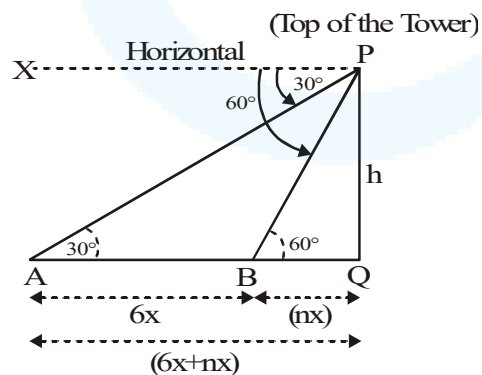
$$= 87 \times \left\{ \sqrt{3} - \frac{1}{\sqrt{3}} \right\} \text{ m}$$

$$= 87 \times \frac{2}{\sqrt{3}} \text{ m} = 87 \times \frac{2}{3} \times \sqrt{3} \text{ m}$$

$$= \frac{174}{3} \sqrt{3} \text{ m} = 58\sqrt{3} \text{ m}$$

**Q15.** A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of  $30^\circ$ , which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be  $60^\circ$ . Find the time taken by the car to reach the foot of the tower.

**Sol.** Let  $PQ = h$  metres be the height of the tower. P is the top of the tower. PX is horizontal line through P. The first and second positions of the car are at A and B respectively.



$$\angle APX = 30^\circ$$

(Angle of depression of the car when observed at A)



and  $\angle BPX = 60^\circ$

(Angle of depression of the car when observed at B)

Then  $\angle PAQ = 30^\circ$  and  $\angle PBQ = 60^\circ$

Let the speed of the car be  $x$  m/second

Then distance  $AB = 6x$  metres.

Let the time taken from B to Q be  $n$  second.

Then distance  $BQ = nx$  metres.

In right  $\triangle PAQ$ ,

|                                                          |                                           |
|----------------------------------------------------------|-------------------------------------------|
| In right $\triangle PAQ$                                 | In right $\triangle PBQ$                  |
| $\frac{h}{6x + nx} = \tan 30^\circ = \frac{1}{\sqrt{3}}$ | $\frac{h}{nx} = \tan 60^\circ = \sqrt{3}$ |
| $\Rightarrow h = \frac{(n+6)x}{\sqrt{3}} \dots(1)$       | $\Rightarrow h = nx\sqrt{3} \dots(2)$     |

From (1) and (2), we have

$$\frac{(n+6)x}{\sqrt{3}} = nx\sqrt{3}$$

$$\Rightarrow n + 6 = n\sqrt{3} \times \sqrt{3}$$

$$\Rightarrow 3n = n + 6$$

$$\Rightarrow 2n = 6$$

$$\Rightarrow n = 3$$

Hence, the time from B to Q = 3 seconds.