CLASS X: MATHS
Chapter 4: Quadratic Solutions

Questions and Solutions | Exercise 4.1 - NCERT Books

Q1. Check whether the following are quadratic equations :
(i) $(x + 1)^2 = 2 (x - 3)$
(ii) $x^2 - 2x = (-2) (3 - x)$
(iii) $(x - 2) (x + 1) = (x - 1) (x + 3)$
(iv) $(x - 3) (2x + 1) = x(x + 5)$
(v) $(2x - 1) (x - 3) = (x + 5) (x - 1)$
(vi) $x^2 + 3x + 1 = (x - 2)^2$
(vii) $(x + 2)^3 = 2x(x^2 - 1)$
(viii) $x^3 - 4x^2 - x + 1 = (x - 2)^3$

Sol. (i) $(x + 1)^2 = 2 (x - 3)$
$\Rightarrow x^2 + 2x + 1 = 2x - 6$
$\Rightarrow x^2 + 2x - 2x + 1 + 6 = 0$
$\Rightarrow x^2 + 0x + 7 = 0$
It is of the form $ax^2 + bx + c = 0$.
Hence, the given equation is a quadratic equation.

(ii) $x^2 - 2x = (-2)(3 - x)$
$\Rightarrow x^2 - 2x = -6 + 2x$
$\Rightarrow x^2 - 4x + 6 = 0$
It is of the form $ax^2 + bx + c = 0$.
Hence, the given equation is a quadratic equation.

(iii) $(x - 2) (x + 1) = (x - 1) (x + 3)$
$\Rightarrow x^2 + x - 2x - 2 = x^2 + 3x - x - 3$
$\Rightarrow x^2 - x - 2 = x^2 + 2x - 3$
$\Rightarrow - x - 2x - 2 + 3 = 0$
$\Rightarrow - 3x + 1 = 0$ or $3x - 1 = 0$
It is not of the form $ax^2 + bx + c = 0$
Hence, the given equation is not a quadratic equation.

(iv) $(x - 3)(2x + 1) = x(x + 5)$

$\Rightarrow\ 2x^2 - 5x - 3 = x^2 + 5x$

$\Rightarrow\ x^2 - 10x - 3 = 0$

It is of the form $ax^2 + bx + c = 0$

Hence, the given equation is a quadratic equation.

(v) $(2x - 1)(x - 3) = (x + 5)(x - 1)$

$\Rightarrow\ 2x^2 - 7x + 3 = x^2 + 4x - 5$

$\Rightarrow\ x^2 - 11x + 8 = 0$

It is of the form $ax^2 + bx + c = 0$.

Hence, the given equation is a quadratic equation.

(vi) $x^2 + 3x + 1 = (x - 2)^2$

$\Rightarrow\ x^2 + 3x + 1 = x^2 + 4 - 4x$

$\Rightarrow\ 7x - 3 = 0$

It is not of the form $ax^2 + bx + c = 0$.

Hence, the given equation is not a quadratic equation.

(vii) $(x + 2)^3 = 2x(x^2 - 1)$

$\Rightarrow\ x^3 + 3 \times x \times 2\,(x + 2) + 2^3 = 2x\,(x^2 - 1)$

$\Rightarrow\ x^3 + 6x\,(x + 2) + 8 = 2x^3 - 2x$

$\Rightarrow\ x^3 + 6x^2 + 12x + 8 = 2x^3 - 2x$

$\Rightarrow\ -x^3 + 6x^2 + 14x + 8 = 0$

$\Rightarrow\ x^3 - 6x^2 - 14x - 8 = 0$

It is a cubic equation and not a quadratic equation.

(viii) $x^3 - 4x^2 - x + 1 = (x - 2)^3$

$\Rightarrow\ x^3 - 4x^2 - x + 1 = x^3 - 8 - 6x^2 + 12x$

$\Rightarrow\ 2x^2 - 13x + 9 = 0$

It is of the form $ax^2 + bx + c = 0$.

Hence, the given equation is a quadratic equation.

Q2. Represent the following situations in the form of quadratic equations :

(i) The area of a rectangular plot is 528 m^2. The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.

(ii) The product of two consecutive positive integers is 306. We need to find the integers.

(iii) Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.

(iv) A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/hr less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.

Sol. (i) Let breadth be = x meters

Then length = (2x + 1) meters.

$x \times (2x + 1) = 528$ (Area of the plot)

or $2x^2 + x - 528 = 0$

(ii) Let the consecutive integers be x and x + 1. It is given that their product is 306.

$\therefore \quad x(x + 1) = 306$

$\Rightarrow \quad x^2 + x - 306 = 0$

(iii) Let Rohan's present age = x years

Then present age of Rohan's mother

= (x + 26) years

After 3 years, it is given that

$(x + 3) \times \{(x + 26) + 3\} = 360$

or $(x + 3)(x + 29) = 360$

$\Rightarrow \quad x^2 + 32x + 87 = 360$

$\Rightarrow \quad x^2 + 32x + 87 - 360 = 0$

$\Rightarrow \quad x^2 + 32x - 273 = 0$

(iv) Let the speed of train be x km/h.

Time taken to travel 480 km $= \frac{480}{x}$ hrs

In second condition,
let the speed of train = (x – 8) km/h
It is also given that the train will take 3 hours more to cover the same distance.

Therefore, time taken to travel 480 km = $\left(\frac{480}{x}+3\right)$ hrs

Speed × Time = Distance

$$(x-8)\left(\frac{480}{x}+3\right)=480$$

$$\Rightarrow 480+3x-\frac{3840}{x}-24=480$$

$$\Rightarrow 3x-\frac{3840}{x}=24$$

$\Rightarrow 3x^2 - 24x + 3840 = 0$
$\Rightarrow x^2 - 8x + 1280 = 0$

Questions and Solutions | Exercise 4.2 - NCERT Books

Q1. Find the roots of the following quadratic equations by factorisation :
(i) $x^2 - 3x - 10 = 0$
(ii) $2x^2 + x - 6 = 0$
(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$
(iv) $2x^2 - x + \frac{1}{8} = 0$
(v) $100x^2 - 20x + 1 = 0$

Sol. (i) $x^2 - 3x - 10 = 0$
$\Rightarrow x^2 - 5x + 2x - 10 = 0$
$\Rightarrow x(x-5) + 2(x-5) = 0$
$\Rightarrow (x+2)(x-5) = 0$
$\Rightarrow x + 2 = 0$ or $x - 5 = 0$
$\Rightarrow x = -2$ or $x = 5$
Hence, the two roots are – 2 and 5.

(ii) $2x^2 + x - 6 = 0$

$\Rightarrow 2x^2 + 4x - 3x - 6 = 0$

$\Rightarrow 2x(x + 2) - 3(x + 2) = 0$

$\Rightarrow (x + 2)(2x - 3) = 0$

$\Rightarrow x + 2 = 0$ or $2x - 3 = 0$

$\Rightarrow x = -2$ or $x = \frac{3}{2}$

(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$\Rightarrow \sqrt{2}x^2 + 5x + 2x + 5\sqrt{2} = 0$

$\Rightarrow x(\sqrt{2}x + 5) + \sqrt{2}(\sqrt{2}x + 5) = 0$

$\Rightarrow (x + \sqrt{2})(\sqrt{2}x + 5) = 0$

$\Rightarrow x = -\sqrt{2}$ or $-\frac{5}{\sqrt{2}}$

Hence, the two roots are $-\sqrt{2}$ and $-\frac{5}{\sqrt{2}}$

(iv) $2x^2 - x + \frac{1}{8} = 0$

or $16x^2 - 8x + 1 = 0$

or $(4x - 1)^2 = 0$

$\Rightarrow$ Both roots are given by $4x - 1 = 0$,

i.e., $x = \frac{1}{4}$. Hence, the roots are $\frac{1}{4}, \frac{1}{4}$.

(v) $100x^2 - 20x + 1 = 0$

$\Rightarrow 100x^2 - 10x - 10x + 1 = 0$

$\Rightarrow 10x(10x - 1) - 1(10x - 1) = 0$

$\Rightarrow (10x - 1)^2 = 0$

$\Rightarrow (10x - 1) = 0$ or $(10x - 1) = 0$

$\Rightarrow x = \frac{1}{10}$ or $x = \frac{1}{10}$

Q2. Solve the problem given in example 1.
(i) - x2 – 45x + 324 = 0 (ii) x2 – 55x + 750 = 0

Sol (i) We found the equation as x2 – 45x + 324 = 0 We factorize by splitting the middle term method x2 – 9x – 36x + 324 = 0 x (x – 9) – 36 (x – 9) = 0 (x – 36) (x – 9) = 0 Thus, x = 36 & x = 9 are the roots of equation

Sol (ii) We found the equation as x2 – 55x + 750 = 0 We factorize this by splitting the middle term method x2 – 30x – 25 x + 750 = 0 x (x – 30) – 25 (x – 30) = 0 (x – 25) (x – 30) = 0 Hence, 25 & 30 are the roots of the equation

Q3. Find two numbers whose sum is 27 and product is 182.

Sol. Let one number be x, then second number = 27 – x

$x \times (27 - x) = 182$

$\Rightarrow 27x - x^2 = 182$

$\Rightarrow x^2 - 27x + 182 = 0$

$\Rightarrow x^2 - 14x - 13x + 182 = 0$

$\Rightarrow x(x - 14) - 13(x - 14) = 0$

$\Rightarrow (x - 13)(x - 14) = 0$

$\Rightarrow x = 13 \text{ or } 14$

$\Rightarrow 27x = 14 \text{ or } 13$

Hence, the two marbles are 13 and 14.

Q4. Find two consecutive positive integers, sum of whose squares is 365.

Sol. Let the consecutive positive integers be x and x + 1.

Given that $x^2 + (x + 1)^2 = 365$

$\Rightarrow x^2 + x^2 + 1 + 2x = 365$

$\Rightarrow 2x^2 + 2x - 364 = 0$

$\Rightarrow x^2 + x - 182 = 0$

$\Rightarrow x^2 + 14x - 13x - 182 = 0$

$\Rightarrow$ $x(x + 14) - 13(x + 14) = 0$

$\Rightarrow$ $(x + 14)(x - 13) = 0$

Either $x + 14 = 0$ or $x - 13 = 0$,

i.e., $x = -14$ or $x = 13$

Since the integers are positive, x can only be 13.

$\therefore$ $x + 1 = 13 + 1 = 14$

Therefore, two consecutive positive integers will be 13 and 14.

Q5. The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

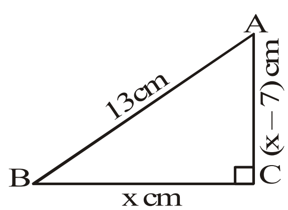
Sol. In ΔABC, base $BC = x$ cm

and altitude $AC = (x - 7)$ cm

$\angle ACB = 90°$

$AB = 13$ cm

By Pythagoras theorem, we have



$BC^2 + AC^2 = AB^2$

$\Rightarrow$ $x^2 + (x - 7)^2 = 13^2$

$\Rightarrow$ $x^2 + x^2 - 14x + 49 = 169$

$\Rightarrow$ $2x^2 - 14x - 120 = 0$

$\Rightarrow$ $x^2 - 7x - 60 = 0$

$\Rightarrow$ $x^2 - 12x + 5x - 60 = 0$

$\Rightarrow$ $x (x - 12) + 5 (x - 12) = 0$

$\Rightarrow$ $(x + 5) (x - 12) = 0$

$\Rightarrow$ x = – 5 or x = 12

We reject x = – 5

$\Rightarrow$ x = 12

Therefore, BC = 12 cm and AC = 5 cm.

Q6. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production of that day was Rs. 90, find the number of articles produced and the cost of each article.

Sol. Let the number of articles produced be x.

Therefore, cost of production of each article = Rs (2x + 3)

It is given that the total production is Rs 90.

$\therefore$ x(2x + 3) = 90

$\Rightarrow 2x^2 + 3x - 90 = 0$

$\Rightarrow 2x^2 + 15x - 12x - 90 = 0$

$\Rightarrow x(2x + 15) - 6(2x + 15) = 0$

$\Rightarrow (2x + 15)(x - 6) = 0$

Either 2x + 15 = 0 or x – 6 = 0,

i.e., $x = \frac{-15}{2}$ or x = 6

As the number of articles produced can only be a positive integer, therefore, x can only be 6.

Hence, number of articles produced = 6

Cost of each article = 2 × 6 + 3 = Rs. 15

Questions and Solutions | Exercise 4.3 - NCERT Books

Q1. Find the nature of the roots of the following quadratic equations. If the real roots exist, find them :

(i) $2x^2 - 3x + 5 = 0$

(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$

(iii) $2x^2 - 6x + 3 = 0$

Sol. (i) $2x^2 - 3x + 5 = 0$

$a = 2, b = -3, c = 5$

Discriminant $D = b^2 - 4ac = 9 - 4 \times 2 \times 5$

$= 9 - 40 = -31$

$\Rightarrow D < 0$

Hence, no real root.

(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$

$a = 3, b = -4\sqrt{3}, c = 4$

Discriminant $D = b^2 - 4ac$

$= (-4\sqrt{3})^2 - 4(3)(4) = 48 - 48 = 0$

$\Rightarrow D = 0$

$\Rightarrow$ Two roots are equal.

The roots are

$$= \frac{-b \pm \sqrt{D}}{2a} = \frac{4\sqrt{3} \pm 0}{2 \times 3} = \frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}$$

Hence, the roots are $\frac{2}{\sqrt{3}}$ and $\frac{2}{\sqrt{3}}$.

(iii) $2x^2 - 6x + 3 = 0$

$a = 2, b = -6, c = 3$

Discriminant $D = b^2 - 4ac = (-6)^2 - 4(2)(3)$

$= 36 - 24 = 12$

As $D > 0$,

Therefore, roots are distinct and real.

The roots are

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$= \frac{6 \pm \sqrt{12}}{4} = \frac{6 \pm 2\sqrt{3}}{4} = \frac{3 \pm \sqrt{3}}{2}$$

Therefore, the roots are $\frac{3 + \sqrt{3}}{2}$ or $\frac{3 - \sqrt{3}}{2}$.

Q2. Find the values of k for each of the following quadratic equations, so that they have two real equal roots.
(i) $2x^2 + kx + 3 = 0$
(ii) $kx(x - 2) + 6 = 0$

Sol. (i) $2x^2 + kx + 3 = 0$
$a = 2, b = k, c = 3$
$D = b^2 - 4ac = k^2 - 4 \times 2 \times 3 = k^2 - 24$
Two roots will be equal
if $D = 0$, i.e., if $k^2 - 24 = 0$
i.e., if $k^2 - 24$, i.e., if $k = \pm\sqrt{24}$
i.e., if $k = \pm 2\sqrt{6}$

(ii) $kx(x - 2) + 6 = 0$
or $kx^2 - 2kx + 6 = 0$
$a = k, b = -2k, c = 6$
Discriminant $D = b^2 - 4ac = (-2k)^2 - 4(k)(6)$
$= 4k^2 - 24k$
Two roots will be equal
if $D = 0$,
$4k^2 - 24k = 0$
$4k(k - 6) = 0$
Either $4k = 0$ or $k - 6 = 0$
$k = 0$ or $k = 6$
However, if $k = 0$, then the equation will not have the terms 'x^2' and 'x'.
Hence $k = 6$.

Q3. Is it possible to design a rectangular mango grove whose length is twice its breadth, and area is $800\ m^2$? If so, find its length and breadth.

Sol. Let x be the breadth and 2x be the length of the rectangle.

$x \times 2x = 800$

$\Rightarrow 2x^2 = 800$

$\Rightarrow x^2 = 400 = (20)^2$

$\Rightarrow x = 20$

Hence, the rectangle is possible and it has breadth = 20 m and length = 40 m.

Q4. Is the following situation possible? If so, determine their present ages.
The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in year was 48.

Sol. Let the age of one friend be x years.
Age of the other friend will be (20 – x) years.
4 years ago, age of 1st friend = (x – 4) years
And, age of 2nd friend = (20 – x – 4)
= (16 – x) years
Given that,
$(x - 4)(16 - x) = 48$
$16x - 64 - x^2 + 4x = 48$
$-x^2 + 20x - 112 = 0$
$x^2 - 20x + 112 = 0$
$a = 1, b = -20, c = 112$
Discriminant $D = b^2 - 4ac = (-20)^2 - 4(1)(112)$
$= 400 - 448 = -48$
As $b^2 - 4ac < 0$,
Therefore, no real root is possible for this equation and hence, this situation is not possible.

Q5. Is it possible to design a rectangular park of perimeter 80 m and area 400 m^2 ? If so, find its length and breadth.

Sol. Perimeter of the rectangular park = 80 m

$\Rightarrow$ Length + Breath of the park = $\frac{80}{2}$ m = 40 m.

Let the breadth be x metres, then length

= (40 – x) m.

Here, x < 40.

$x \times (40 - x) = 400$ [Each = area of the park]

i.e., $-x^2 + 40x - 400 = 0$

i.e., $x^2 - 40x + 400 = 0$

i.e., $(x - 20)^2 = 0$

$\Rightarrow$ x = 20

Thus, we have length = breadth = 20 m

Therefore, the park is a square having 20 m side.