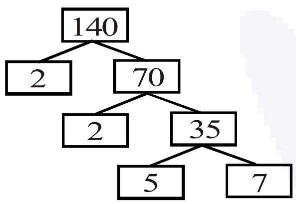
CLASS X: MATHS
Chapter 1: Real Numbers

Questions and Solutions | Exercise 1.1 - NCERT Books

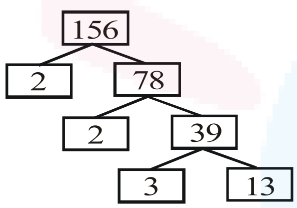
Q1. Express each number as product of its prime factors :
(i) 140 (ii) 156(iii) 3825 (iv) 5005 (v) 7429

Sol. (i) 140



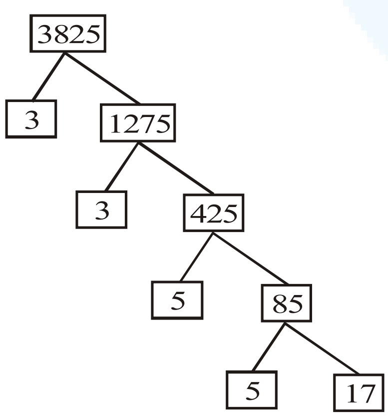
So, $140 = 2 \times 2 \times 5 \times 7 = 2^2 \times 5 \times 7$

(ii) 156



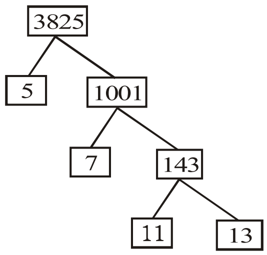
So, $156 = 2 \times 2 \times 3 \times 13 = 2^2 \times 3 \times 13$

(iii)



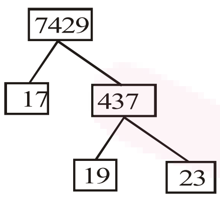
So, $3825 = 3^2 \times 5^2 \times 17$

(iv) 5005



So, 5005 = 5 × 7 × 11 × 13

(v) 7429

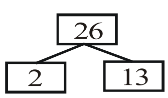


So, 7429 = 17 × 19 × 23

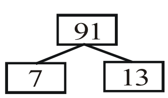
Q2. Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of two numbers.

(i) 26 and 91 (ii) 510 and 92 (iii) 336 and 54

Sol. (i) 26 and 91



So, 26 = 2 × 13



So, 91 = 7 × 13

Therefore,

LCM (26, 91) = 2 × 7 × 13 = 182

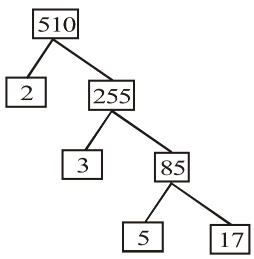
HCF (26, 91) = 13

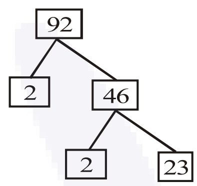
Verification : LCM × HCF = 182 × 13 = 2366

and 26 × 91 = 2366

i.e., LCM × HCF = product of two numbers.

(ii) 510 and 92





$510 = 2 \times 3 \times 5 \times 17, 92 = 2^2 \times 23$

LCM (510, 92) = $2^2 \times 3 \times 5 \times 17 \times 23 = 23460$

HCF = (510, 92) = 2

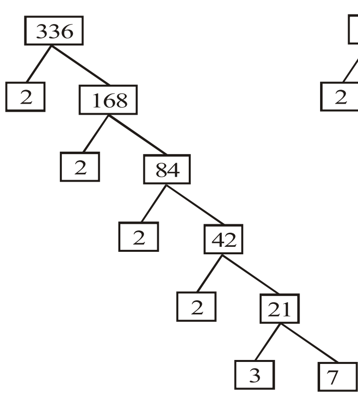
Verification :–

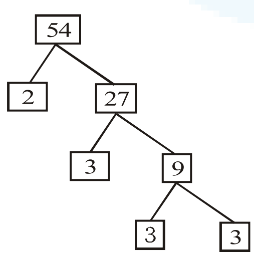
LCM × HCF = 23460 × 2 = 46920

and 510 × 92 = 46920

i.e., LCM × HCF = product of two numbers.

(iii) 336 and 54





$336 = 2^4 \times 3 \times 7$ $54 = 2 \times 3^3$

LCM $= 2^4 \times 3^3 \times 7 = 3024$

HCF $= 2 \times 3 = 6$

Verfication,

LCM $\times$ HCF $= 2^4 \times 3^3 \times 7 \times 2 \times 3 = 18144$

Product of two numbers $= 336 \times 54 = 18144$

i.e., LCM $\times$ HCF = product of two numbers.

Q3. Find the LCM and HCF of the following integers by applying the prime factorisation method.
(i) 12, 15 and 21 (ii) 17, 23 and 29 (iii) 8, 9 and 25

Sol. (i) 12, 15 and 21

So, $12 = 2 \times 2 \times 3 = 2^2 \times 3$

So, $15 = 3 \times 5$

So, $21 = 3 \times 7$

Therefore,

HCF (12, 15, 21) = 3 ;

LCM = (12, 15, 21) $= 2^2 \times 3 \times 5 \times 7 = 420$

(ii) 17, 23, 29

$17 = 1 \times 17$

$23 = 1 \times 23$

$29 = 1 \times 29$

LCM $= 1 \times 17 \times 23 \times 29$

HCF = 1

(iii) 8, 9, 25

$8 = 2 \times 2 \times 2$

$9 = 3 \times 3$

$25 = 5 \times 5$

LCM = $2^3 \times 3^2 \times 5^2$

HCF = 1

Q4. Given that HCF (306, 657) = 9, find LCM (306, 657).

Sol. LCM (306, 657)

$$= \frac{306 \times 657}{\text{HCF}(306, 657)} = \frac{306 \times 657}{9} = 22338.$$

Q5. Check whether 6^n can end with the digit 0 for any natural number n.

Sol. If the number 6^n, for any natural number n, ends with digit 0, then it would be divisible by 5. That is, the prime factorisation of 6^n would contain the prime number 5. This is not possible because $6^n = (2 \times 3)^n = 2^n \times 3^n$; so the only primes in the factorisation of 6^n are 2 and 3 and the uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorisation of 6^n. So, there is no natural number n for which 6^n ends with the digit zero.

Q6. Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

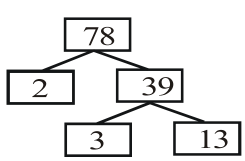
Sol. (i) $7 \times 11 \times 13 + 13 = (7 \times 11 + 1) \times 13$

$= (77 + 1) \times 13$

$= 78 \times 13 = (2 \times 3 \times 13) \times 13$

So, $78 = 2 \times 3 \times 13$

$78 \times 13 = 2 \times 3 \times 13^2$



Since, $7 \times 11 \times 13 + 13$ can be expressed as a product of primes, therefore, it is a composite number.

(ii) $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$

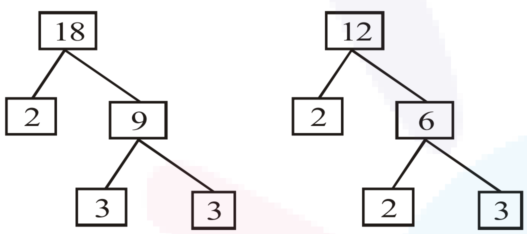
$= (7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1) \times 5$

$= 1009 \times 5$

Since, $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ can be expressed as a product of primes, therfore it is a composite number.

Q7. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

Sol. LCM of 18 & 12.



$18 = 2 \times 3^2$ $\qquad$ $12 = 2^2 \times 3$

LCM $(18, 12) = 2^2 \times 3^2 = 36$

Thus, after 36 minutes they will meet again at the starting point.

Questions and Solutions | Exercise 1.2 - NCERT Books

Q1. Prove that $\sqrt{5}$ is irrational.

Sol. Let us assume, to the contrary, that $\sqrt{5}$ is rational.

So, we can find coprime integers a and b ($\neq 0$) such that

$$\sqrt{5} = \frac{a}{b}$$

$\Rightarrow \sqrt{5}\, b = a$

Squaring on both sides, we get

$$5b^2 = a^2$$

Therefore, 5 divides a^2.

Therefore, 5, divides a

So, we can write a = 5c for some integer c.
Substituting for a, we get

$5b^2 = 25c^2$

$\Rightarrow\ b^2 = 5c^2$

This means that 5 divides b^2, and so 5 divides b.
Therefore, a and b have at least 5 as a common factor.
But this contradicts the fact that a and b have no common factor other than 1.

This contradiction arose because of our incorrect assumption that $\sqrt{5}$ is rational.

So, we conclude that $\sqrt{5}$ is irrational.

Q2. Prove that $3 + 2\sqrt{5}$ is irrational.

Sol. Let us assume, to the contrary, that $3 + 2\sqrt{5}$ is rational. That is, we can find coprime integers a and b (b ≠ 0) such that $3 + 2\sqrt{5} = \frac{a}{b}$

Therefore, $\frac{a}{b} - 3 = 2\sqrt{5}$

$\Rightarrow \frac{a-3b}{b} = 2\sqrt{5}$

$\Rightarrow \frac{a-3b}{2b} = \sqrt{5} \Rightarrow \frac{a}{2b} - \frac{3}{2} = \sqrt{5}$

Since a and b are integers, we get $\frac{a}{2b} - \frac{3}{2}$ is rational, and so $\frac{a-3b}{2b} = \sqrt{5}$ is rational.

But this contradicts the fact that $\sqrt{5}$ is irrational. This contradiction has arisen because of our incorrect assumption that $3 + 2\sqrt{5}$ is rational.
So, we conclude that $3 + 2\sqrt{5}$ is irrational.

Q3. Prove that the following are irrationals :

(i) $\frac{1}{\sqrt{2}}$ (ii) $7\sqrt{5}$ (iii) $6 + \sqrt{2}$

Sol. (i) Let us assume, to the contrary, that $\frac{1}{\sqrt{2}}$ is rational. That is we can find coprime integers a and b (b ≠ 0) such that,

$\frac{1}{\sqrt{2}} = \frac{p}{q}$

Therefore, $q = \sqrt{2}p$

Squaring on both sides, we get

$q^2 = 2p^2$ ...(i)

Therefore, 2 divides q^2

so, 2 divides q

so we can write q = 2r for some integer r

squaring both sides, we get

$q^2 = 4r^2$ ...(ii)

From (i) & (ii), we get

$2p^2 = 4r^2$

$p^2 = 2r^2$

Therefore, 2 divides p^2

So, 2 divides p

So, p & q have atleast 2 as a common factor.

But this contradict the fact that p & q have no common factor other than 1.

This contradict our assumption that $\frac{1}{\sqrt{2}}$ is rational. So, we condude that $\frac{1}{\sqrt{2}}$ is irrational.

(ii) Let us assume, to the contrary, that $7\sqrt{5}$ is rational.

That is, we can find coprime integers a and b (b ≠ 0) such that $7\sqrt{5} = \frac{a}{b}$

Therefore, $\frac{a}{7b} = \sqrt{5}$

Since a and b are integers, we get $\frac{a}{7b}$ is rational, and so $\frac{a}{7b} = \sqrt{5}$ is rational.

But this contradicts the fact that $\sqrt{5}$ is irrational. This contradiction has arisen because of our incorrect assumption that $7\sqrt{5}$ is rational.

So, we conclude that $7\sqrt{5}$ is irrational.

(iii) Let us assume, to the contrary, that $6 + \sqrt{2}$ is rational.

That is, we can find coprime integers a and b (b ≠ 0) such that $6 + \sqrt{2} = \frac{a}{b}$

Therefore, $\frac{a}{b} - 6 = \sqrt{2}$

$\Rightarrow \frac{a-6b}{b} = \sqrt{2}$

Since a and b are integers, we get $\frac{a}{b} - 6$ is rational, and so $\frac{a-6b}{b} = \sqrt{2}$ is rational.

But this contradicts the fact that $\sqrt{2}$ is irrational. This contradiction has arisen because of our incorrect assumption that

$6+\sqrt{2}$ is rational.

So, we conclude that $6+\sqrt{2}$ is irrational.