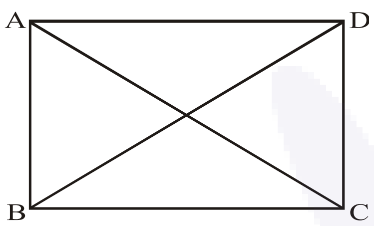
CLASS IX: MATHS

Chapter 8: Quadrilaterals

Questions and Solutions | Exercise 8.1 - NCERT Books

Q1. If the diagonals of a parallelogram are equal, then show that it is a rectangle.

Sol. **Given :** ABCD is a parallelogram with diagonal AC = diagonal BD



To prove : ABCD is a rectangle.

Proof : In triangle ABC and ABD,

	AB = AB	[Common]
	AC = BD	[Given]
	AD = BC	[Opp. Sides of a ||gm]
∴	ΔABC ≅ BAD	[By SSS congruency]
⇒	∠DAB = ∠CBA	[By C.P.C.T.] ....(i)

[∵ AD||BC and AB cuts them, the sum of the interior angle of the same side of transversal is 180°]

∠DAB + ∠CBA = 180° .....(ii)

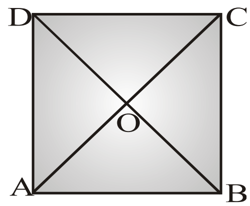
From eq. (i) and (ii), ∠DAB = ∠CBA = 90°

Hence, ABCD is a rectangle

Q2. Show that the diagonals of a square are equal and bisect each other at right angles.

Sol. **Given:** ABCD is a square.

To Prove : (i) AC = BD (ii) AC and BD bisect each other at right angles.



Proof: In ΔABC and ΔBAD,

$AB = BA$ [Common]

$BC = AD$ [Opp. sides of square ABCD]

$\angle ABC = \angle BAD$ [Each = 90° ($\because$ ABCD is a square]

$\therefore$ $\Delta ABC \cong \Delta BAD$ [SAS Rule]

$\therefore$ $AC = BD$ ... (i) [C.P.C.T.]

In ΔAOD and ΔBOC

$AD = CB$ [Opp. sides of square ABCD]

$\angle OAD = \angle OCB$

[Alternate angles as $AD \parallel BC$ and transversal AC intersects them]

$\angle ODA = \angle OBC$

[Alternate angles as $AD \parallel BC$ and transversal BD intersects them]

$\Delta AOD \cong \Delta BOC$ [ASA Rule]

$\therefore$ $OA = OC$ and $OB = OD$ ...(ii) [C.P.C.T.]

So, O is the mid point of AC and BD.

Now, In ΔAOB and ΔCOB

$AB = BC$ [Given]

$OA = OC$ [from (ii)]

$OB = OB$ [Common]

$\therefore$ $\Delta AOB \cong \Delta COB$ [By SSS Rule]

$\therefore$ $\angle AOB = \angle BOC$ [C.P.C.T]

But $\angle AOB + \angle BOC = 180°$ [Linear pair]

$\angle AOB + \angle AOB = 180°$

[AOB = BOC proved earlier]

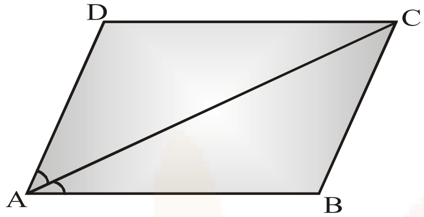
$\Rightarrow$ $2\angle AOB = 180°$

$\Rightarrow$ $\angle AOB = \frac{180°}{2} = 90°$

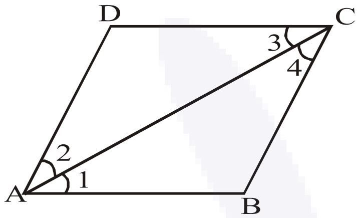
$\therefore$ $\angle AOB = \angle BOC = 90°$

$\therefore$ AC and BD bisect each other at right angles.

Q3. In figure, ABCD is a parallelogram. Diagonal AC bisects ∠A. Show that
(i) it bisects ∠C also (ii) ABCD is a rhombus.



Sol. **Given :**



Diagonal AC bisects ∠A of the parallelogram ABCD.

To prove :

(i) AC bisects ∠C

(ii) ABCD is a rhombus

Proof :

(i) Since AB||DC and AC intersects them.

∴ ∠1 = ∠3 [Alternate angles] ...(i)

Similarly ∠2 = ∠4 ...(ii)

But ∠1 = ∠2 [Given] ...(iii)

∴ ∠3 = ∠4 [Using eq. (i), (ii) and (iii)]

Thus AC bisects ∠C.

(ii) ∠2 = ∠3 = ∠4 = ∠1

⇒ AD = CD [Sides opposite to equal angles]

∴ AB = CD = AD = BC

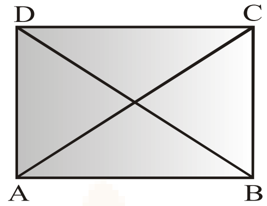
Hence, ABCD is a rhombus.

Q4. ABCD is a rectangle in which diagonal AC bisects ∠A as well as ∠C. Show that

(i) ABCD is a square

(ii) diagonal BD bisects ∠B as well as ∠D.

Sol. **Given :** ABCD is a rectangle in which diagonal AC bisects ∠A as well as ∠C.



To prove : (i) ABCD is a square

(ii) diagonal BD bisects ∠B as well as ∠D.

Proof :

(i) ∵ AB || DC and transversal AC intersects them.

so, ∠BAC = ∠DCA [Alternate angles]

But ∠BAC = ∠DAC [∵ AC bisects ∠A]

∴ ∠ DCA = ∠DAC

⇒ DA = CD

[Sides opposite to equal angles of a triangle are equal]

But AB = CD and DA = BC [Opposite side of a rectangle]

∴ AB = BC = CD = DA

Also ∠A = ∠B = ∠C = ∠D = 90°

[∵ ABCD is a rectangle]

Hence, ABCD is a square

(ii) In Δ BAD and Δ BCD,

BA = BC [∵ ABCD is a square]

AD = CD [∵ ABCD is a square]

BD = BD [Common]

∴ ΔBAD ≅ ΔBCD [By SSS congruence rule]

∴ ∠ ABD = ∠ CBD [By C.P.C.T.]

∠ADB = ∠CDB [By C.P.C.T.]

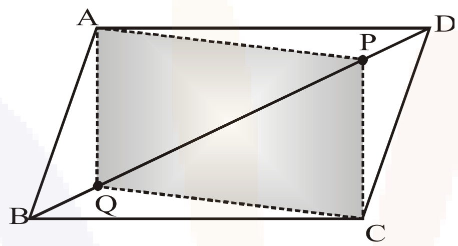
Hence, diagonal BD bisect ∠B as well as ∠D

Q5. In parallelogram ABCD, two points P and Q are taken on diagonal BD such that DP = BQ. Show that :

(i) $\Delta APD \cong \Delta CQB$ (ii) AP = CQ

(iii) $\Delta AQB \cong \Delta CPD$ (iv) AQ = CP

(v) APCQ is a parallelogram



Sol. (i) In ΔAPD and ΔCQB, we have

DP = BQ [Given]

AD = CB

[Opposite sides of parallelogram ABCD]

$\angle ADP = \angle CBQ$ [Pair of alternate angles]

$\Rightarrow$ $\Delta APD \cong \Delta CQB$ [SAS congruence criteria]

(ii) Then, by CPCT, we have AP = CQ

(iii) We can prove

$\Delta AQB \cong \Delta CPD$ [as we have done in (i)]

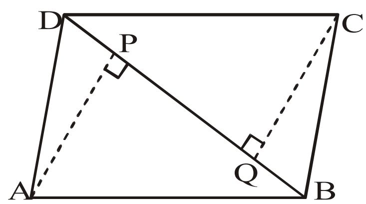
(iv) By CPCT, we have AQ = CP

(v) Now, we have AP = CQ and AQ = CP

Hence, APCQ is a parallelogram.

Q6. ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD. Show that

(i) $\Delta APB \cong \Delta CQD$ (ii) AP = CQ



Sol. **Given :** ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD respectively.

To prove : (i) $\Delta APB \cong \Delta CQD$ (ii) AP = CQ

Proof :

(i) In ΔAPB and ΔCQD,

AB = CD	[Opp. side of || gm ABCD]
$\angle ABP = \angle CDQ$	[$\therefore$ AB || DC and transversal BD intersect them]
$\angle APB = \angle CQD$	[Each = 90°]
$\therefore \Delta APB \cong \Delta CQD$	[AAS Rule]
(ii) $\therefore$ AP = CQ	[C.P.C.T.]

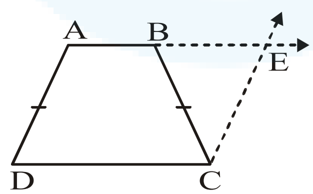
Q7. ABCD is a trapezium in which AB||CD and AD = BC. Show that (fig)

(i) $\angle A = \angle B$

(ii) $\angle C = \angle D$

(iii) $\Delta ABC \cong \Delta BAD$

(iv) diagonal AC = diagonal BD



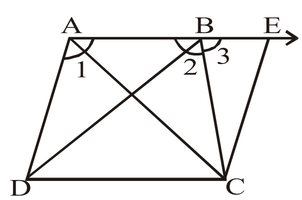
Sol. **Given :** ABCD is a trapezium.

AB || CD and AD = BC

To Prove :

(i) $\angle A = \angle B$

(ii) $\angle C = \angle D$



(iii) Δ ABC $\cong$ Δ BAD

(iv) Diagonal AC = Diagonal BD

Construction : Draw CE || AD and extend AB to intersect CE at E.

Proof :

(i) As AECD is a parallelogram. [By construction]

$\therefore$ AD = EC

But AD = BC [Given]

$\therefore$ BC = EC

$\Rightarrow$ $\angle 3 = \angle 4$ [Angles opposite to equal sides are equal]

Now, $\angle 1 + \angle 4 = 180°$ [Interior angles]

and $\angle 2 + \angle 3 = 180°$ [Linear pair]

$\Rightarrow$ $\angle 1 + \angle 4 = \angle 2 + \angle 3$

$\Rightarrow$ $\angle 1 = \angle 2$ [$\because$ $\angle 3 = \angle 4$]

$\Rightarrow$ $\angle A = \angle B$

(ii) $\angle 3 = \angle BCD$ [Alternate interior angles]

$\angle D = \angle 4$ [Opposite angles of a parallelogram]

But $\angle 3 = \angle 4$ [Δ BCE is an isosceles triangle]

$\therefore$ $\angle BCD = \angle ADC$

$\therefore$ $\angle C = \angle D$

(iii) In ΔABC and ΔBAD,

AB = AB [Common]

$\angle 1 = \angle 2$ [Proved]

AD = BC [Given]

$\therefore$ $\Delta ABC \cong \Delta BAD$ [By SAS congruency]

$\Rightarrow$ AC = BD [By C.P.C.T.]

Questions and Solutions | Exercise 8.2 - NCERT Books

Q1. ABCD is a quadrilateral in which P, Q, R and S are mid points of the sides AB, BC, CD and DA (fig.) AC is a diagonal. Show that

(i) SR||AC and SR = 1/2 C

(ii) PQ = SR

(iii) PQRS is a parallelogram.

Sol. **Given :** ABCD is a quadrilateral in which P, Q, R and S are mid-points of AB, BC, CD and DA. AC is a diagonal.

To prove : (i) SR || AC and SR = $\frac{1}{2}$ AC

(ii) PQ = SR

(iii) PQRS is a parallelogram.

Proof : (i) In ΔDAC,

∵ S is the mid-point of DA and R is the mid-point of DC

∴ SR || AC and SR = $\frac{1}{2}$ AC [By Mid-point theorem]

(ii) In ΔBAC,

∵ P is the mid-point of AB and Q is the mid-point of BC

∴ PQ || AC and PQ = $\frac{1}{2}$ AC [By Mid-point theorem]

But from (i) SR = $\frac{1}{2}$ AC & (ii) PQ = $\frac{1}{2}$ AC

⇒ PQ = SR

(iii) PQ || AC [From (ii)]

SR || AC [From (i)]

∴ PQ || SR

[Two lines parallel to the same line are parallel to each other]

Also, PQ = SR [From (ii)]

∴ PQRS is a parallelogram.

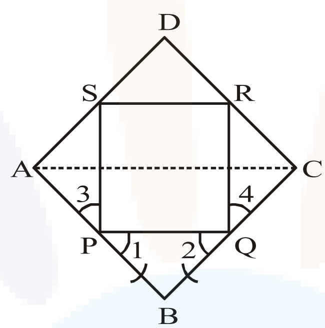
[A quadrilateral is a parallelogram if a pair of opposite sides is parallel and is of equal length]

Q2. ABCD is a rhombus and P, Q, R and S are the mid points of sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.

Sol. **Given :** P, Q, R and S are the mid-points of respective sides AB, BC, CD and DA of rhombus. PQ, QR, RS and SP are joined.

To prove : PQRS is a rectangle.

Construction : Join A and C.



Proof : In ΔABC, P is the mid-point of AB and Q is the mid-point of BC.

$\therefore$ PQ $\|$ AC and PQ = $\frac{1}{2}$ AC .....(i)

In ΔADC, R is the mid-point of CD and S is the mid-point of AD.

$\therefore$ SR $\|$ AC and SR = $\frac{1}{2}$ AC ....(ii)

From eq. (i) and (ii), PQ $\|$ SR and PQ = SR

$\therefore$ PQRS is a parallelogram.

Now ABCD is a rhombus [Given]

$\therefore$ AB = BC

$\Rightarrow$ $\frac{1}{2}AB = \frac{1}{2}BC \Rightarrow PB = BQ$

$\therefore$ $\angle 1 = \angle 2$ [Angles opposite to equal sides are equal]

Now in triangles APS and CQR, we have,

AP = CQ [P and Q are the mid-points of AB and BC and AB = BC]

Similarly, AS = CR and PS = QR

[Opposite sides of a parallelogram]

$\therefore$ $\Delta APS \cong \Delta CQR$ [By SSS congruency]

$\Rightarrow \quad \angle 3 = \angle 4$ [By C.P.C.T.]

Now, we have $\angle 1 + \angle SPQ + \angle 3 = 180°$

and $\quad \angle 2 + \angle PQR + \angle 4 = 180°$

$\therefore \quad \angle 1 + \angle SPQ + \angle 3 = \angle 2 + \angle PQR + \angle 4$

Since $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$ [Proved above]

$\therefore \quad \angle SPQ = \angle PQR$ .....(iii)

Now PQRS is a parallelogram [Proved above]

$\therefore \quad \angle SPQ + \angle PQR = 180°$ .....(iv) [Interior angles]

Using eq. (iii) and (iv),

$\angle SPQ + \angle SPQ = 180°$

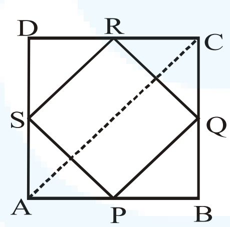
$\Rightarrow \quad 2\angle SPQ = 180° \Rightarrow \angle SPQ = 90°$

Hence, PQRS is a rectangle.

Q3. ABCD is a rectangle and P,Q,R and S are mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus.

Sol. **Given :** A rectangle ABCD in which P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. PQ, QR, RS and SP are joinned.

To prove : PQRS is a rhombus.



Construction : Join AC.

Proof : In Δ ABC, P and Q are the mid-points of sides AB, BC respectively.

$\therefore \quad PQ \parallel AC$ and $PQ = \frac{1}{2} AC$ ....(i)

In Δ ADC, R and S are the mid-points of sides CD, AD respectively.

$\therefore \quad SR \parallel AC$ and $SR = \frac{1}{2} AC$ ...(ii)

From eq.(i) and (ii), $PQ \parallel SR$ and $PQ = SR$ ...(iii)

$\therefore$ PQRS is a parallelogram.

Now ABCD is a rectangle. [Given]

$\therefore$ AD = BC

$\Rightarrow$ $\frac{1}{2}AD = \frac{1}{2}BC \Rightarrow$ AS = BQ ...(iv)

In triangles APS and BPQ,

AP = BP [P is the mid-point of AB]

$\angle PAS = \angle PBQ$ [Each 90°]

and AS = BQ [From eq. (iv)]

$\therefore$ $\Delta APS \cong \Delta BPQ$ [By SAS congruency]

$\Rightarrow$ PS = PQ [By C.P.C.T.] .....(v)

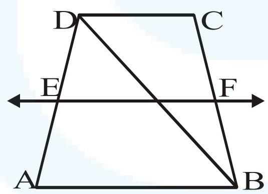
From eq.(iii) and (v), we get that PQRS is a parallelogram.

$\Rightarrow$ PS = PQ

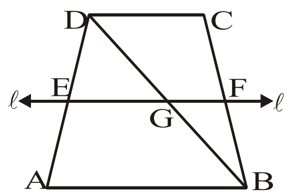
$\Rightarrow$ Two adjacent sides are equal.

Hence, PQRS is a rhombus.

Q4. ABCD is a trapezium in which AB||DC, BD is a diagonal and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F (fig.). Show that F is the mid-point of BC.



Sol. Line $\ell \parallel AB$ and passes through E.



Line ℓ meets BC in F and BD in G.

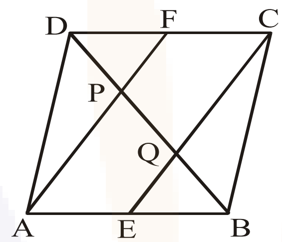
In ΔABD, E is mid-point of AD and EG || AB.

$\Rightarrow$ G is mid-point of BD.

Also, ℓ || AB and AB || CD $\Rightarrow$ ℓ || CD

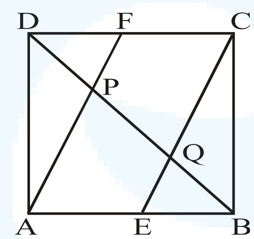
$\Rightarrow$ F is mid-point of BC. [$\because$ G is mid-point of BD]

Q5. In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively (fig.). Show that the line segments AF and EC trisect the diagonal BD.



Sol. Since E and F are the mid-points of AB and CD respectively.

Given : ABCD is a parallelogram. E and F are midpoints of AB and AC respectively.



To prove : DP = PQ = QB

Proof : -

$\therefore$ $AE = \frac{1}{2}AB$ and $CF = \frac{1}{2}CD$ ....(i)

But ABCD is a parallelogram.

$\therefore$ AB = CD and AB || DC

$\Rightarrow$ $\frac{1}{2}AB = \frac{1}{2}CD$ and AB || DC

$\Rightarrow$ AE = FC and AE || FC [From eq. (i)]

$\therefore$ AECF is a parallelogram.

⇒ FA || CE ⇒ FP || CQ

[FP is a part of FA and CQ is a part of CE] ..... (ii)

Since the line segment drawn through the mid-point of one side of a triangle and parallel to the other side bisects the third side.

In ΔDCQ, F is the mid-point of CD and

⇒ FP || CQ

∴ P is the is mid-point of DQ.

⇒ DP = PQ ....(iii)

Similarly, In ΔABP, E is the mid-point of AB and

⇒ EQ || AP

∴ Q is the mid-point of BP.

⇒ BQ = PQ ....(iv)

From eq.(iii) and (iv),

DP = PQ = BQ ....(v)

Now, BD = BQ + PQ + DP = BQ + BQ + BQ = 3BQ

⇒ $BQ = \frac{1}{3} BD$ .....(vi)

From eq (v) and (vi), $DP = PQ = BQ = \frac{1}{3} BD$

⇒ Points P and Q trisects BD. So AF and CE trisects BD.

Q6. ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that

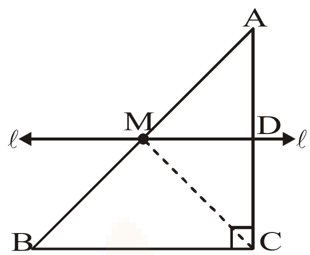
(i) D is the mid-point of AC

(ii) MD ⊥ AC

(iii) CM = MA = 1/2 AB

Sol. (i) Through M, we draw line ℓ || BC. ℓ intersects AC at D.

⇒ D is mid-point of AC.



(ii) $\angle ADM = \angle ACB = 90°$

[Corresponding angles]

$\Rightarrow$ $\angle ADM = 90°$ $\Rightarrow$ $MD \perp AC$.

(iii) In ΔCMD and ΔAMD;

$CD = AD$, $MD = MD$

and $\angle CDM = \angle ADM$ [Each = 90°]

Therefore, $\Delta CMD \cong \Delta AMD$

$\Rightarrow$ $CM = AM$; Also $AM = 1/2\ AB$.