Ex - 13.4

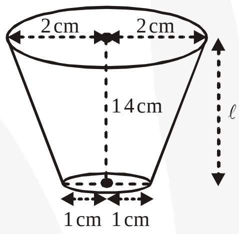
NOTE: Use $\pi = \frac{22}{7}$, unless stated otherwise.

Q1. A drinking glass is in the shape of a frustum of a cone of height 14 cm. The diameters of its two circular ends are 4 cm and 2 cm. Find the capacity of the glass.

Sol. R = 2 cm, r = 1 cm, h = 14 cm

Capacity of the glass = volume of the frustum with radii of ends as 2 cm and 1 cm and height 14 cm

$= \frac{1}{3}\pi h\,\{R^2 + r^2 + Rr\}$



$= \frac{1}{3}\pi \times 14 \times \{(2)^2 + (1)^2 + 2(1)\}\ cm^3$

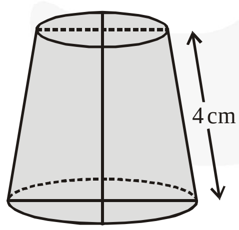
$= \frac{1}{3} \times \frac{22}{7} \times 14 \times 7\ cm^3$

$= \frac{308}{3} = 102\frac{2}{3} cm^3$

Q2. The slant height of a frustum of a cone is 4 cm and the perimeters (circumference) of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum.

Sol. We have

Slant height (ℓ) = 4 cm



$2\pi r_1 = 18$ cm and $2\pi r_2 = 6$ cm

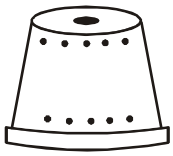
$\Rightarrow \pi r_1 = \frac{18}{2} = 9$ cm and $\pi r_2 = \frac{6}{2} = 3$ cm

∴ Curved surface area of the frustum of the cone

$= \pi(r_1 + r_2)\,\ell = (\pi r_1 + \pi r_2)\,\ell = (9 + 3) \times 4\ cm^2$

$= 12 \times 4\ cm^2 = 48\ cm^2$.

Q3. A fez, the cap used by the Turks, is shaped like the frustum of a cone (see fig.). If its radius on the open side is 10 cm, radius at the upper base is 4 cm and its slant height is 15 cm, find the area of material used for making it.



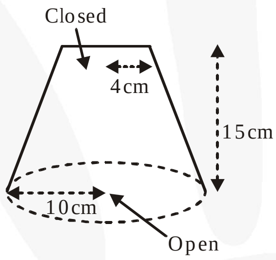
Sol. R = 10 cm, r = 4 cm, ℓ = 15 cm

Curved surface area = $\pi \times \ell \times \{R + r\}$

$= \pi \times 15 \times \{10 + 4\}\ cm^2$

$= \frac{22}{7} \times 15 \times 14\ cm^2 = 660\ cm^2$

Area of the closed side



$= \pi r^2 = \frac{22}{7} \times (4)^2$

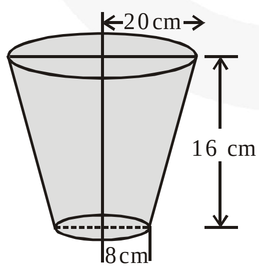
$= \frac{352}{7} = 50\frac{2}{7}\ cm^2$

total area of the material used

$= 600 + 50\ \frac{2}{7}\ cm^2 = 710\ \frac{2}{7}\ cm^2$

Q4. A container, opened from the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm, respectively. Find the cost of the milk which can completely fill the container, at the rate of Rs. 20 per litre. Also find the cost of metal sheet used to make the container, if it costs Rs. 8 per 100 cm^2. (Take π = 3.14).

Sol. We have : r_1 = 20 cm,



r_2 = 8 cm and h = 16 cm

$\therefore$ Volume of the frustum

$= \frac{1}{3}\pi h[r_1^2 + r_2^2 + r_1 r_2]$

$= \frac{1}{3} \times \frac{314}{100} \times 16[20^2 + 8^2 + 20 \times 8]\ cm^3$

$= \frac{1}{3} \times \frac{314}{100} \times 16 \times [400 + 64 + 160]\ cm^3$

$= \frac{1}{3} \times \frac{314}{100} \times 16 \times 624\ cm^3 = \left[\frac{314}{100} \times 16 \times 208\right] cm^3$

$= \left[\frac{314}{100} \times 16 \times 208\right] \div 1000$ litres

$= \frac{314 \times 16 \times 208}{100000}$ litres

$\therefore$ Cost of milk = ` 20 × $\frac{314 \times 16 \times 208}{100000}$ litres

= ` 208.998 ≈ 209

Now, slant height of the given frustum

$\ell = \sqrt{h^2 + (r_1 - r_2)^2} = \sqrt{16^2 + (20 - 8)^2}$

$= \sqrt{16^2 + 12^2} = \sqrt{256 + 144} = \sqrt{400} = 20$ cm

$\therefore$ Curved surface area = $\pi(r_1 + r_2)\ell$

$= \frac{314}{100}(20 + 8) \times 20\ cm^2$

$= \frac{314}{100} \times 28 \times 20\ cm^2 = 1758.4\ cm^2$

Area of the bottom = πr^2

$= \frac{314}{100} \times 8 \times 8\ cm^2 = 200.96\ cm^2$

$\therefore$ Total area of metal required

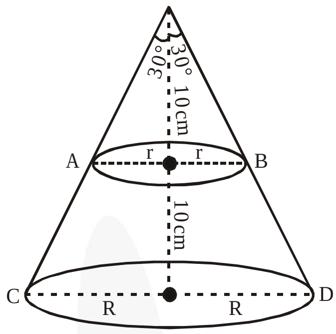
$= 1758.4\ cm^2 + 200.96\ cm^2 = 1959.36\ cm^2$

Cost of metal required = ` $\frac{8}{100} \times 1959.36\ cm^2$

= ` 156.75

Q5. A metallic right circular cone 20 cm high and whose vertical angle is 60° is cut into two parts at the middle of its height by a plane parallel to its base. If the frustum so obtained be drawn into a wire of diameter 1/16 cm, find the length of the wire.

Sol. From the figure



$$\frac{R}{20} = \tan 30° = \frac{1}{\sqrt{3}}, \text{ i.e., } R = \frac{20}{\sqrt{3}} \text{ cm}$$

$$\frac{r}{10} = \tan 30° = \frac{1}{\sqrt{3}}, \text{ i.e., } r = \frac{10}{\sqrt{3}} \text{ cm}$$

h = 10 cm is the height of the frust.

Volume of the material in the frustum ACDB

$$= \frac{1}{3}\pi \times h \times \{R^2 + r^2 + Rr\}$$

$$= \frac{1}{3}\pi \times 10 \times \left\{\frac{400}{3} + \frac{100}{3} + \frac{200}{3}\right\} \text{cm}^3$$

$$= \frac{7000}{9}\pi \text{cm}^3$$

Now let us suppose wire of diameter $\frac{1}{16}$ cm is made of length x cm.

$$\text{Then, } \pi \times \left(\frac{1}{32}\right)^2 \times x = \frac{7000}{9}\pi$$

$$x = \frac{7000 \times 32 \times 32}{9} \text{ cm}$$

$$x = \frac{71680}{9} \text{ m} = 7964.4 \text{ m}$$