CHAPTER 12

SURFACE AREAS AND VOLUMES

(A) Main Concepts and Results

- The surface area of an object formed by combining any two of the basic solids, namely, cuboid, cone, cylinder, sphere and hemisphere.
- The volume of an object formed by combining any two of the basic solids namely, cuboid, cone, cylinder, sphere and hemisphere.
- The formulae involving the frustum of a cone are:

  (i) Volume of the frustum of the cone = $\frac{1}{3}\pi h[r_1^2 + r_2^2 + r_1 r_2]$

  (ii) Curved surface area of the frustum of the cone = $\pi(r_1+r_2)l$,

  (iii) Total surface area of the frustum of the solid cone

  $= \pi l(r_1+r_2) + \pi r_1^2 + \pi r_2^2$, where $l = \sqrt{h^2 + (r_1 - r_2)^2}$,

  h = vertical height of the frustum, l = slant height of the frustum and r_1 and r_2 are radii of the two bases (ends) of the frustum.

- Solid hemisphere: If r is the radius of a hemisphere, then
  curved surface area = $2\pi r^2$
  total surface area = $3\pi r^2$, and volume = $\frac{2}{3}\pi r^3$

- Volume of a spherical shell = $\frac{4}{3}\pi\left(r_1^3 - r_2^3\right)$, where r_1 and r_2 are respectively its external and internal radii.

  Throughout this chapter, take $\pi = \frac{22}{7}$, if not stated otherwise.

(B) Multiple Choice Questions :

Choose the correct answer from the given four options:

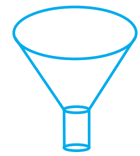

Fig. 12.1

Sample Question 1 : A funnel (see Fig.12.1) is the combination of

(A) a cone and a cylinder (B) frustum of a cone and a cylinder
(C) a hemisphere and a cylinder (D) a hemisphere and a cone

Solution : Answer (B)

Sample Question 2 : If a marble of radius 2.1 cm is put into a cylindrical cup full of water of radius 5cm and height 6 cm, then how much water flows out of the cylindrical cup?

(A) 38.8 cm^3 (B) 55.4 cm^3 (C) 19.4 cm^3 (D) 471.4 cm^3

Solution : Answer (A)

Sample Question 3 : A cubical ice cream brick of edge 22 cm is to be distributed among some children by filling ice cream cones of radius 2 cm and height 7 cm upto its brim. How many children will get the ice cream cones?

(A) 163 (B) 263 (C) 363 (D) 463

Solution : Answer (C)

Sample Question 4 : The radii of the ends of a frustum of a cone of height h cm are r_1 cm and r_2 cm. The volume in cm^3 of the frustum of the cone is

(A) $\frac{1}{3}\pi h[r_1^2 + r_2^2 + r_1 r_2]$ (B) $\frac{1}{3}\pi h[r_1^2 + r_2^2 - r_1 r_2]$

(C) $\frac{1}{3}\pi h[r_1^2 - r_2^2 + r_1 r_2]$ (D) $\frac{1}{3}\pi h[r_1^2 - r_2^2 - r_1 r_2]$

Solution : Answer (A)

Sample Question 5 : The volume of the largest right circular cone that can be cut out from a cube of edge 4.2 cm is

(A) 9.7 cm^3 (B) 77.6 cm^3 (C) 58.2 cm^3 (D) 19.4 cm^3

Solution : Answer (D)

EXERCISE 12.1

Choose the correct answer from the given four options:

1. A cylindrical pencil sharpened at one edge is the combination of

(A) a cone and a cylinder
(B) frustum of a cone and a cylinder
(C) a hemisphere and a cylinder
(D) two cylinders.

2. A *surahi* is the combination of

(A) a sphere and a cylinder
(B) a hemisphere and a cylinder
(C) two hemispheres
(D) a cylinder and a cone.

3. A plumbline (*sahul*) is the combination of (see Fig. 12.2)

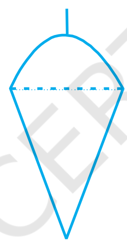

Fig. 12.2

(A) a cone and a cylinder
(B) a hemisphere and a cone
(C) frustum of a cone and a cylinder
(D) sphere and cylinder

4. The shape of a glass (tumbler) (see Fig. 12.3) is usually in the form of

(A) a cone
(B) frustum of a cone
(C) a cylinder
(D) a sphere

Fig. 12.3

5. The shape of a *gilli*, in the *gilli-danda* game (see Fig. 12.4), is a combination of

(A) two cylinders
(B) a cone and a cylinder
(C) two cones and a cylinder
(D) two cylinders and a cone

Fig. 12.4

6. A shuttle cock used for playing badminton has the shape of the combination of

(A) a cylinder and a sphere
(B) a cylinder and a hemisphere
(C) a sphere and a cone
(D) frustum of a cone and a hemisphere

7. A cone is cut through a plane parallel to its base and then the cone that is formed on one side of that plane is removed. The new part that is left over on the other side of the plane is called

(A) a frustum of a cone
(B) cone
(C) cylinder
(D) sphere

8. A hollow cube of internal edge 22cm is filled with spherical marbles of diameter 0.5 cm and it is assumed that $\frac{1}{8}$ space of the cube remains unfilled. Then the number of marbles that the cube can accomodate is

(A) 142296
(B) 142396
(C) 142496
(D) 142596

9. A metallic spherical shell of internal and external diameters 4 cm and 8 cm, respectively is melted and recast into the form a cone of base diameter 8cm. The height of the cone is

(A) 12cm
(B) 14cm
(C) 15cm
(D) 18cm

10. A solid piece of iron in the form of a cuboid of dimensions 49cm × 33cm × 24cm, is moulded to form a solid sphere. The radius of the sphere is

(A) 21cm
(B) 23cm
(C) 25cm
(D) 19cm

11. A mason constructs a wall of dimensions 270cm× 300cm × 350cm with the bricks each of size 22.5cm × 11.25cm × 8.75cm and it is assumed that $\frac{1}{8}$ space is covered by the mortar. Then the number of bricks used to construct the wall is

(A) 11100 (B) 11200 (C) 11000 (D) 11300

12. Twelve solid spheres of the same size are made by melting a solid metallic cylinder of base diameter 2 cm and height 16 cm. The diameter of each sphere is

(A) 4 cm (B) 3 cm (C) 2 cm (D) 6 cm

13. The radii of the top and bottom of a bucket of slant height 45 cm are 28 cm and 7 cm, respectively. The curved surface area of the bucket is

(A) 4950 cm^2 (B) 4951 cm^2 (C) 4952 cm^2 (D) 4953 cm^2

14. A medicine-capsule is in the shape of a cylinder of diameter 0.5 cm with two hemispheres stuck to each of its ends. The length of entire capsule is 2 cm. The capacity of the capsule is

(A) 0.36 cm^3 (B) 0.35 cm^3 (C) 0.34 cm^3 (D) 0.33 cm^3

15. If two solid hemispheres of same base radius r are joined together along their bases, then curved surface area of this new solid is

(A) $4\pi r^2$ (B) $6\pi r^2$ (C) $3\pi r^2$ (D) $8\pi r^2$

16. A right circular cylinder of radius r cm and height h cm (h>2r) just encloses a sphere of diameter

(A) r cm (B) $2r$ cm (C) h cm (D) $2h$ cm

17. During conversion of a solid from one shape to another, the volume of the new shape will

(A) increase (B) decrease (C) remain unaltered (D) be doubled

18. The diameters of the two circular ends of the bucket are 44 cm and 24 cm. The height of the bucket is 35 cm. The capacity of the bucket is

(A) 32.7 litres (B) 33.7 litres (C) 34.7 litres (D) 31.7 litres

19. In a right circular cone, the cross-section made by a plane parallel to the base is a

(A) circle (B) frustum of a cone (C) sphere (D) hemisphere

20. Volumes of two spheres are in the ratio 64:27. The ratio of their surface areas is

(A) 3 : 4 (B) 4 : 3 (C) 9 : 16 (D) 16 : 9

(C) Short Answer Questions with Reasoning

Write 'True' or 'False' and justify your answer.

Sample Question 1 : If a solid cone of base radius r and height h is placed over a solid cylinder having same base radius and height as that of the cone, then the curved surface area of the shape is $\pi r\sqrt{h^2 + r^2} + 2\pi rh$.

Solution : True. Since the curved surface area taken together is same as the sum of curved surface areas measured separately.

Sample Question 2 : A spherical steel ball is melted to make eight new identical balls. Then, the radius of each new ball be $\frac{1}{8}$th the radius of the original ball.

Solution : False. Let r be the radius of the original steel ball and r_1 be the radius of the new ball formed after melting.

Therefore, $\frac{4}{3}\pi r^3 = 8 \times \frac{4}{3}\pi r_1^3$. This implies $r_1 = \frac{r}{2}$.

Sample Question 3 : Two identical solid cubes of side a are joined end to end. Then the total surface area of the resulting cuboid is $12a^2$.

Solution : False. The total surface area of a cube having side a is $6a^2$. If two identical faces of side a are joined together, then the total surface area of the cuboid so formed is $10a^2$.

Sample Question 4 : Total surface area of a *lattu* (top) as shown in the Fig. 12.5 is the sum of total surface area of hemisphere and the total surface area of cone.

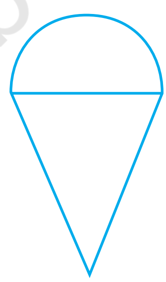

Fig. 12.5

Solution : False. Total surface area of the lattu is the sum of the curved surface area of the hemisphere and curved surface area of the cone.

Sample Question 5 : Actual capacity of a vessel as shown in the Fig. 12.6 is equal to the difference of volume of the cylinder and volume of the hemisphere.

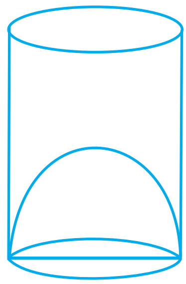

Fig. 12.6

Solution : True. Actual capacity of the vessel is the empty space inside the glass that can accomodate something when poured in it.

EXERCISE 12.2

Write 'True' or 'False' and justify your answer in the following:

1. Two identical solid hemispheres of equal base radius r cm are stuck together along their bases. The total surface area of the combination is $6\pi r^2$.

2. A solid cylinder of radius r and height h is placed over other cylinder of same height and radius. The total surface area of the shape so formed is $4\pi rh + 4\pi r^2$.

3. A solid cone of radius r and height h is placed over a solid cylinder having same base radius and height as that of a cone. The total surface area of the combined solid is $\pi r\left[\sqrt{r^2+h^2}+3r+2h\right]$.

4. A solid ball is exactly fitted inside the cubical box of side a. The volume of the ball is $\frac{4}{3}\pi a^3$.

5. The volume of the frustum of a cone is $\frac{1}{3}\pi h[r_1^2 + r_2^2 - r_1 r_2]$, where h is vertical height of the frustum and r_1, r_2 are the radii of the ends.

6. The capacity of a cylindrical vessel with a hemispherical portion raised upward at the bottom as shown in the Fig. 12.7 is $\frac{\pi r^2}{3}[3h-2r]$.

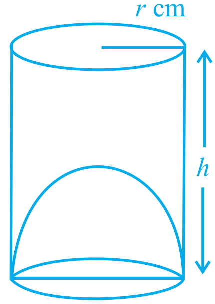

Fig. 12.7

7. The curved surface area of a frustum of a cone is $\pi l\ (r_1+r_2)$, where $l=\sqrt{h^2+(r_1+r_2)^2}$, r_1 and r_2 are the radii of the two ends of the frustum and h is the vertical height.

8. An open metallic bucket is in the shape of a frustum of a cone, mounted on a hollow cylindrical base made of the same metallic sheet. The surface area of the metallic sheet used is equal to

curved surface area of frustum of a cone + area of circular base + curved surface area of cylinder

(C) Short Answer Questions

Sample Question 1 : A cone of maximum size is carved out from a cube of edge 14 cm. Find the surface area of the cone and of the remaining solid left out after the cone carved out.

Solution : The cone of maximum size that is carved out from a cube of edge 14 cm will be of base radius 7 cm and the height 14 cm.

Surface area of the cone = $\pi rl + \pi r^2$

$$=\frac{22}{7}\times 7\times\sqrt{7^2+14^2}+\frac{22}{7}(7)^2$$

$$=\frac{22}{7}\times 7\times\sqrt{245}+154=(154\sqrt{5}+154)\text{cm}^2=154\left(\sqrt{5}+1\right)\text{cm}^2$$

Surface area of the cube = $6 \times (14)^2 = 6 \times 196 = 1176\ \text{cm}^2$

So, surface area of the remaining solid left out after the cone is carved out

$$=\left(1176-154+154\sqrt{5}\right)\text{cm}^2=(1022+154\sqrt{5})\ \text{cm}^2.$$

Sample Question 2 : A solid metallic sphere of radius 10.5 cm is melted and recast into a number of smaller cones, each of radius 3.5 cm and height 3 cm. Find the number of cones so formed.

Solution : The volume of the solid metallic sphere = $\frac{4}{3}\pi(10.5)^3$ cm^3

Volume of a cone of radius 3.5 cm and height 3 cm = $\frac{1}{3}\pi(3.5)^2 \times 3$ cm^3

Number of cones so formed = $\dfrac{\frac{4}{3}\pi \times 10.5 \times 10.5 \times 10.5}{\frac{1}{3}\pi \times 3.5 \times 3.5 \times 3.5} = 126$

Sample Question 3 : A canal is 300 cm wide and 120 cm deep. The water in the canal is flowing with a speed of 20 km/h. How much area will it irrigate in 20 minutes if 8 cm of standing water is desired?

Solution : Volume of water flows in the canal in one hour = width of the canal × depth of the canal × speed of the canal water = 3 × 1.2 × 20 × 1000m^3 = 72000m^3.

In 20 minutes the volume of water = $\frac{72000 \times 20}{60}$ m^3 $= 24000$m^3.

Area irrigated in 20 minutes, if 8 cm, i.e., 0.08 m standing water is required

$= \frac{24000}{0.08}$ m^2 $= 300000$ m^2 = 30 hectares.

Sample Question 4 : A cone of radius 4 cm is divided into two parts by drawing a plane through the mid point of its axis and parallel to its base. Compare the volumes of the two parts.

Solution : Let h be the height of the given cone. On dividing the cone through the mid-point of its axis and parallel to its base into two parts, we obtain the following (see Fig. 12.8):

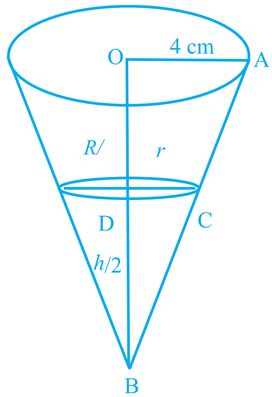

Fig. 12.8

In two similar triangles OAB and DCB, we have $\frac{\text{OA}}{\text{CD}}=\frac{\text{OB}}{\text{BD}}$. This implies $\frac{4}{r}=\frac{h}{\frac{h}{2}}$.

Therefore, $r = 2$.

$$\text{Therefore, } \frac{\text{Volume of the smaller cone}}{\text{Volume of the frustum of the cone}} = \frac{\frac{1}{3}\pi\times(2)^2\times\left(\frac{h}{2}\right)}{\frac{1}{3}\pi\times\left(\frac{h}{2}\right)[4^2+2^2+4\times 2]} = \frac{1}{7}$$

Therefore, the ratio of volume of the smaller cone to the volume of the frustum of the cone is 1: 7.

Sample Question 5 : Three cubes of a metal whose edges are in the ratio 3:4:5 are melted and converted into a single cube whose diagonal is $12\sqrt{3}$ cm. Find the edges of the three cubes.

Solution : Let the edges of three cubes (in cm) be $3x$, $4x$ and $5x$, respectively.

Volume of the cubes after melting is = $(3x)^3 + (4x)^3 + (5x)^3 = 216x^3$ cm^3

Let a be the side of new cube so formed after melting. Therefore, $a^3 = 216x^3$

So, $a = 6x$, Diagonal = $\sqrt{a^2+a^2+a^2} = a\sqrt{3}$

But it is given that diagonal of the new cube is $12\sqrt{3}$ cm. Therefore, $a\sqrt{3}=12\sqrt{3}$, i.e., $a = 12$.

This gives $x = 2$. Therefore, edges of the three cubes are 6 cm, 8 cm and 10 cm, respectively.

EXERCISE 12.3

1. Three metallic solid cubes whose edges are 3 cm, 4 cm and 5 cm are melted and formed into a single cube.Find the edge of the cube so formed.

2. How many shots each having diameter 3 cm can be made from a cuboidal lead solid of dimensions $9\text{cm} \times 11\text{cm} \times 12\text{cm}$?

3. A bucket is in the form of a frustum of a cone and holds 28.490 litres of water. The radii of the top and bottom are 28 cm and 21 cm, respectively. Find the height of the bucket.

4. A cone of radius 8 cm and height 12 cm is divided into two parts by a plane through the mid-point of its axis parallel to its base. Find the ratio of the volumes of two parts.

5. Two identical cubes each of volume 64 cm^3 are joined together end to end. What is the surface area of the resulting cuboid?

6. From a solid cube of side 7 cm, a conical cavity of height 7 cm and radius 3 cm is hollowed out. Find the volume of the remaining solid.

7. Two cones with same base radius 8 cm and height 15 cm are joined together along their bases. Find the surface area of the shape so formed.

8. Two solid cones A and B are placed in a cylinderical tube as shown in the Fig.12.9. The ratio of their capacities are 2:1. Find the heights and capacities of cones. Also, find the volume of the remaining portion of the cylinder.

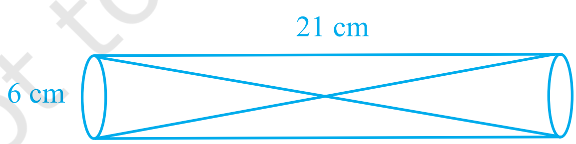

Fig. 12.9

9. An ice cream cone full of ice cream having radius 5 cm and height 10 cm as shown in the Fig.12.10. Calculate the volume of ice cream, provided that its $\frac{1}{6}$ part is left unfilled with ice cream.

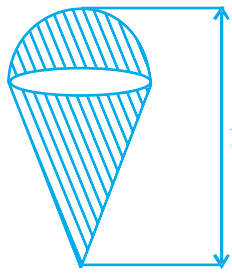

Fig. 12.10

10. Marbles of diameter 1.4 cm are dropped into a cylindrical beaker of diameter 7 cm containing some water. Find the number of marbles that should be dropped into the beaker so that the water level rises by 5.6 cm.

11. How many spherical lead shots each of diameter 4.2 cm can be obtained from a solid rectangular lead piece with dimensions 66 cm, 42 cm and 21 cm.

12. How many spherical lead shots of diameter 4 cm can be made out of a solid cube of lead whose edge measures 44 cm.

13. A wall 24 m long, 0.4 m thick and 6 m high is constructed with the bricks each of dimensions 25 cm × 16 cm × 10 cm. If the mortar occupies $\frac{1}{10}$th of the volume of the wall, then find the number of bricks used in constructing the wall.

14. Find the number of metallic circular disc with 1.5 cm base diameter and of height 0.2 cm to be melted to form a right circular cylinder of height 10 cm and diameter 4.5 cm.

(E) Long Answer Questions

Sample Question 1 : A bucket is in the form of a frustum of a cone of height 30 cm with radii of its lower and upper ends as 10 cm and 20 cm, respectively. Find the capacity and surface area of the bucket. Also, find the cost of milk which can completely fill the container, at the rate of Rs 25 per litre (use $\pi = 3.14$).

Solution : Capacity (or volume) of the bucket = $\frac{\pi h}{3}[r_1^2 + r_2^2 + r_1 r_2]$.

Here, $h = 30$ cm, $r_1 = 20$ cm and $r_2 = 10$ cm.

So, the capacity of bucket = $\frac{3.14 \times 30}{3}[20^2 + 10^2 + 20 \times 10]$ cm^3 = 21.980 litres.

Cost of 1 litre of milk = Rs 25

Cost of 21.980 litres of milk = Rs 21.980 × 25 = Rs 549.50

Surface area of the bucket = curved surface area of the bucket
+ surface area of the bottom

$$= \pi l(r_1 + r_2) + \pi r_2^2,\ l = \sqrt{h^2 + (r_1 - r_2)^2}$$

Now, $l = \sqrt{900 + 100}$ cm = 31.62 cm

Therefore, surface area of the bucket $= 3.14 \times 31.62(20 + 10) + \frac{22}{7}(10)^2$

$= 3.14\ [948.6 + 100]$ cm^2

$= 3.14\ [1048.6]$ cm^2 = 3292.6 cm^2 (approx.)

Sample Question 2 : A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 4 cm and the diameter of the base is 8 cm. Determine the volume of the toy. If a cube circumscribes the toy, then find the difference of the volumes of cube and the toy. Also, find the total surface area of the toy.

Solution : Let r be the radius of the hemisphere and the cone and h be the height of the cone (see Fig. 12.11).

Volume of the toy = Volume of the hemisphere + Volume of the cone

$$= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$$

$$= \left(\frac{2}{3} \times \frac{22}{7} \times 4^3 + \frac{1}{3} \times \frac{22}{7} \times 4^2 \times 4\right)\text{cm}^3 = \frac{1408}{7}\ \text{cm}^3.$$

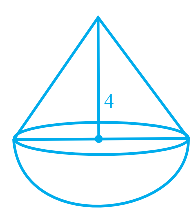

Fig. 12.11

A cube circumscribes the given solid. Therefore, edge of the cube should be 8 cm.

Volume of the cube = 8^3 cm^3 = 512 cm^3.

Difference in the volumes of the cube and the toy = $\left(512 - \frac{1408}{7}\right)$ cm^3 = 310.86 cm^3

Total surface area of the toy = Curved surface area of cone + curved surface area of hemisphere

$= \pi r l + 2\pi r^2$, where $l = \sqrt{h^2 + r^2}$

$= \pi r\ (\ l + 2r)$

$= \frac{22}{7} \times 4 \ \ \sqrt{4^2 + 4^2} + 2 \times 4 \ \ \text{cm}^2$

$= \frac{22}{7} \times 4 \ \ 4\sqrt{2} + 8 \ \ \text{cm}^2$

$= \frac{88 \times 4}{7} \ \ \sqrt{2} + 2 \ \ \text{cm}^2$

= 171.68 cm^2

Sample Question 3 : A building is in the form of a cylinder surmounted by a hemispherical dome (see Fig. 12.12). The base diameter of the dome is equal to $\frac{2}{3}$ of the total height of the building. Find the height of the building, if it contains $67\frac{1}{21}$ m^3 of air.

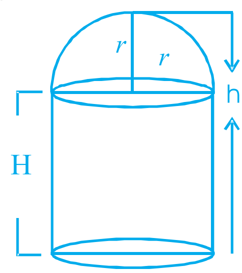

Fig. 12.12

Solution : Let the radius of the hemispherical dome be r metres and the total height of the building be h metres.

Since the base diameter of the dome is equal to $\frac{2}{3}$ of the total height, therefore $2r = \frac{2}{3}h$. This implies $r = \frac{h}{3}$. Let H metres be the height of the cylindrical portion.

Therefore, H = $h - \frac{h}{3} = \frac{2}{3}h$ metres.

Volume of the air inside the building = Volume of air inside the dome + Volume of the air inside the cylinder $= \frac{2}{3}\pi r^3 + \pi r^2 \mathrm{H}$, where H is the height of the cylindrical portion

$$= \frac{2}{3}\pi \left(\frac{h}{3}\right)^3 + \pi \left(\frac{h}{3}\right)^2 \left(\frac{2}{3}h\right) = \frac{8}{81}\pi h^3 \text{ cu. metres}$$

Volume of the air inside the building is $67\frac{1}{21}$ m^3. Therefore, $\frac{8}{81}\pi h^3 = \frac{1408}{21}$. This gives $h = 6$ m.

EXERCISE 12.4

1. A solid metallic hemisphere of radius 8 cm is melted and recasted into a right circular cone of base radius 6 cm. Determine the height of the cone.

2. A rectangular water tank of base 11 m × 6 m contains water upto a height of 5 m. If the water in the tank is transferred to a cylindrical tank of radius 3.5 m, find the height of the water level in the tank.

3. How many cubic centimetres of iron is required to construct an open box whose external dimensions are 36 cm, 25 cm and 16.5 cm provided the thickness of the iron is 1.5 cm. If one cubic cm of iron weighs 7.5 g, find the weight of the box.

4. The barrel of a fountain pen, cylindrical in shape, is 7 cm long and 5 mm in diameter. A full barrel of ink in the pen is used up on writing 3300 words on an average. How many words can be written in a bottle of ink containing one fifth of a litre?

5. Water flows at the rate of 10m/minute through a cylindrical pipe 5 mm in diameter. How long would it take to fill a conical vessel whose diameter at the base is 40 cm and depth 24 cm?

6. A heap of rice is in the form of a cone of diameter 9 m and height 3.5 m. Find the volume of the rice. How much canvas cloth is required to just cover the heap?

7. A factory manufactures 120000 pencils daily. The pencils are cylindrical in shape each of length 25 cm and circumference of base as 1.5 cm. Determine the cost of colouring the curved surfaces of the pencils manufactured in one day at Rs 0.05 per dm^2.

8. Water is flowing at the rate of 15 km/h through a pipe of diameter 14 cm into a cuboidal pond which is 50 m long and 44 m wide. In what time will the level of water in pond rise by 21 cm?

9. A solid iron cuboidal block of dimensions 4.4 m × 2.6 m × 1m is recast into a hollow cylindrical pipe of internal radius 30 cm and thickness 5 cm. Find the length of the pipe.

10. 500 persons are taking a dip into a cuboidal pond which is 80 m long and 50 m broad. What is the rise of water level in the pond, if the average displacement of the water by a person is $0.04 m^3$?

11. 16 glass spheres each of radius 2 cm are packed into a cuboidal box of internal dimensions 16 cm × 8 cm × 8 cm and then the box is filled with water. Find the volume of water filled in the box.

12. A milk container of height 16 cm is made of metal sheet in the form of a frustum of a cone with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find the cost of milk at the rate of Rs. 22 per litre which the container can hold.

13. A cylindrical bucket of height 32 cm and base radius 18 cm is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, find the radius and slant height of the heap.

14. A rocket is in the form of a right circular cylinder closed at the lower end and surmounted by a cone with the same radius as that of the cylinder. The diameter and height of the cylinder are 6 cm and 12 cm, respectively. If the the slant height of the conical portion is 5 cm, find the total surface area and volume of the rocket [Use $\pi = 3.14$].

15. A building is in the form of a cylinder surmounted by a hemispherical vaulted dome and contains $41\frac{19}{21}$ m^3 of air. If the internal diameter of dome is equal to its total height above the floor, find the height of the building?

16. A hemispherical bowl of internal radius 9 cm is full of liquid. The liquid is to be filled into cylindrical shaped bottles each of radius 1.5 cm and height 4 cm. How many bottles are needed to empty the bowl?

17. A solid right circular cone of height 120 cm and radius 60 cm is placed in a right circular cylinder full of water of height 180 cm such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is equal to the radius of the cone.

18. Water flows through a cylindrical pipe, whose inner radius is 1 cm, at the rate of 80 cm/sec in an empty cylindrical tank, the radius of whose base is 40 cm. What is the rise of water level in tank in half an hour?

19. The rain water from a roof of dimensions 22 m $\times$ 20 m drains into a cylindrical vessel having diameter of base 2 m and height 3.5 m. If the rain water collected from the roof just fill the cylindrical vessel, then find the rainfall in cm.

20. A pen stand made of wood is in the shape of a cuboid with four conical depressions and a cubical depression to hold the pens and pins, respectively. The dimension of the cuboid are 10 cm, 5 cm and 4 cm. The radius of each of the conical depressions is 0.5 cm and the depth is 2.1 cm. The edge of the cubical depression is 3 cm. Find the volume of the wood in the entire stand.