Instructions

Print the Mind Maps in

Landscape Mode

Units and Dimension

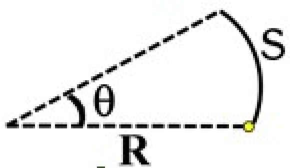
System of Units

	Length	Mass	Time
CGS	Centimetre	Gram	Second
FPS	Foot	Pound	Second
MKS	Metre	Kilogram	Second

Fundamental Quantities

	SI unit	Symbol
Length	metre	m
Mass	kilogram	kg
Time	second	s
Temperature	Kelvin	K
Electric Current	Ampere	A
Luminous Intensity	Candela	cd
Amount of Substance	mole	mol

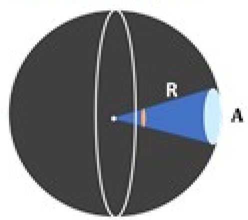
Plane angle



$$\text{Plane Angle} = \frac{\text{Arc Length}}{\text{Radius}}$$

	SI unit	Symbol
Plane Angle	radian	rad
Solid Angle	Steradian	sr

Solid angle



$$\text{Solid Angle} = \frac{\text{Area}}{(\text{Radius})^2}$$

Applications of Dimensions

1. Principle of Homogeneity

Only same type of physical quantities can be added or subtracted $x = A + B$ $x = A - B$

$$[x] = [A] = [B]$$

2. Conversion of Units

$1\text{ m} = 100\text{ cm} = 1000\text{ mm}$

$n_1u_1 = n_2u_2 = n_3u_3$ $\quad nu = \text{constant}$

When unit become smaller, numerical value increase.

3. To derive relationship between physical quantities

Limits of Dimensions

1. The relation derived from this method gives no information about the dimensionless constants.
2. The dimensional quantity to be derived must be of multiplication type. We cannot derive relations if addition or subtractions are involved.
3. This method is not applicable when unknown variables are more than equations present.

A physical quantity is expressed as

$$n\,u$$

n → Numerical Value; u → Unit

Derived Quantities

Quantities which can be derived from fundamental quantities.

Dimensions

Quantity	Dimension Symbol
Length	L
Mass	M
Time	T
Temperature	θ, K
Electric Current	I, A
Amount of Substance	Mol
Luminous Intensity	cd

✓ Angles and trigonometric functions are dimensionless quantity

$[\theta] = [L^0]$ $\qquad [\sin\theta] = M^0L^0T^0$

✓ All exponents are dimensionless.

✓ Logarithmic functions and it's arguments are dimensionless

$[a] = M^0L^0T^0$

$[\log a] = M^0L^0T^0$

★ Dimensionally correct equation may or may not be correct but dimensionally incorrect equation must be incorrect.

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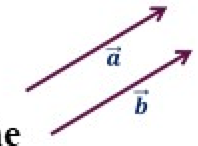
Vectors

Two vectors are said to be equal if, and only if, they have same magnitude and the same direction.



$$\vec{a} = \vec{b}$$

$0° \le$ Angle between two vectors $\le 180°$

Parallel Vectors

Same direction and angle between vectors is 0°

Anti-Parallel Vectors

Opposite direction and angle between vectors is 180°

Scalar Product (Dot Product) of two Vectors

$$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta$$

$$\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$$

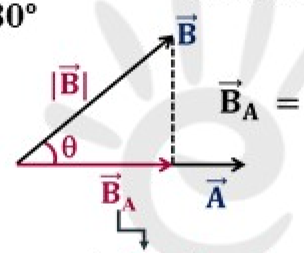
$$\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$$

$$\vec{A}\cdot\vec{A} = |\vec{A}|^2$$

$\hat{i}\cdot\hat{i} = 1 \quad \hat{j}\cdot\hat{j} = 1 \quad \hat{k}\cdot\hat{k} = 1$

$\hat{i}\cdot\hat{j} = 0 \quad \hat{i}\cdot\hat{k} = 0 \quad \hat{k}\cdot\hat{j} = 0$

To find angle between two vectors $\cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}$



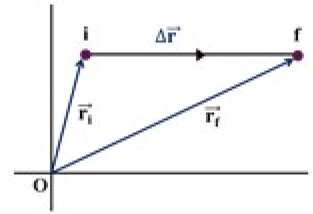
$$\vec{B}_A = |\vec{B}|\frac{(\vec{A}\cdot\vec{B})}{|\vec{A}||\vec{B}|}\hat{A}$$

component of B along A

$|\vec{A}|$ = Magnitude of $\vec{A}$

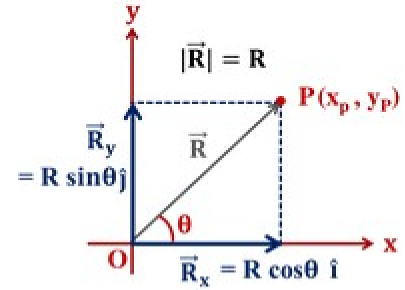
Displacement vector

$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$$



Resolution of a Vector into Components

Rectangular Components



$$\vec{R} = \vec{R}_x + \vec{R}_y$$

$$\vec{R} = R\cos\theta\,\hat{i} + R\sin\theta\,\hat{j}$$

$$|\vec{R}| = \sqrt{R_x^2 + R_y^2}$$

$$|\vec{R}| \ge 0$$

$$\tan\theta = \frac{R_y}{R_x} \qquad \hat{R} = \frac{\vec{R}}{|\vec{R}|} = \frac{R_x\hat{i} + R_y\hat{j}}{\sqrt{R_x^2 + R_y^2}}$$

Position Vector $\overrightarrow{OP} = \vec{r} = x_p\hat{i} + y_p\hat{j}$

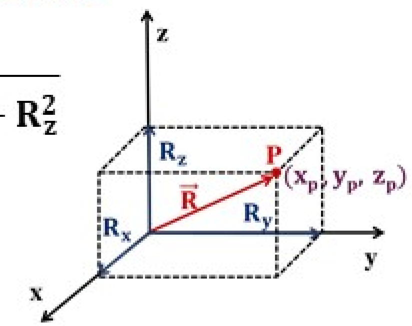
Representation of vector in 3D

$$\vec{R} = R_x\hat{i} + R_y\hat{j} + R_z\hat{k}$$

$$R = |\vec{R}| = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

Position Vector

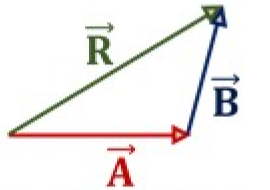
$$\vec{r} = x_p\hat{i} + y_p\hat{j} + z_p\hat{k}$$



Triangle rule of vector addition

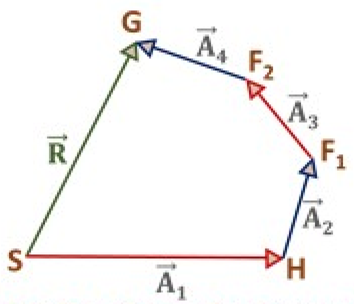
$$\vec{A} + \vec{B} = \vec{R}$$

$$|\vec{R}| \ne |\vec{A}| + |\vec{B}|$$



Polygon rule of vector addition

$$\vec{R} = \vec{A}_1 + \vec{A}_2 + \vec{A}_3 + \vec{A}_4$$



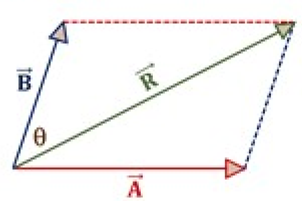
Parallelogram rule of vector addition

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

When $\theta = 0°$ $\quad R_{max} = A + B$

When $\theta = 180°$ $\quad R_{min} = |A - B|$

$$|\vec{R}|_{min} \le |\vec{R}| \le |\vec{R}|_{max}$$



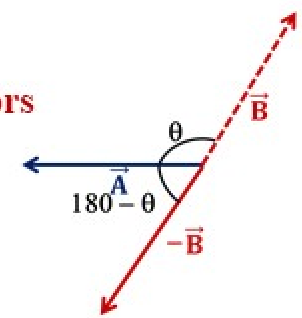
Zero vector

$$\vec{A} - \vec{A} = \vec{0}$$

Subtraction of vectors

$$|\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

$$|A - B| \le |\vec{A} - \vec{B}| \le A + B$$



Unit vector

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$$

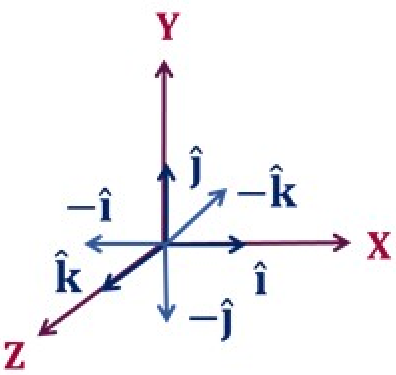
magnitude = 1

purpose is to describe a direction in space

$$|\hat{i}| = 1 = |-\hat{i}|$$

$$|\hat{j}| = 1 = |-\hat{j}|$$

$$|\hat{k}| = 1 = |-\hat{k}|$$



Multiplication of vector with real number

magnitude $|\lambda|\,|\vec{A}|$, direction remains the same.

y	differentiation	Integration
x^n	nx^{n-1}	$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$
ℓnx	$\frac{1}{x}$	$\int \frac{1}{x}\,dx = \ln x + c$
e^x	e^x	$\int e^x\,dx = e^x + c$
$\sin x$	$\cos x$	$\int \sin x\,dx = -\cos x + c$
$\cos x$	$-\sin x$	$\int \cos x\,dx = \sin x + c$
$\tan x$	$\sec^2 x$	
$\cot x$	$-\text{cosec}2x$	
C	0	
$y_1 + y_2$	$\frac{dy_1}{dx} + \frac{dy_2}{dx}$	$\int (f(x) + g(x))dx$
$y_1 - y_2$	$\frac{dy_1}{dx} - \frac{dy_2}{dx}$	$= \int f(x)dx + \int g(x)dx$
$y_1 \cdot y_2$	$y_1\frac{dy_2}{dx} + y_2\frac{dy_1}{dx}$	
$\frac{y_1}{y_2}$	$\frac{y_2\frac{dy_1}{dx} - y_1\frac{dy_2}{dx}}{y_2^2}$	$\int kf(x)dx = k\int f(x)dx$

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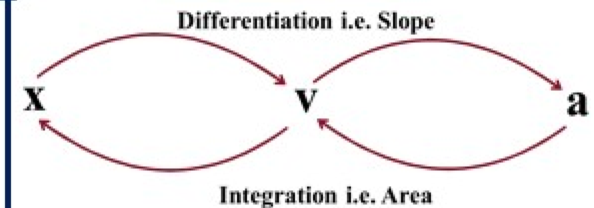
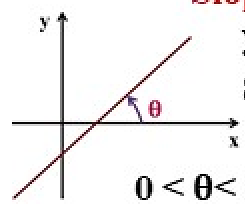


If sign of a and v are same then speed is increasing and if sign are opposite then speed is decreasing

Kinematics 1D

Slope



$y = (m)x + c$

Slope $(m) = \tan\theta$

$0 < \theta < 90°$ slope = +ve

$90° < \theta < 180°$ slope = −ve

$\theta = 0°$ slope = 0

For particle under constant acceleration

(1) $v = u + at$

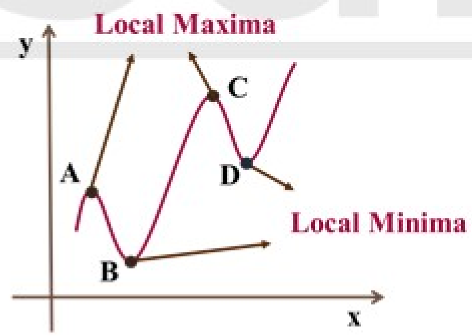
(2) $s = ut + \frac{1}{2}at^2$

(3) $v^2 = u^2 + 2as$

(4) $s = \left(\frac{v+u}{2}\right)t$

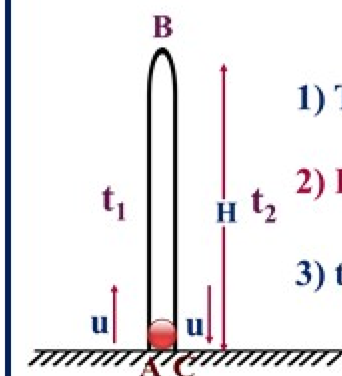
(5) $s = vt - \frac{1}{2}at^2$

Local maxima and Local minima



	$\frac{dy}{dx}$	$\frac{d^2y}{dx^2}$
Local maxima	0	-ve
Local minima	0	+ve

Motion under gravity



1) $T_{ime\ of\ flight} = \frac{2u}{g}$

2) $H_{max} = \frac{u^2}{2g}$

3) $t_1 = t_2 = \sqrt{\frac{2H}{g}} = \frac{u}{g}$

Speed

Average: $< Speed > = \frac{s}{\Delta t}$

Instantaneous: Inst. Speed $= \frac{ds}{dt}$

s is distance

Velocity

Average: $< \vec{v} > = \frac{\Delta\vec{r}}{\Delta t}$

Instantaneous: $\vec{v}_{inst} = \frac{d\vec{r}}{dt}$

r is displacement
(minimum distance between initial and final point)

Acceleration

Average: $< \vec{a} > = \frac{\Delta\vec{v}}{\Delta t}$

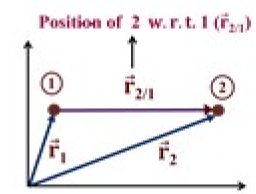
Instantaneous: $\vec{a} = \frac{d\vec{v}}{dt}$

Relative motion

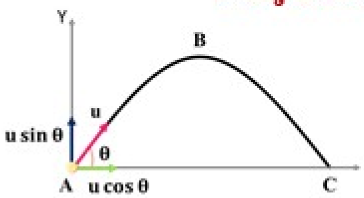
$\vec{r}_{2/1} = \vec{r}_2 - \vec{r}_1$

$\vec{v}_{2/1} = \vec{v}_2 - \vec{v}_1$

$\vec{a}_{2/1} = \vec{a}_2 - \vec{a}_1$



Projectile motion



$$T = \frac{2u\sin\theta}{g} = \frac{2u_y}{g}$$

$$H = \frac{u^2\sin^2\theta}{2g} = \frac{u_y^{\,2}}{2g}$$

Range is same for the complimentary angles.

Range is maximum when θ is 45 °

$$R = \frac{u^2\sin 2\theta}{g}$$

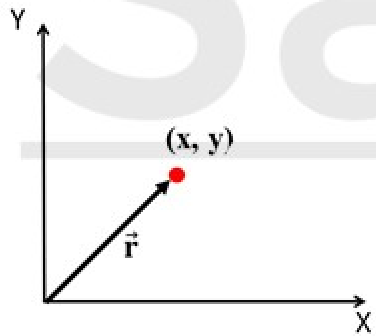
Equation of trajectory $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$

Path of particle having constant acceleration

Initial Velocity (u)	Path
u = 0	Straight Line
u ≠ 0, u and a collinear	Straight Line
u ≠ 0, u and a non-collinear	Parabolic

If two particles are projected in gravitational field then during the time at flight of both particles, trajectory of one particle w.r.t. other particle will be a straight line.

Kinematics 2D

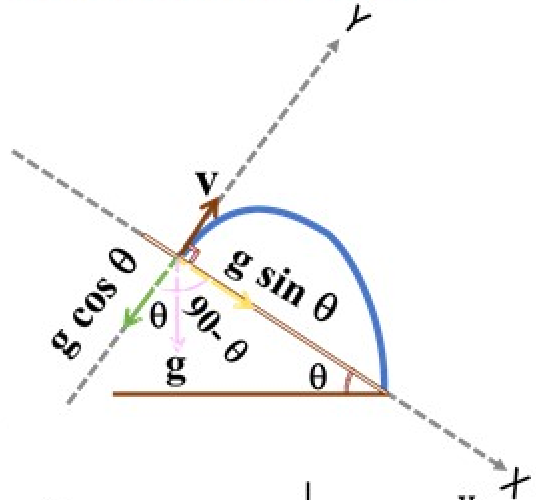


$$\vec{r} = x\hat{i} + y\hat{j}$$

$$\vec{v} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$$

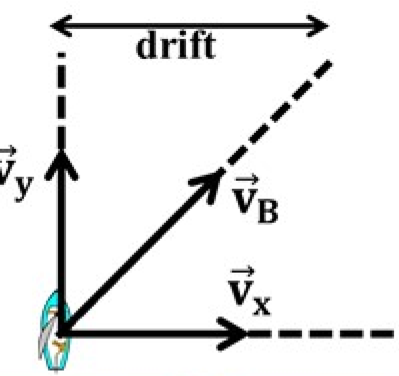
$$\vec{a} = \frac{d^2x}{dt^2}\hat{i} + \frac{d^2y}{dt^2}\hat{j}$$

Projectile on inclined plane



x	y
$u_x = 0$	$u_y = v$
$a_x = g\sin\theta$	$a_y = -g\cos\theta$
$t = \frac{2v}{g\cos\theta}$	$s_y = 0$
$s_x = \frac{1}{2}g\sin\theta\left(\frac{2v}{g\cos\theta}\right)^2$	$0 = ut + \left(\frac{g\cos\theta t^2}{2}\right)$
	$t = \frac{2v}{g\cos\theta}$

River and Boat:



1. x component of $\vec{v}_B$ (i.e. along river flow) is responsible for drift.
2. y component of $\vec{v}_B$ (i.e. perpendicular river flow) is responsible for crossing the river.

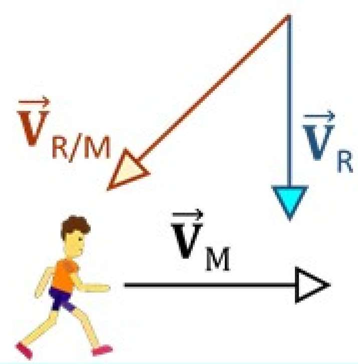
$$\vec{v}_B = \vec{v}_{B/r} + \vec{v}_r$$

Rain Man situation:

1 Man observes rain in direction of velocity of rain w.r.t. man ($\vec{v}_{R/M}$).

2. Umbrella is held in direction opposite to the velocity of rain w.r.t. man ($-\vec{v}_{R/M}$).

$$\vec{V}_{R/M} = \vec{V}_R - \vec{V}_M$$



Newton's First Law of Motion (Law of Inertia)

Every body preserves its state of rest, or of uniform motion in a straight line, unless it is compelled to change that state by external forces impressed on it.

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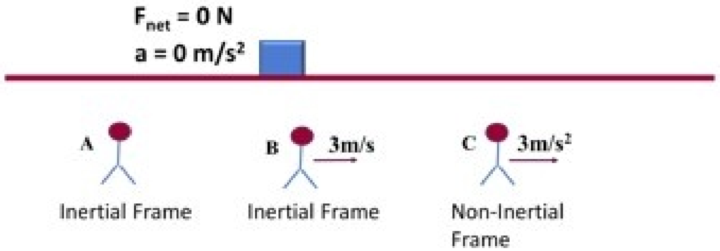
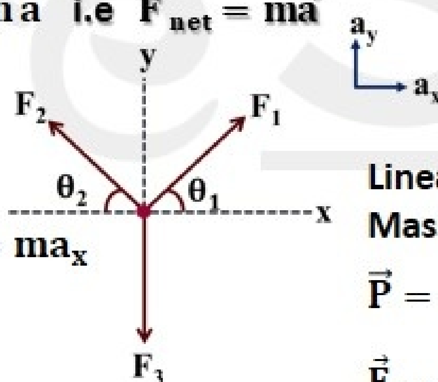
Inertial and Non-Inertial Reference Frame



If the net force acting on a body is zero, it is possible to find a reference frame in which that body has zero acceleration. Such reference frame is called Inertial Reference Frame.

Newton's Second Law of Motion

In Inertial Frame, $\sum \vec{F} = m\vec{a}$ i.e $\vec{F}_{net} = m\vec{a}$



2^{nd} Law for x-axis

$$\sum \vec{F}_x = m\vec{a}_x$$

$$\Rightarrow F_1\cos\theta_1 - F_2\cos\theta_2 = ma_x$$

2^{nd} Law for y-axis

$$\sum \vec{F}_y = m\vec{a}_y$$

$$\Rightarrow F_1\sin\theta_1 + F_2\sin\theta_2 - F_3 = ma_y$$

Normal Contact Force

- It is an electromagnetic type of force.
- It always acts along the common normal of the two surfaces in contact i.e. perpendicular to the surfaces.
- It is always directed towards the system.

Linear Momentum

Linear Momentum is the product of Mass and Velocity

$\vec{P} = m\vec{v}$ (kg m/s) or (N-s)

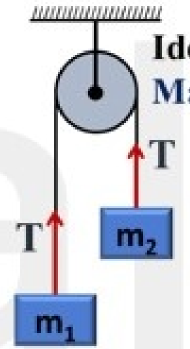
$$\vec{F}_{net} = \frac{d\vec{P}}{dt}$$

The rate of change of Linear Momentum of a body = net Force on a body.

Tension Force

- It is an electromagnetic type of force.
- This is a force applied by a string on an object or Force applied by one part of string on the remaining part of string.
- It acts along the string and away from the system on which it acts.

Tension in a massless string remains constant throughout the string if no tangential force acts along the string.



Ideal Pulley: Massless and Frictionless

Ideal String: Massless and Inextensible

Newton's Third Law of Motion

$\vec{F_1} = -\vec{F_2}$

Every action has an equal and opposite reaction.

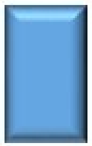
Action and Reaction act on different bodies and not on the same body.

Action and Reaction forces are of same type.

NLM

Analysis of Translational Motion using NLM

In translational motion of a body, velocity of each point of the body is equal to velocity of every other point of the body.



In translation motion system can be treated as a particle.

Steps To Follow

(1) Define a System
(2) Define the Environment of the System
(3) Draw Free Body Diagram (FBD) of the system. Take only forces ON the system (not By the system).
(4) Select appropriate axis and apply Newton's 2nd Law along each axis.

$\sum \vec{F} = m\vec{a}$ where $\sum \vec{F}$ is net force acting ON the system along the chosen axis

Note:
'ma' is not a Force therefore, during the listing of forces in FBD in inertial frame, 'ma' should not be included.

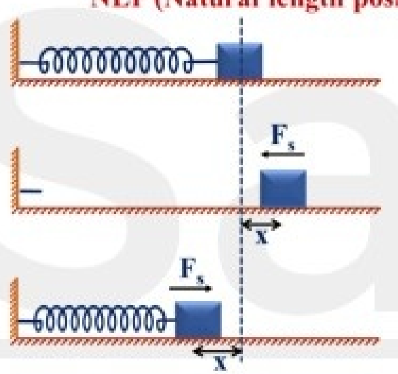
Translational Equilibrium

A system is said to be in Translational Equilibrium when net force on the system is zero. $\sum \vec{F} = 0$

Note:
If magnitude of acceleration of each particle connected with a string is same then

$$a = \frac{\text{Net pulling force on string}}{\text{total mass}}$$

Spring Force:



$\vec{F}_s = -K\vec{x}$

$|F_s| = K\,|x|$

Variation of 'k' with natural length

natural length $= \ell$ natural length $= \ell/2$

$k\ell = \text{Constant}$ $k\ell = 2k\dfrac{\ell}{2}$

If both ends of a spring are attached with inertial mass then sudden change in length of spring is not possible.

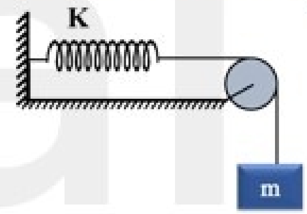
Reading of Weighing Machine

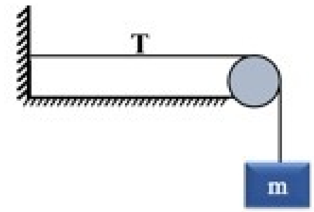
If an object is put on a weighing machine and 'N' is normal contact force b/w object & machine then reading of weighing machine will be given by

$$\text{Reading} = \frac{N}{g}$$

Key Point

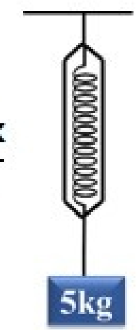
If a spring is connected with a string then replace spring with string and find tension (T) in string and equate T with Kx to find elongation.





Reading of Spring Balance

$$\text{Reading of spring balance} = \frac{kx}{g}$$



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Pseudo Force

- Second law of motion is not valid in non-inertial frame.
- To use Newton second law equation, one additional factor is added in F.B.D. of the system.
- This additional factor is called Pseudo Force.

Magnitude of Pseudo force is equal to 'ma'

'm' is mass of system
'a' is acceleration of non-inertial frame (in which analysis is done) w.r.t. inertial frame.

Pseudo force is applied in direction opposite to direction of 'a'

Pseudo force is not a force.

Constraint Motion

For a system,

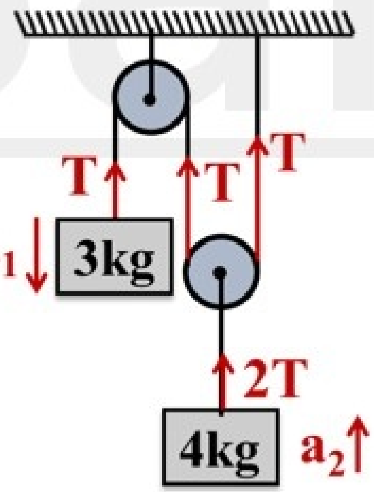
$$\sum \vec{T}.\vec{x} = 0 \qquad \sum \vec{T}.\vec{v} = 0$$

$$\sum \vec{T}.\vec{a} = 0$$

(if $\vec{a}$ are collinear with $\vec{v}$)

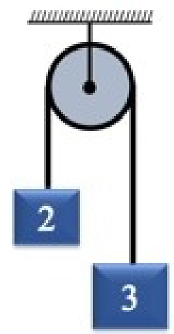
Steps To Follow to Solve Problems Involving Constraint Motion

1. Take acceleration of each block in some direction.
2. Make FBD of each block and write Newton's 2nd law equation as per direction taken in step-I
3. Write constraint equations. Solve and get values of all accelerations. If acceleration comes positive – same direction as assumed in STEP-I and if acceleration comes negative then it is opposite to the direction assumed in STEP-I



Keypoint

Constraint relation in string will also hold for string having mass provided it is inextensible.



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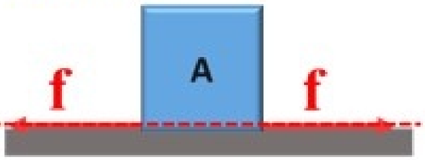
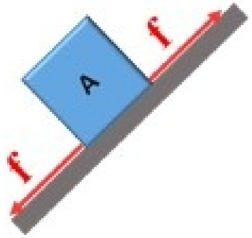
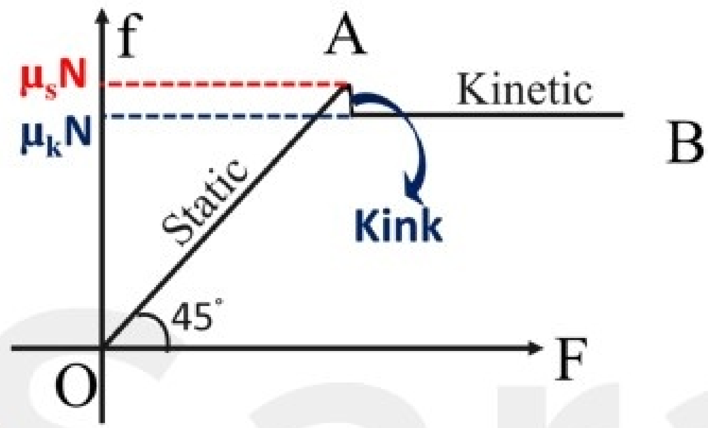
Friction

Friction does not oppose motion instead it opposes Relative Motion to the surface applying friction force.

Friction acts along the surface in contact.



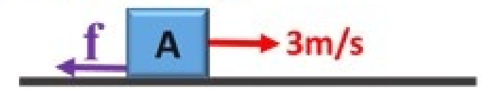




Kinetic Friction

Kinetic friction comes into picture when relative slipping occurs

It acts in direction opposite to relative velocity.



Laws Of Kinetic Friction

1) Value of kinetic friction is directly proportional to Normal force acting between the two surfaces in contact

2) Kinetic friction force acting between two surfaces in contact does not depend on area of contact.

$$\boxed{f_k = \mu_k N}$$

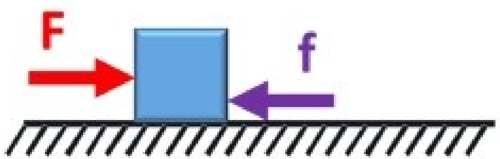
f_k → Kinetic friction

μ_k → coefficient of kinetic friction

Static Friction

If there is a tendency of relative slipping (only tendency and not actual) between two surfaces in contact then the friction force acting between them is called Static Friction force.

It is a variable force whose value is equal to requirement to stop relative slipping till it reaches its limiting value known as Limiting Friction.



Laws Of Static Friction

1) Maximum value of static friction (Limiting friction) is directly proportional to Normal force acting between the two surfaces in contact.

2) Static friction force acting between two surfaces in contact does not depend on area of contact.

$$\boxed{(f_s)_{max} = \mu_s N} \qquad \boxed{f_s \le \mu_s N}$$

$(f_s)_{max}$ → Limiting friction

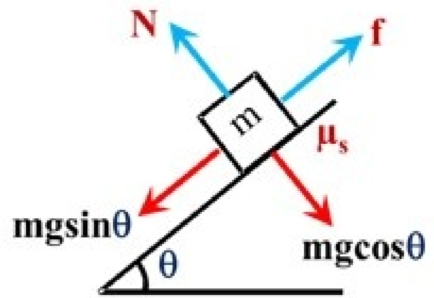
μ_s → coefficient of static friction

Generally $\mu_k < \mu_s$
(from experimental observation)

Value of μ_s and μ_k depends on nature of surfaces in contact.

Friction

Angle of Response



At some angle of inclination θ the body starts sliding down the plane due to gravity. This angle of inclination is called angle of repose (θ).

$$\boxed{\theta = \tan^{-1}(\mu_s) \text{ is angle of repose.}}$$

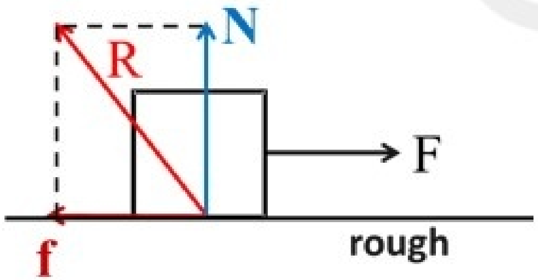
$$f_{min} = 0 \quad \leq \quad f \quad \leq \quad f_{max} = \mu N$$

$$R_{min} = N \quad \leq \quad R \quad \leq \quad R_{max.} = N\sqrt{1+\mu^2}$$

$$\tan\phi = 0 \quad \leq \quad \tan\phi \quad \leq \quad \tan\phi = \mu$$

$$\phi_{min.} = 0 \quad \leq \quad \phi \quad \leq \quad \phi_{max.} = \tan^{-1}(\mu)$$

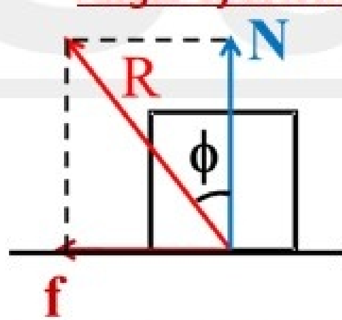
Net Contact Force:



Resultant of Normal and Friction force is the Net Contact Force.

$$\boxed{R = \sqrt{N^2 + f^2}}$$

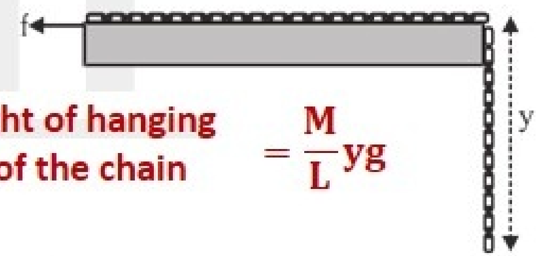
Angle of Friction



$$\boxed{\tan\phi = \frac{f}{N}}$$

The Angle of Friction (ϕ) is the angle between Net Contact Force and Normal Reaction

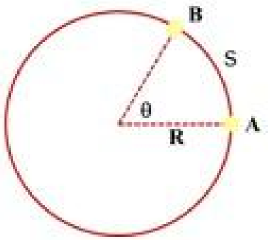
$\phi = 0$ for smooth surface



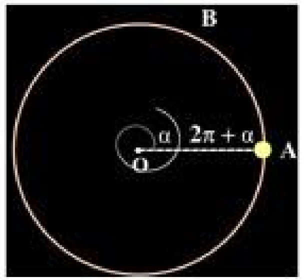
Weight of hanging part of the chain $= \frac{M}{L}yg$

For equilibrium with maximum portion hanging,
limiting friction = weight of hanging part of the chain

$$\boxed{y = \frac{\mu L}{1+\mu}}$$



Angle $\theta = \frac{S}{R} = \frac{\textit{arc length}}{\textit{radius}}$ radian



Angular Displacement $= 2\pi + \alpha$

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Circular Motion

If α = constant, then

$$\omega_f = \omega_i + \alpha t$$

$$\theta = \omega_i t + \frac{1}{2}\alpha t^2$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\theta$$

$$\theta = \left(\frac{\omega_i + \omega_f}{2}\right) t$$

$$\theta = \omega_f t - \frac{1}{2}\alpha t^2$$

Angular Velocity (ω)

Angular Velocity is defined as the rate of change of angular position w.r.t time.

Angular Velocity

Average: $<\omega> = \frac{\Delta\theta}{\Delta t}$

Instantaneous: $\omega = \frac{d\theta}{dt}$

SI unit is rad/s

Dimension is $[T^{-1}]$

$$1 \text{ RPM} = \frac{\pi}{30} \text{ rad/sec}$$

Angular Acceleration (α)

Angular Acceleration is defined as rate of change of angular velocity w.r.t. time.

Angular Acceleration

Average: $<\alpha> = \frac{\Delta\omega}{\Delta t}$

Instantaneous: $\alpha = \frac{d\omega}{dt}$

SI unit is rad/s^2

Dimension is $[T^{-2}]$

Time Period (T)

The time taken by an object to make one revolution is known as its Time Period

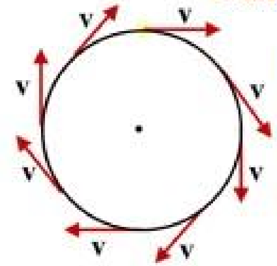
$$T = \frac{2\pi}{\omega}$$

Frequency (f)

The number of revolutions made in one second is known as Frequency.

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

Uniform Circular Motion



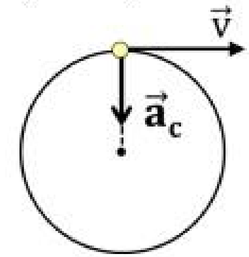
If a particle is moving in a circle with constant speed then its motion is called Uniform Circular Motion (UCM).

$$|\vec{a}| = v\omega$$

$$|\vec{a}| = \frac{v^2}{R} \left(\because \omega = \frac{v}{R}\right)$$

$$|\vec{a}| = \omega^2 R$$

$$a_c = \frac{v^2}{R}$$

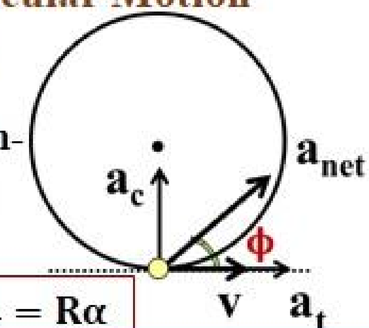


This acceleration acts towards the centre, so it is called Centripetal Acceleration (a_c)

- Centripetal acceleration is perpendicular to the velocity and is responsible for changing the direction of the velocity.
- In U.C.M, $\vec{a}_c$ is not constant as its magnitude is constant but direction is changing.

Non–Uniform Circular Motion

If a particle is moving in a circle with variable speed then its motion is called Non-Uniform Circular Motion.



$$a_{net} = \sqrt{(a_c)^2 + (a_t)^2}$$

$$\tan\phi = \frac{a_c}{a_t}$$

$$a_t = R\alpha$$

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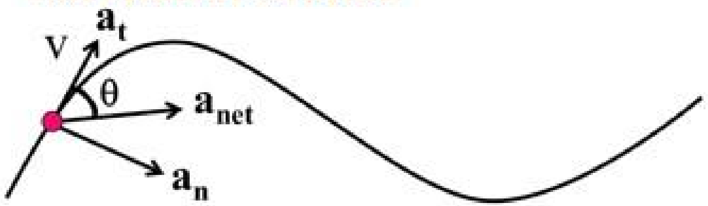
Circular Motion

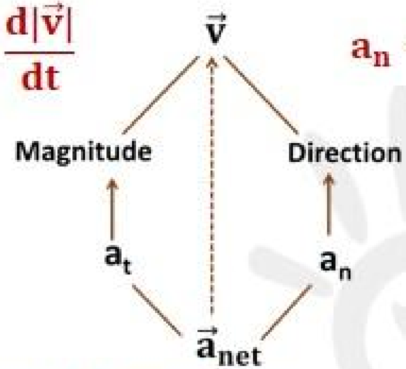
Curvilinear Motion



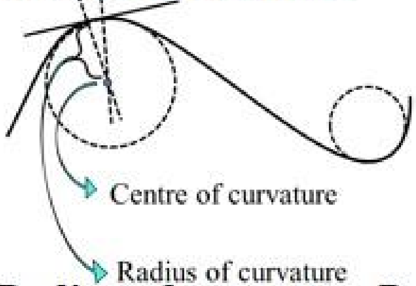
a_t is responsible for changing the magnitude of velocity i.e. speed of particle.

a_n is responsible for changing the direction of velocity. Its direction is towards the CONCAVE side.

$$a_t = \frac{d|\vec{v}|}{dt} \qquad a_n = \frac{v^2}{R}$$



Radius Of Curvature



$$\text{Radius of curvature } R = \frac{v^2}{a_n}$$

Radius of curvature is property of curve & not of motion of particle.

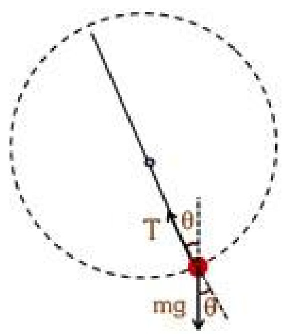
Dynamics of Circular Motion

The net resultant force providing the centripetal acceleration is called Centripetal Force.

$$\boxed{\sum F_c = m\,a_c = \frac{mv^2}{R} = m\omega^2 R}$$

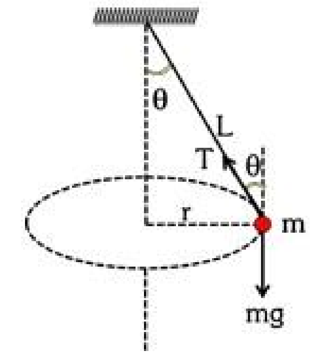
It should not be included in FBD drawn in inertial frame.

Vertical Circular Motion



$$a_t = g\sin\theta$$

Conical Pendulum



Time period of pendulum 'T'

$$T = \frac{2\pi r}{v} = 2\pi\sqrt{\frac{L\cos\theta}{g}}$$

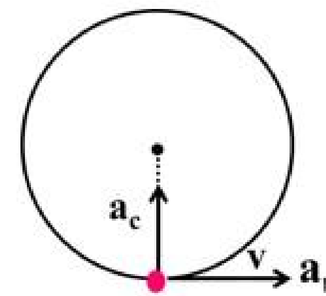
Dynamics of Non-UCM

Along radial

$$\sum F_C = ma_C = \frac{mv^2}{R} = m\omega^2 R$$

Along tangential $\sum F_t = ma_t$

$$F_{net} = \sqrt{\left(\sum F_C\right)^2 + \left(\sum F_t\right)^2}$$



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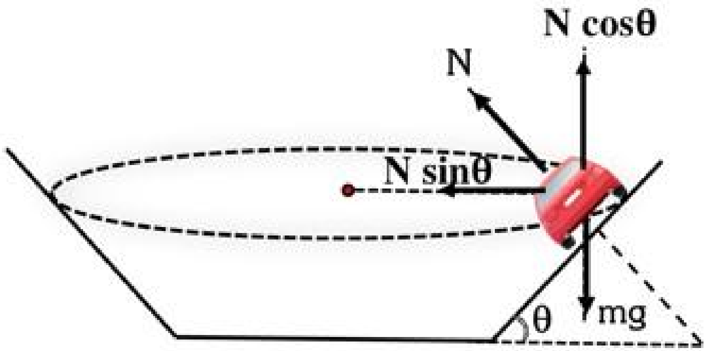
Circular Motion

Banking of Roads



No friction

$$\tan\theta = \frac{v^2}{Rg} \qquad V_0 = \sqrt{Rg\tan\theta}$$

Friction is present

$v < v_0$

For minimum speed, $f = \mu N$

$$v_{min} = \sqrt{Rg\left(\frac{\tan\theta - \mu}{1 + \mu\tan\theta}\right)}$$

$v > v_0$

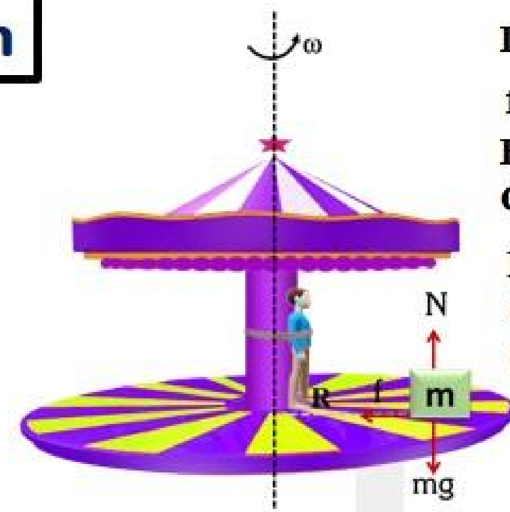
For maximum speed, $f = \mu N$

$$v_{max} = \sqrt{Rg\left(\frac{\tan\theta + \mu}{1 - \mu\tan\theta}\right)}$$

vehicle can successfully turn on a banked road in a circle of radius R for

$$v_{min} \leq v \leq v_{max}$$

Centrifugal Force



In ground frame

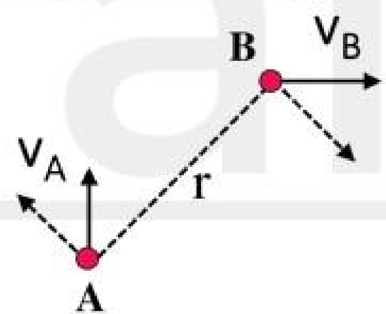
$f = m\omega^2R$

Friction is the Centripetal Force.

In man's frame

$m\omega^2R$ Pseudo Force (Centrifugal Force)

Angular Velocity in General



$$\omega = \frac{(v_\perp)_{rel}}{r}$$

$$= \frac{\text{relative velocity} \perp \text{ to line joining two particles}}{\text{separation between two particles}}$$

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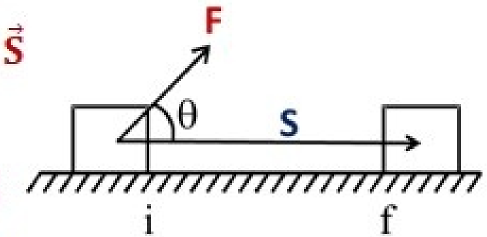
WPE

Work done by force $\vec{F}$ on an object is defined as

$$W_F = \int \vec{F}.d\vec{s}$$

$\vec{F}$ is the force on object $d\vec{s}$ is displacement of point of application of force ($\vec{F}$)

If $\vec{F}$ is constant, then $= \vec{F}.\vec{S}$



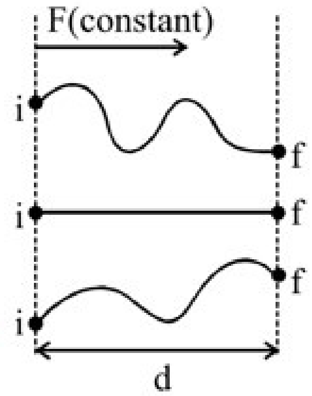
$W_F = \vec{F}.\vec{S} = FS\cos\theta$

θ is angle between $\vec{F}$ and $\vec{S}$

$0^\circ < \theta < 90^\circ \Rightarrow W_F = +ve$

$\theta = 90^\circ \Rightarrow W_F = 0$

$\theta > 90^\circ \Rightarrow W_F = -ve$



$W_1 = W_2 = W_2 = Fd$

Work done by constant force depends only on initial and final position and not on the path taken

SI unit of work is Joule (J).
Cgs unit is erg.

$\boxed{1\ \text{erg} = 10^{-7}\ \text{joule}}$

Work is a scalar quantity.
It can be –ve , zero or +ve.

Work Done by Variable Force

If the force applying on a body is changing its direction or magnitude or both, the force is said to be variable.

$$\boxed{W = \int \vec{F}.d\vec{r}}$$

$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$W = \int F_x dx + Fydy + Fzdz$$

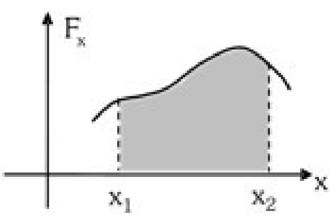
If particle goes from $(x_i\ y_i\ z_i)$ to $(x_f\ y_f\ z_f)$

$$\boxed{W = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy + \int_{z_i}^{z_f} F_z dz}$$

$\boxed{W_{Net}}$

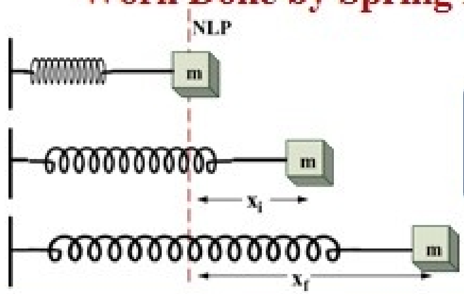
- Get F_{net} and calculate its work to get W_{net} (for translation motion
- Get work from all the forces by applying $W = \int \vec{F}.d\vec{s}$ and add them to get net work

Work as Area Under Curve



$$W = \int F_x dx$$

Work Done by Spring Force



$$\boxed{W_{sp} = \frac{1}{2}k\left(x_i^2 - x_f^2\right)}$$

Where x_i and x_f are initial and final change in lengths from Natural Length of spring

Work Energy Theorem

Work done by all the forces (external or internal) acting on a system is equal to the change in kinetic energy of the system.

$W_{net} = \Delta KE = KE_f - KE_i$

KE of a particle $= \frac{1}{2}mv^2 = \frac{p^2}{2m}$

where p is momentum of particle

$W_{net} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

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WPE

- Work energy theorem is valid in inertial frame.
- But we can use its equation in <u>non-inertial frame</u> by considering work of Psuedo Force.

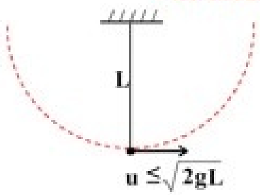
Work Energy Theorem For System of Particle

$$\boxed{(W_{net})_{sys} = (KE_f)_{sys} - (KE_i)_{sys}}$$

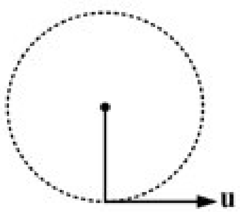
- $(W_{net})_{sys} = \sum W_F$ on system
- W_F on system = $\sum W_F$ on each part of system
- KE of system = $\sum$ KE of each particle of system.

- Work done by an Internal Force on the system is independent of the reference frame
- If distance between particles in a system along internal force remains unchanged always then work done by internal force on the system is 'zero'.

Vertical Circular Motion



$u \le \sqrt{2gl}$ (circular motion but not complete)



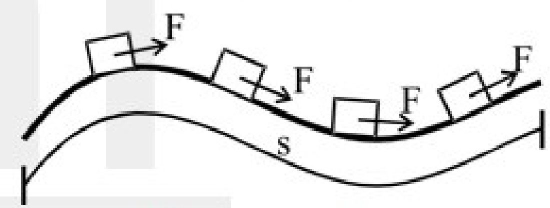
$u \ge \sqrt{5gl}$ (complete vertical circular motion)

Central Force

Force on an object whose magnitude only depends on the distance \`r\` of the object from a fixed point and is directed along the line joining object and the fixed point is referred as Central Force.

$$\boxed{\vec{F} = f(r)\hat{r}}$$

Work Done By Tangential Force



$W_F = \int \vec{F}.d\vec{s} = \int F\,ds\,\text{Cos}0^\circ$

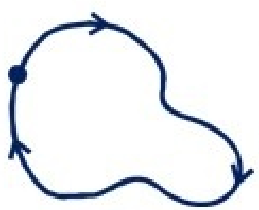
If $\vec{F}$ is constant in magnitude

$W_F = F\int ds = Fs$ → Length of curve

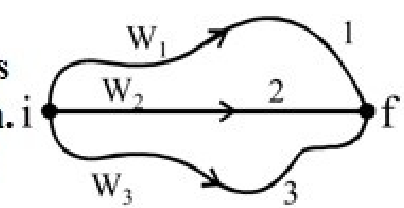
Conservative Force

A Force is a Conservative Force when work done by it in any closed path (Loop) is zero.

$W_F = 0 \Rightarrow$ F is conservative

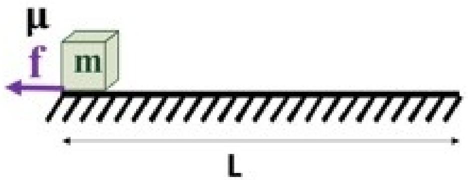


Work done by conservative force is independent of path. It depends only on initial & final position.



$W_1 = W_2 = W_3$ by F

- Constant Forces are conservative.
- Central forces are conservative.



W_{fric} = - μmgL

Kinetic friction is an example of non-conservative force.

Potential Energy (U)

$\Delta U = -W_{int,C}$

$W_{int,C}$ is work done by internal conservative force

- In any system, change in potential energy is equal to the negative of work done by internal conservative force.
- PE is defined only for conservative forces.
- PE cannot be defined for single particle system. It is always defined for system of more than one particle.

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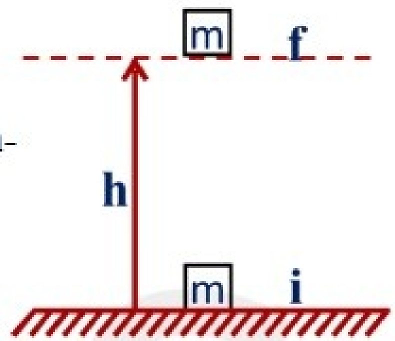
WPE

Gravitational Potential Energy



$U_f = U_i + mgh$

If we take U at ground = 0 as reference value then

$U = mgh$

$W_g = -\Delta U = U_i - U_f$

$= mgh_i - mgh_f$

$= mg(h_i - h_f)$

Potential energy in terms of Force (conservative)

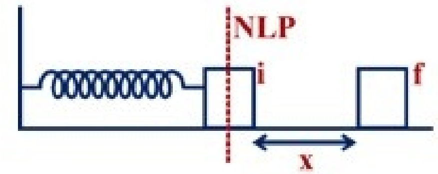
$$U_f - U_i = -\left(\int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy + \int_{z_i}^{z_f} F_z dz\right)$$

Mechanical Energy & its Conservation

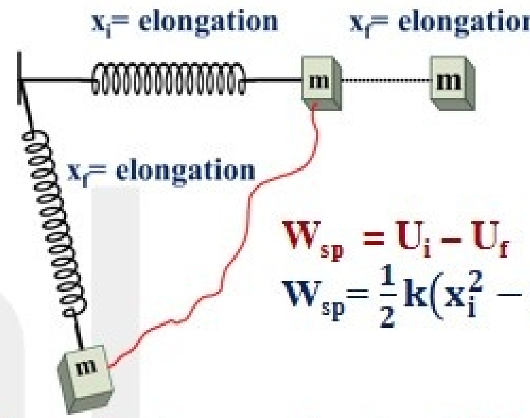
Mechanical Energy =
Potential Energy + Kinetic energy

ME = U + KE

Potential energy associated with Spring Force



$$U_f - U_i = \frac{1}{2} K x^2$$



$W_{sp} = U_i - U_f$

$W_{sp} = \frac{1}{2} k(x_i^2 - x_f^2)$

Force in terms of potential energy

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

$$\vec{F} = -\left[\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} + \frac{\partial U}{\partial z}\hat{k}\right]$$

A body is said to be in equilibrium when

$F_{net} = 0$

Stable Equilibrium

- It is an equilibrium where on slight displacement of particle from equilibrium position a force acts on particle which try to bring the particle back to equilibrium position.
- Such force is restoring and opposite to displacement.

$$\frac{dU}{dx} = 0 \quad \text{and} \quad \frac{d^2U}{dx^2} = +ve \quad \text{Stable Equilibrium}$$

$$\frac{dU}{dx} = 0 \quad \text{and} \quad \frac{d^2U}{dx^2} = -ve \quad \text{Unstable Equilibrium}$$

Unstable Equilibrium

- It is an equilibrium where on slight displacement of particle from equilibrium position a force acts on particle which tries to take the particle away from the equilibrium position.

Neutral Equilibrium

- It is an equilibrium where on slight displacement of particle from equilibrium position the particle remains in equilibrium position

Power

Power of a force is equal to rate of work done by that force.

$$P = \frac{dW}{dt}$$

$$P_{avg} = \langle P \rangle = \frac{\Delta W}{\Delta t}$$

S.I. unit of power is Watt (W)

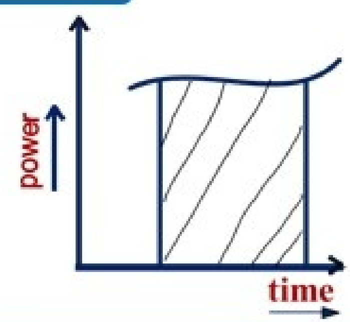
Other unit is Horse Power (HP)

1 HP = 746 W.

$P = \frac{dW}{dt}$ Slope of W-t graph gives instantaneous Power

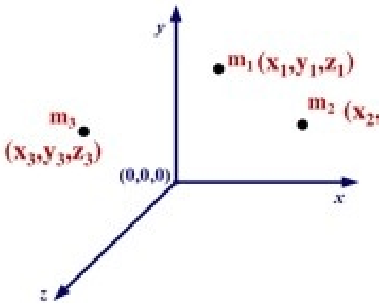
$W = \int dW = \int Pdt$

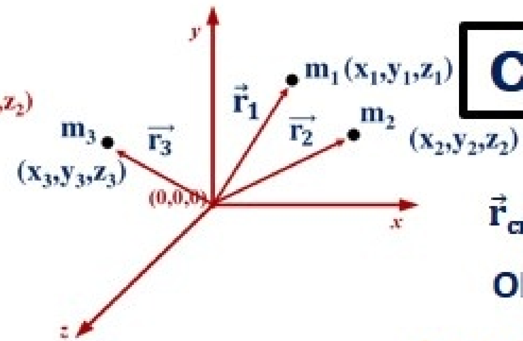
W is given by area under P-t graph



Center of Mass

Centre of mass of discrete point masses





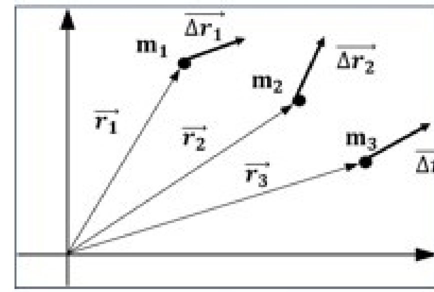
$$\vec{r}_{cm} = \frac{\sum m_i \vec{r}_i}{\sum m_i} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots}{m_1 + m_2 + \ldots}$$

OR $\vec{r}_{CM} = X_{CM}\hat{i} + Y_{CM}\hat{j}$

$$X_{cm} = \left(\frac{m_1x_1 + m_2x_2 + \ldots}{m_1 + m_2 + \ldots}\right) = \frac{\sum m_i x_i}{\sum m_i}$$

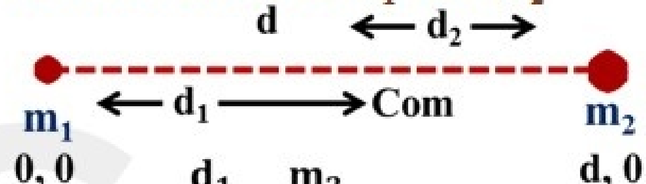
$$Y_{cm} = \left(\frac{m_1y_1 + m_2y_2 + \ldots}{m_1 + m_2 + \ldots}\right) = \frac{\sum m_i y_i}{\sum m_i}$$

Displacement of COM due to displacement of particles of system



$$\Delta\vec{r}_{cm} = \frac{m_1\Delta\vec{r}_1 + m_2\Delta\vec{r}_2 + \ldots\ldots}{m_1 + m_2 + \ldots\ldots}$$

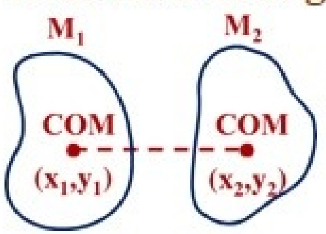
Locate COM of m_1 and m_2



$$\frac{d_1}{d_2} = \frac{m_2}{m_1}$$

Com of two particles divides internally the line joining two particles in inverse ratio of their masses.

Com of several groups of particles



$$X_{CM} = \frac{M_1x_1 + M_2x_2}{M_1 + M_2}$$

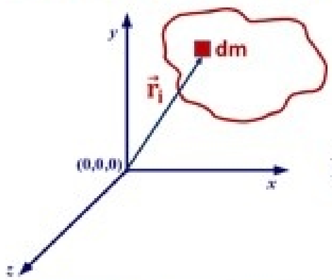
$$Y_{CM} = \frac{M_1y_1 + M_2y_2}{M_1 + M_2}$$

Velocity and Acceleration of COM

$$\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots\ldots}{m_1 + m_2 + \ldots\ldots}$$

$$\vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + \ldots\ldots}{m_1 + m_2 + \ldots\ldots}$$

Com of continuous bodies



$$\vec{r}_{cm} = \frac{\int \vec{r}_i\, dm}{\int dm}$$

Linear mass density (λ)

Mass per unit length is called linear mass density λ $\quad \lambda = \frac{M}{L}$

For uniform object, λ is same for every element

Surface mass density (σ)

Mass per unit area

For uniform objects $\sigma = \frac{M}{A}$

Volumetric mass density (ρ)

Mass per unit volume

If uniform then $\rho = \frac{M}{V}$

For uniform objects COM is at geometrical centre.

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Center of Mass

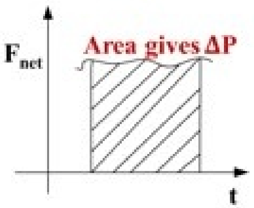
Linear Momentum

Linear Momentum is the product of Mass and Velocity $\vec{P} = m\vec{v}$

SI Unit is (kg m/s) or (N-s)

For a particle.

$$\vec{F}_{net} = \frac{d\vec{P}}{dt} \qquad <\vec{F}_{avg}> = \frac{\Delta\vec{P}}{\Delta t}$$



$$\int \vec{F}_{net}\, dt = \Delta\vec{P}$$

Kinetic Energy of a particle

$$\text{KE of particle} = \frac{P^2}{2m}$$

Linear Momentum for system of particles

It is vector sum of L.M. of all the particles in system.

$$\vec{P}_{sys.} = \sum \vec{P}_i = m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots\ldots = M\vec{V}_{cm}$$

M = Mass of System

$\vec{V}_{cm}$ = Velocity of COM of System

For calculation of linear momentum of a system we can assume whole mass to be concentrated at COM moving with $\vec{V}_{cm}$.

Kinetic Energy of System of particles

KE of system of particles is the algebraic summation of KE of all its constituent particles

$$KE = \sum \frac{1}{2} m_i v_i^2 \neq \frac{1}{2} M (V_{cm})^2$$

For calculation of Kinetic Energy of a system we CANNOT assume whole mass to be concentrated at COM moving with $\vec{V}_{cm}$.

Principle of conservation of L.M.

If net external force acting on a system is zero for a time interval then in that interval linear momentum of the system is conserved (i.e. remains constant)

Impulse-Momentum Theorem

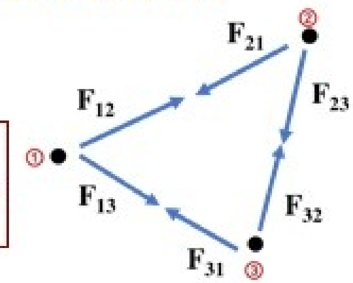
$$\boxed{\vec{I}_{net} = \vec{P}_f - \vec{P}_i}$$

Impulse is a vector quantity whose unit is same as that of momentum (kgm/s) or (N-s.)

(i) If linear momentum of a system is zero, then its KE may or may not be zero.
(ii) If KE of system is zero then its linear momentum must be zero.

Newton's Second Law for System of Particles

$$\boxed{\sum \vec{F}_{ext.} = M\vec{a}_{cm}}$$



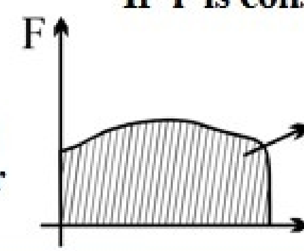
$$M\vec{a}_{cm} = F_{1ext} + F_{1int}\ F_{2ext} + F_{2int} + \ldots\ldots$$

Impulse : Impulse of a force for a time interval "t_1" to "t_2" is defined as

$$\boxed{\vec{I} = \int_{t_1}^{t_2} \vec{F}dt}$$

Area under F-t graph gives Impulse of Force F

If $\vec{F}$ is constant, $\vec{I} = \vec{F}\,\Delta t$



Center of Mass

Impulsive Force

Force which acts for a very small time duration and whose magnitude is very large is called Impulsive Force.

In presence of impulsive forces, non-impulsive forces (like mg, spring force) can be neglected.

Collision

Head On Collision

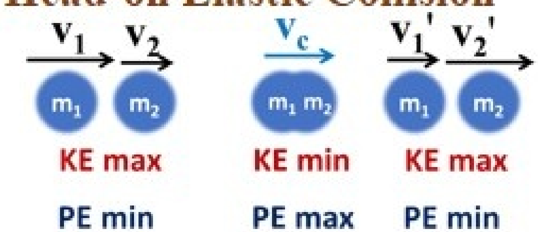
When during collision, velocities of both the objects are along the common normal of colliding objects

Oblique Collision

If velocities of any of the colliding objects is not along the common normal

Elastic bodies are bodies which regain their original shape without any loss of energy.

Head-on Elastic Collision



(i) Total ME of system remains constant.

(ii) Initial KE & Final KE are equal but it is not constant throughout the process.

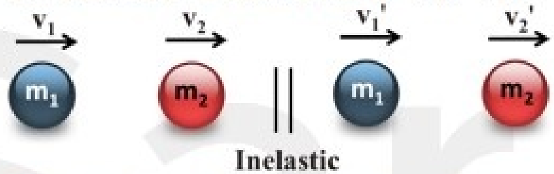
(iii) At the instant when velocities of both the bodies are same then PE is maximum & KE is minimum.

Coefficient of Restitution (e)

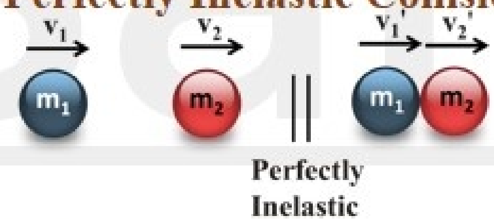
$$\boxed{e = \frac{\text{Velocity of separation}}{\text{Velocity of approach}}}$$

$$e = \frac{v_2' - v_1'}{v_1 - v_2}$$ For elastic collision e = 1

Inelastic Collision $0 < e < 1$



Perfectly Inelastic Collision $e = 0$



Both bodies stick together and move with same velocity after collision

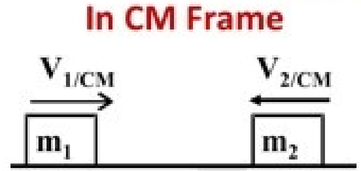
COM Frame

$\vec{P}_{sys} = M\vec{v}_{cm}$ $\quad \vec{P}_{sys/cm} = M\vec{v}_{cm/cm}$

$\vec{P}_{sys/cm} = 0$

In Centre of Mass frame, Linear Momentum of a System is Zero

Kinetic Energy of Two Particle System in CM Frame



$$\frac{m_1 m_2}{m_1 + m_2} = \mu$$

(reduced mass)

$$\boxed{KE_{sys/CM} = \frac{1}{2}\mu\,(v_{rel})^2}$$

$$\boxed{KE_{sys} = KE_{sys/CM} + \frac{1}{2}(M)\,V_{cm}^2}$$

M → total mass of system

If $m_1 = m_2$

$v_2' = v_1 \quad v_1' = v_2$

$\Rightarrow$ velocities are interchanged.

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Center of Mass

KE loss in collision

$$KE_{loss} = KE_i - KE_f$$

$$= \frac{1}{2}\mu v_{app.}^2 (1 - e^2) \quad \left(\because e = \frac{v_{sep}}{v_{app}}\right)$$

Variable Mass

$v_{rel.}$ is known as exhaust speed which is the speed of exhaust relative to the rocket

$$M(\Delta v) = \Delta m\, v_{rel.}$$

$$F_{thrust} = M\frac{\Delta v}{\Delta t} = \frac{\Delta m}{\Delta t} v_{rel.}$$

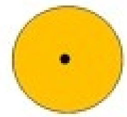
Recoil of Gun

$$\boxed{F_{thrust} = \frac{m_{bullet} v_{rel.}}{\Delta t}}$$

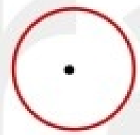
$v_{rel.}$ is known as muzzle velocity i.e. velocity of bullet w.r.t gun.

Center of Mass of some Shape

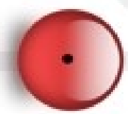
Disk



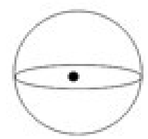
Ring



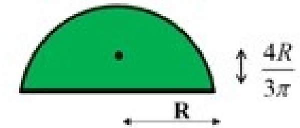
Solid Sphere



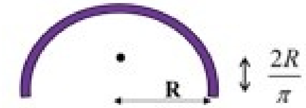
Hollow Sphere



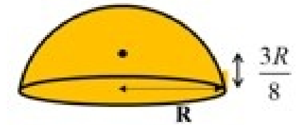
Semi circular plate



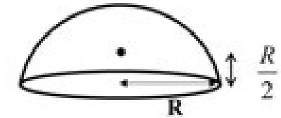
Semi circular ring



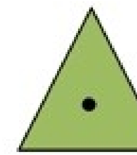
Solid hemisphere



Hollow hemisphere

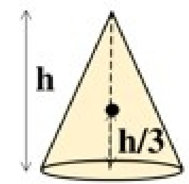


Triangle

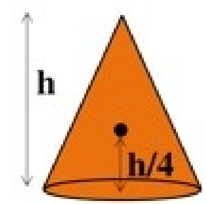


Centroid

Hollow cone

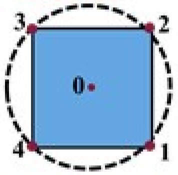


Solid cone



Rotational Motion

CRTM



1. Since distance between two particles of a rigid body remains constant, so the relative motion of one particle w.r.t. other particle is circular motion.

2. Angular velocity of all the particles about a given point of a rigid body is same

$\omega_{1/0} = \omega_{2/0} = \omega_{3/0} = \omega_{4/0} = \omega_0$

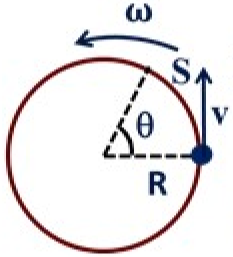
3. This angular velocity of a rigid body about all the point of rigid body is same.

$\omega_{0/1} = \omega_{2/1} = \omega_{3/1} = \omega_{4/1}$
$\parallel$
$\omega_{1/0} = \omega_{2/0} = \omega_{3/0} = \omega_{4/0}$ $= \omega_0$ → 'ω' of rigid body

Kinematics of Pure rotation motion

$\boxed{S = R\theta}$

$\boxed{|v| = R\omega}$



If $\alpha =$ constant

$\omega_f = \omega_i + \alpha t$

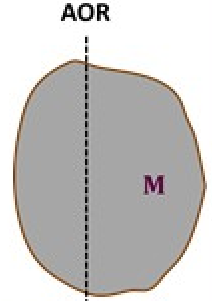
$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$\omega_f^2 = \omega_i^2 + 2\alpha\theta$

$\theta = \left(\frac{\omega_i + \omega_f}{2}\right)t$

$\theta = \omega_f t - \frac{1}{2}\alpha t^2$

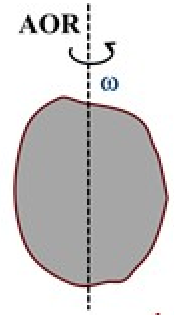
Radius of Gyration



$K = \sqrt{\frac{I}{M}}$ → RoG about an axis

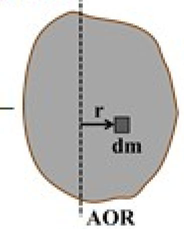
$I = MK^2$

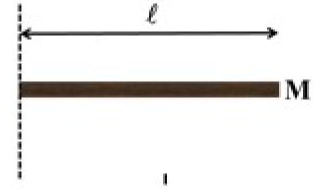
Moment of Inertia



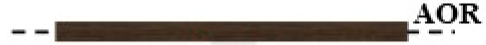
$KE_{sys} = \frac{1}{2} I\omega^2$

$I = \int r^2\, dm$

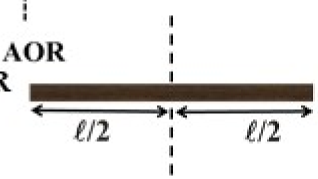




$I = \frac{M\ell^3}{3}$

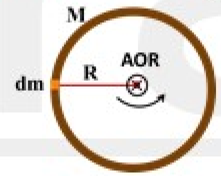


$I = 0$



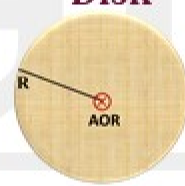
$I = \frac{M\ell^2}{12}$

Uniform Ring



$I = MR^2$

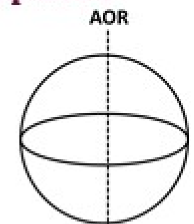
Disk



$I = \frac{MR^2}{2}$

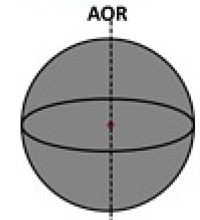
Moving mass parallel to AOR does not change moment of inertia.

Uniform Hollow Sphere



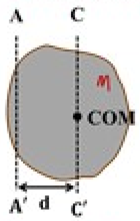
$I = \frac{2}{3}MR^2$

Uniform Solid Sphere



$I = \frac{2}{5}MR^2$

Parallel – axis theorem:

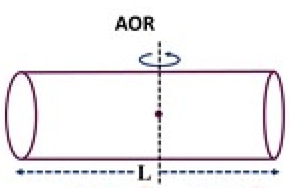


Among the given parallel axis MOI will be minimum about the axis passing through COM.

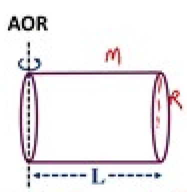
MOI about the parallel axis having same distance from COM is equal.

$I_{AA'} = I_{CC'} + Md^2$

Hollow Cylinder

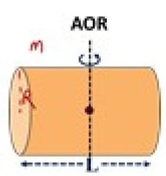


$I = \frac{MR^2}{2} + \frac{ML^2}{12}$

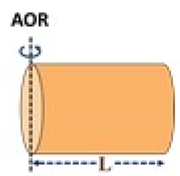


$I = \frac{MR^2}{2} + \frac{ML^2}{3}$

Solid Cylinder

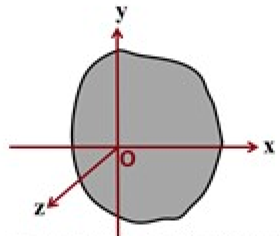


$I = \frac{MR^2}{4} + \frac{ML^2}{12}$



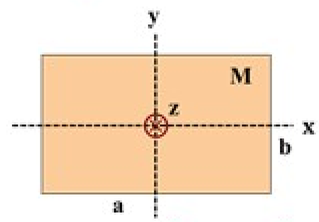
$I = \frac{MR^2}{4} + \frac{ML^2}{3}$

Perpendicular axis theorem:



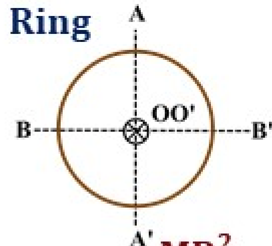
Planar body in (x-y)plane

$I_z = I_x + I_y$



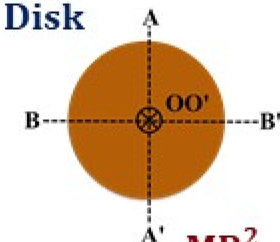
$I_z = \frac{Mb^2}{12} + \frac{Ma^2}{12}$

Ring



$I_{AA'} = \frac{MR^2}{2}$

Disk



$I_{AA'} = \frac{MR^2}{4}$

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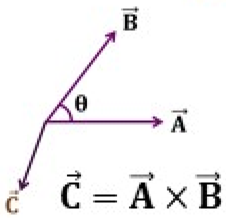
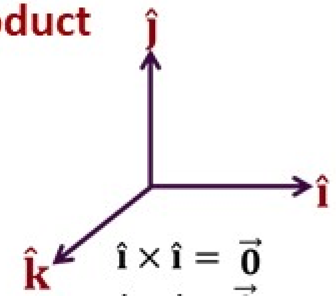
Rotation Motion

Cross Product





$\vec{C} = \vec{A} \times \vec{B}$

$|\vec{C}| = |\vec{A}||\vec{B}| \,\text{Sin}\theta$

θ is angle b/w $\vec{A}$ & $\vec{B}$

Properties

$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$

$\vec{A} \times \vec{B} = -(\vec{B} \times \vec{A})$

$\vec{A} \times \vec{A} = \vec{0}$

$\hat{i} \times \hat{i} = \vec{0}$

$\hat{j} \times \hat{j} = \vec{0}$

$\hat{k} \times \hat{k} = \vec{0}$

$\hat{i} \times \hat{j} = \hat{k}$

$\hat{j} \times \hat{k} = \hat{i}$

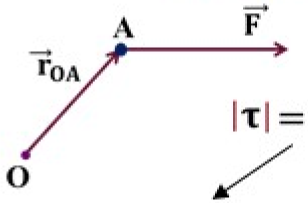
$\hat{k} \times \hat{i} = \hat{j}$

$\hat{j} \times \hat{i} = -\hat{k}$

$\hat{k} \times \hat{j} = -\hat{i}$

$\hat{i} \times \hat{k} = -\hat{j}$

Torque

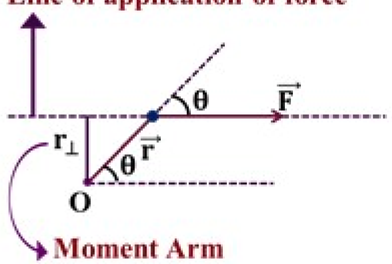


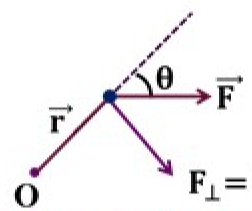
$\vec{\tau}_O = \vec{r}_{OA} \times \vec{F}$

$|\tau| = rF\sin\theta$

$(r\sin\theta)F$ $\quad$ $r(F\sin\theta)$

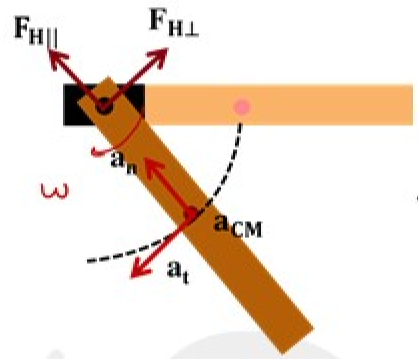
$\boxed{|\tau| = r_\perp F}$ $\quad$ $\boxed{|\tau| = rF_\perp}$





$F_\perp$ is perpendicular component of $\vec{F}$

Calculation of Hinge Force



$$F_H = \sqrt{F_{H||}^2 + F_{H\perp}^2}$$

$(F_{H\perp})$ Perpendicular to rod

$$\sum F_\perp = m(a_{CM})_t = m\alpha\frac{L}{2}$$

$\tau = I\alpha \rightarrow \alpha \rightarrow (a_{CM})_t \rightarrow F_{H\perp}$

$(F_{H||})$ Parallel i.e. along the rod

$$\sum F_{||} = m(a_{CM})_n = m\omega^2\frac{L}{2}$$

W-E Theorem $\rightarrow \omega \rightarrow (a_{CM})_n \rightarrow F_{H||}$

Condition of Equilibrium

Translation Equilibrium $\quad$ **Rotational Equilibrium**

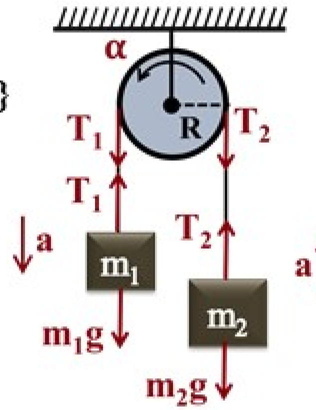
1) $\sum \vec{F} = 0 \rightarrow \sum \vec{F}_x = 0,\ \sum \vec{F}_y = 0,\ \sum \vec{F}_z = 0$

2) $\sum \vec{\tau} = 0$

If $\vec{F}_{net} = 0$ on a rigid body then $\vec{\tau}_{net}$ is same about every point of space.

Inertial Pulley

{No slipping of rope on disk}



$T_1R - T_2R = I\alpha$ ....(1)

$m_1g - T_1 = m_1a$ ....(2)

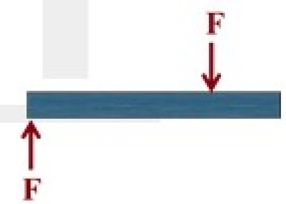
$T_2 - m_2g = m_2a$ ....(3)

$\alpha R = a$ ....(4)

Work done in terms of Torque

$$W = \int \tau d\theta \qquad P = \tau\,\omega$$

Force Couple



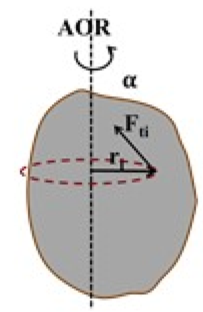
A pair of equal & opposite forces is called force couple.

Torque of force couple is same about any point of the space.

$$\sum \tau_{int} = 0$$

$$\left(\sum \tau_{ext}\right)_{AOR} = (I)_{AOR}\alpha$$

Valid only in inertial frame.



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Rotational Motion

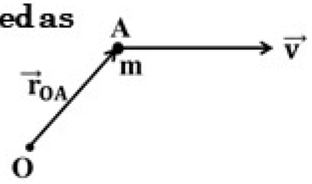
Angular Momentum of Particle

Angular Momentum of Particle about point O is defined as

$\vec{L}_O = \vec{r}_{OA} \times \vec{P}$

$= \vec{r}_{OA} \times (m\vec{v})$

$= m\,\vec{r}_{OA} \times \vec{v}$



where $\vec{P}$ is linear momentum of the particle present at A.

$\vec{L}_O$ is angular momentum about O.

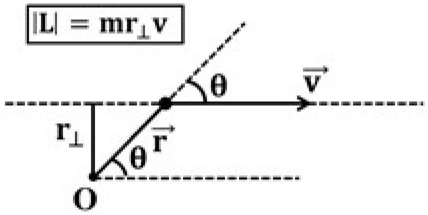
$$\vec{L} = \vec{r} \times \vec{P}$$

AM is also known as moment of LM.

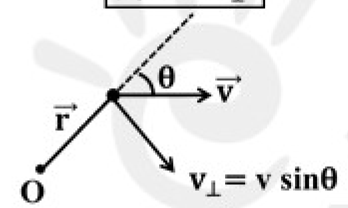
$|L| = mrv\sin\theta$

$m(r\sin\theta)v \Rightarrow \vec{L} = \text{Constant}$ $\quad$ $mr(v\sin\theta)$

$\boxed{|L| = mr_\perp v}$ $\quad$ $\boxed{|L| = mrv_\perp}$

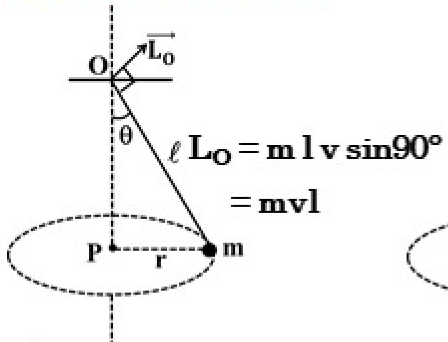


$r_\perp$ is the shortest distance from O to line of velocity.



$v_\perp$ is perpendicular to $\vec{r}$

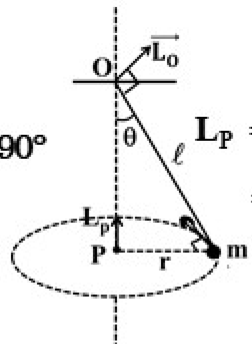
Conical Pendulum



$L_O = m\,l\,v\sin 90°$

$= mvl$

$\vec{L}_O$ is not constant but its magnitude is constant.



$L_P = m\,\ell\sin\theta\, v\sin 90°$

$= mv\ell\sin\theta$

$\vec{L}_P$ is constant

Angular Momentum Conservation Principle

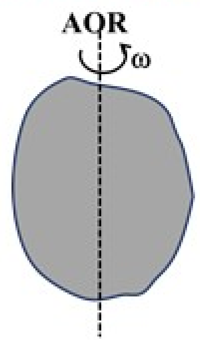
$\vec{\tau}_{net} = \dfrac{d\vec{L}}{dt}$ $\quad$ If $\vec{\tau}_{net} = 0 \Rightarrow \vec{L} = \text{Constant}$

If $\vec{\tau}_{net}$ is zero then its A.M. is conserved.

Angular Momentum of System of Particles

$$\vec{L}_{sys} = \sum \vec{L}_i$$

A.M. of Rigid Body Performing Pure Rotation About Fixed Axis



$$(L_{sys})_{AOR} = I_{AOR}\,\omega$$

Relation Between Torque and Angular Momentum

$\vec{\tau}_{Net} = \dfrac{d\vec{L}}{dt}$ valid only in inertial frame.

$\vec{\tau}_{Net} = \dfrac{d\vec{L}}{dt}$

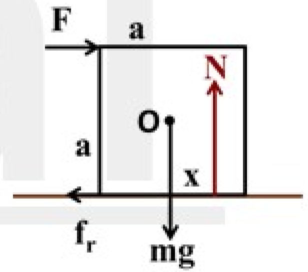
Particle: $\vec{\tau}_{Net} = \dfrac{d\vec{L}}{dt}$

System: $\vec{\tau}_{Net} = \dfrac{d\vec{L}_{sys}}{dt}$

$\sum \tau_{ext} + \sum \tau_{int} = 0$

$(\vec{\tau}_{ext})_{Net} = \dfrac{d\vec{L}_{sys}}{dt}$

Condition of Toppling



$x > \dfrac{a}{2}$ Toppling will occur

$x = \dfrac{a}{2}$ On verge of toppling

$x < \dfrac{a}{2}$ No toppling

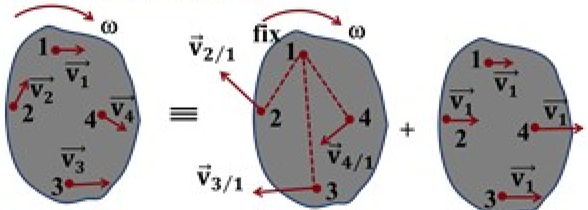
Angular Impulse

Linear Impulse $(\vec{I})$	Angular Impulse $(\vec{J})$
$\vec{I} = \int \vec{F}\,dt$	$\vec{J} = \int \vec{\tau}\,dt$
Unit - Ns	Unit - Nms
$\vec{I}_{net} = \vec{P}_f - \vec{P}_i$	$\vec{J}_{net} = \vec{L}_f - \vec{L}_i$

$\boxed{\vec{J} = \vec{r} \times \vec{I}}$

Combined Rotation + Translational Motion [CRTM]

Velocity Analysis



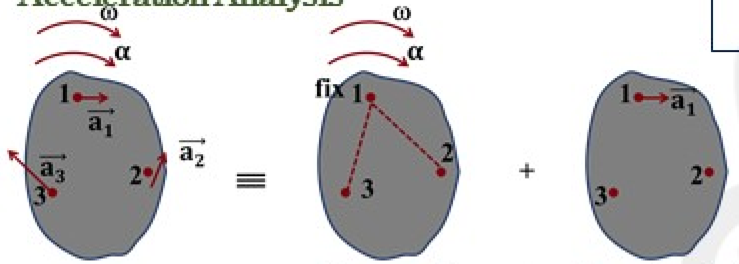
CRTM ≡ Pure rotation + Pure translation

$\vec{v}_2 = \vec{v}_{2/1} + \vec{v}_1$

$\vec{v}_3 = \vec{v}_{3/1} + \vec{v}_1$

$\vec{v}_4 = \vec{v}_{4/1} + \vec{v}_1$

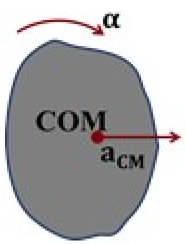
Acceleration Analysis



CRTM ≡ Pure rotation + Pure translation

Dynamics of CRTM

For analysing its motion we apply two equation



$\sum \vec{F}_{ext} = M\vec{a}_{CM}$

$\sum \vec{\tau}_{ext} = I\vec{\alpha}$

Dynamics of CRTM

Second equation is valid in inertial frame.

1) $\sum \vec{F} = M\vec{a}_{CM}$ 2) $\sum \vec{\tau} = I\vec{\alpha}$

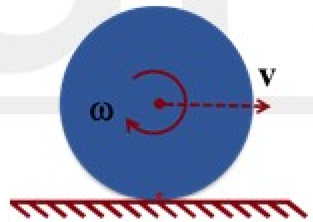
$\sum \vec{F}_{ext} = M\vec{a}_{CM}$ $\sum \vec{\tau}_{ext} = I\vec{\alpha}$

To apply second equation about non-inertial point, Pseudo force is applied at COM of body & τ of Pseudo force is also taken into account.

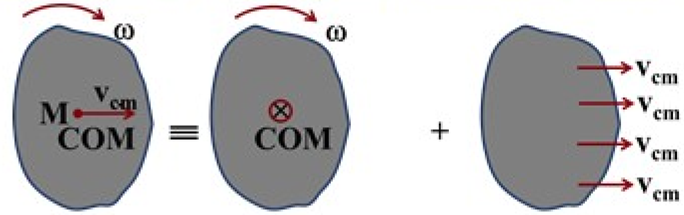
Rolling Motion

If during the motion, there is no relative slipping between the points of contact, then motion is called Rolling.

$$v = \omega R$$



Kinetic Energy of Rigid Body Performing CRTM

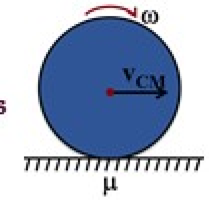


CRTM ≡ Rotational KE + Translational KE

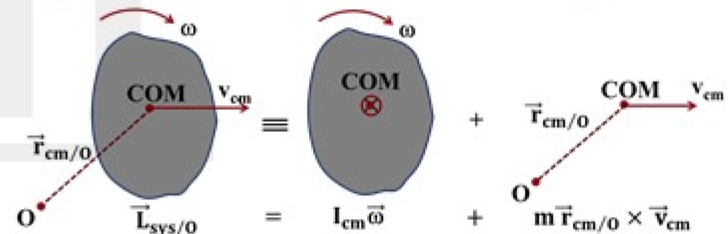
$$KE = \frac{1}{2}I_{CM}\omega^2 + \frac{1}{2}Mv_{CM}^2$$

Keypoint

For a body performing rolling motion over a fixed surface, work done by friction force on the body will be zero as velocity of point of application of friction always zero.



Angular Momentum of R.B. Performing CRTM



$\vec{L}_{sys/O} = I_{cm}\vec{\omega} + m\vec{r}_{cm/O} \times \vec{v}_{cm}$

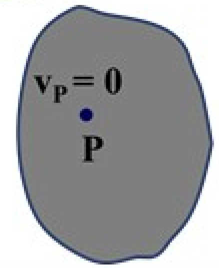
Rotational as rigid body; Pure rotation about COM

Translation as a particle

$$\vec{L}_{sys/O} = I_{cm}\vec{\omega} + m\vec{r}_{cm/O} \times \vec{v}_{cm}$$

Instantaneous Center of Rotation (I_{COR}) & Axis of Rotation (I_{AOR})

- Let at an instant of time velocity of point P is zero.
- To calculate velocity of other points of rigid body, rigid body can be assumed to perform pure rotation about an axis passing through point P at that instant. This point is called I_{COR} and axis passing through it is called I_{AOR}.



GRAVITATION

ESCAPE VELOCITY

$$V_e = \sqrt{\frac{2GM_e}{R}}$$

AT SURFACE $= \sqrt{\frac{GM_e}{R}} = \sqrt{2gR}$

$= 11.2$ km/s (earth)

$$U = -\frac{GM_1M_2}{r}$$

r or r

SATELLITE

$V = \sqrt{\frac{GM_e}{r}}$ $\quad T = 2\pi\sqrt{\frac{r^3}{GM_e}}$

$$\frac{|PE|}{2} = |KE| = |ME|$$

GEOSTATIONARY

$T = 24$ hrs, West to East, $\omega = 0$ (w.r.t. earth)

Parking Orbit, telecomm, weather forecast

$h = 3600$ km ; $V = 3.08$ km/sec

TUNNEL

$$T = 2\pi\sqrt{\frac{R^3}{GM}} \quad \left(2\pi\sqrt{\frac{L}{g}}\right)$$

g

$g_0\left(1+\frac{h}{R}\right)^2 \approx g_0\left(1-\frac{2h}{R}\right)$

d, R

$g_0\left(1-\frac{d}{R}\right)$ $\quad g_0 = \frac{GM_e}{R^2}$

$V < V_e$ Ellipse $\left(\frac{V_e}{\sqrt{2}}\right)$

$V = V_e$ Parabola

$V > V_e$ Hyperbola

6.67×10^{-11}

$$\frac{GM_1M_2}{r^2}$$

Binary Star about COM

$g_{app} = g_0$ at pole max

$g_{app} = g_0 - \omega^2 R_e$ (min) (at Equator)

g_0 more at poles than Equator (Non Sph)

FIELD

M, r: $\frac{GM}{r^2}$

HOLLOW: ZERO, $\frac{GM}{r^2}$

SOLID: R, r, $\frac{GMr}{R^3}$, $\frac{GM}{R^2}$

KEPLER'S LAWS

① SUN AT FOCI

② EQUAL AREA IN EQUAL TIME

AREAL VELOCITY $\leftarrow \frac{dA}{dt} = \frac{L}{2M_p}$

③ $T = 2\pi\sqrt{\frac{a^3}{GM_{sun}}}$

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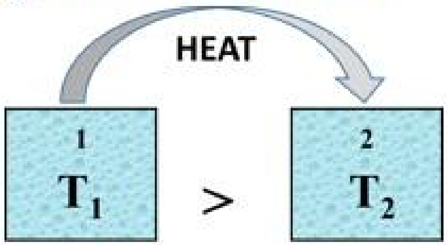
Calorimetry

Heat

Transfer of energy that takes place solely due to temperature difference is called Heat flow or Heat transfer and the amount of energy transferred is called Heat.



Unit of Heat is

Joule & Calorie

1 Calorie = 4.18 J

Dimensions are ML^2T^{-2}

Calorie

1 calorie is the amount of heat required to the raise the temperature of 1 gm of water from 14.5°C to 15.5°C at 1 atm pressure.

Thermal Equilibrium

Two bodies are said to be in thermal equilibrium when no heat flows between them.

If two bodies are in thermal equilibrium with each other then their temperature is same.

Zeroth Law of Thermodynamics

If bodies 'A' & 'B' are in thermal equilibrium with each other & bodies 'B' & 'C' are in thermal equilibrium with each other then bodies 'A' & 'C' must be in thermal equilibrium with each other.

If two bodies are in thermal equilibrium with each other then their temperature is same.

$\Delta Q \propto \Delta T$ $\qquad$ $\Delta Q \propto m$

$$\Delta Q = m\underset{\substack{\downarrow \\ \text{Specific Heat Capacity}}}{s}\Delta T$$

Specific Heat Capacity(s)

$$\Delta Q = ms\Delta T \qquad s = \frac{\Delta Q}{m\Delta T}$$

Specific Heat Capacity of a substance is defined as heat required to raise a unit temperature of a unit mass of the substance.

$s = \dfrac{\Delta Q}{m\Delta T}$ SI unit : Joule/kg–K

$s = \dfrac{\Delta Q}{m\Delta T}$ CGS unit : cal/g –°C $\quad \rho = \dfrac{M}{V}$

Specific heat capacity of water

s_{water} = 1 cal/g – °C
= 1 cal / g – K
= 4200 joule / kg – K

Heat Capacity

Heat capacity (C) = mass × specific heat capacity

$C = ms$

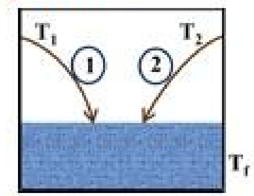
$\Delta Q = C\Delta T$

Molar Heat Capacity

Heat required to raise the temperature of 1 mole of substance by 1°C.

In an Isolated System

$T_2 > T_1$



Isolated System

A system is said to be isolated if no exchange or transfer of heat can take place to and from the surrounding.

Principle of Calorimetry

According to this- In an isolated system heat lost by the part at higher temperature is equal to heat gained by the part at lower temperature.

This is based on energy conservation principle

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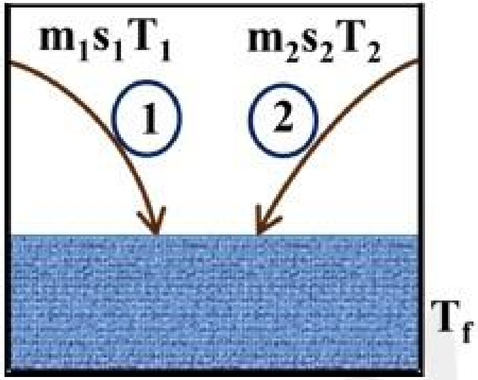
Calorimetry

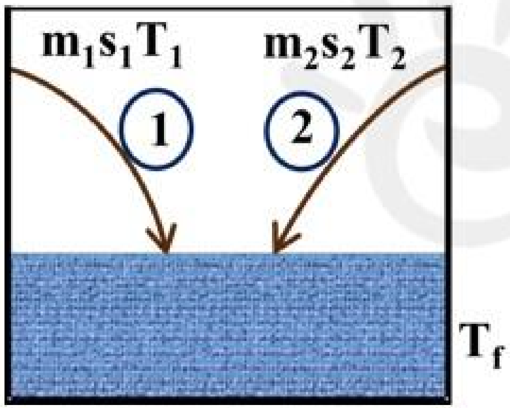
In an Isolated System

$T_2 > T_1$

M-1 : Heat gain = Heat loss

$$m_1 s_1 (T_f - T_1) = m_2 s_2 (T_2 - T_f)$$





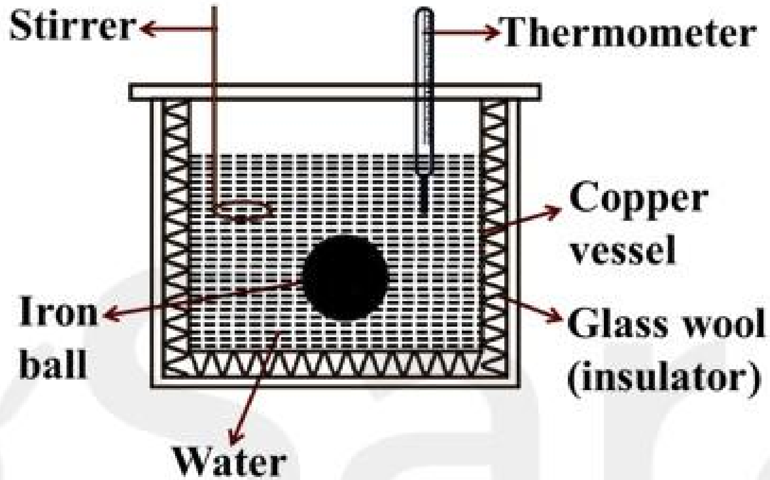
M-1 : Heat gain = Heat loss

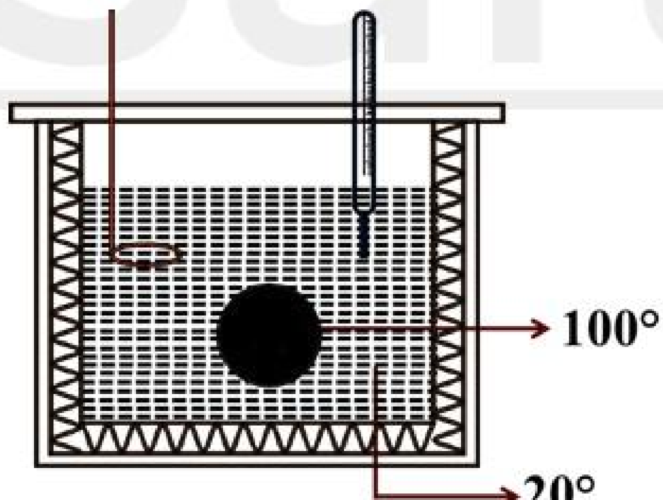
$$m_1 s_1 (T_f - T_1) = m_2 s_2 (T_2 - T_f)$$

M-2 : Σ Heat gain = 0

$$m_1 s_1 (T_f - T_1) + m_2 s_2 (T_f - T_2) = 0$$

Calorimeter





$$m_w s_w (T_f - 20) + m_c s_c (T_f - 20) = m_i s_i (100 - T_f)$$

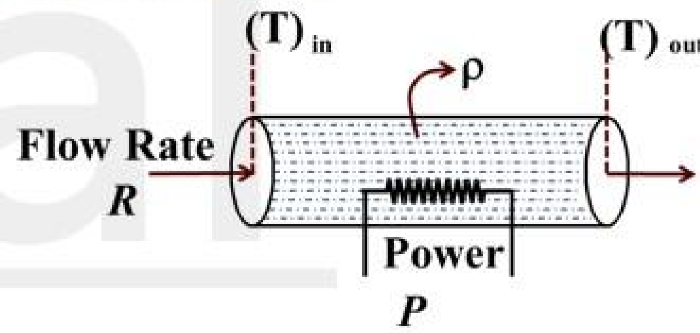
$$m_c s_c = m_{w\,eq} s_w$$

Water Equivalent

$$m_w s_w (T_f - 20) + m_{w\,eq} s_w (T_f - 20) = m_i s_i (100 - T_f)$$

$$s_w (T_f - 20)(m_w + m_{w\,eq}) = m_i s_i (100 - T_f)$$

Flow Calorimeter



$$\Delta Q = m s_L \Delta T \qquad \frac{\Delta Q}{\Delta t} = \frac{m s_L \Delta T}{\Delta t}$$

$$P = \underbrace{\rho\left(\frac{\text{volume}}{\Delta t}\right)}_{\text{Flow Rate}} s_L(\Delta T)$$

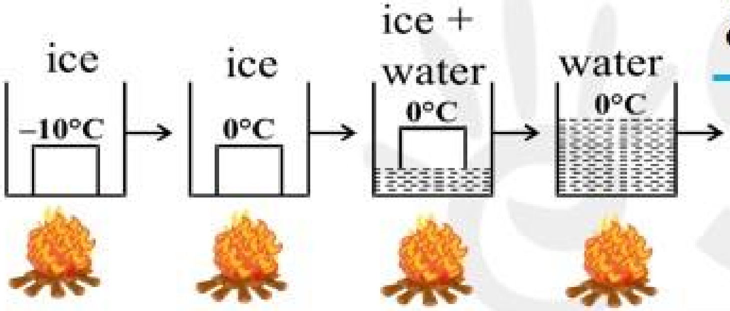
$$\boxed{P = \rho R s L(\Delta T)}$$

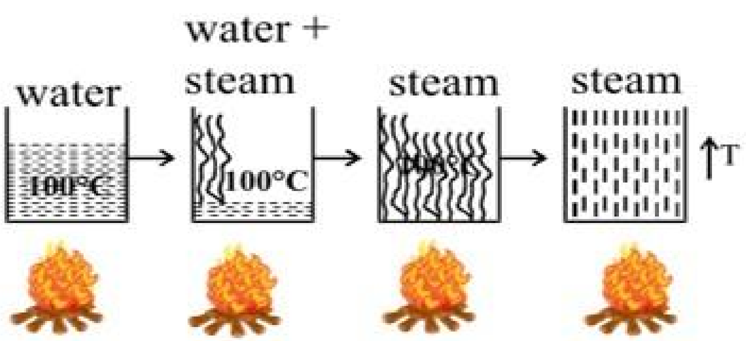
Calorimetry

Change of State

During the change of state temperature of substance do not change on supplying the heat.

Both solid & liquid state (or liquid & vapor state) of the substance co-exist at thermal equilibrium during the change of state.





Latent Heat

$\Delta Q \propto m$

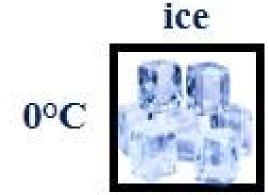
$\Delta Q = mL$ Latent Heat

$$L = \frac{\Delta Q}{m}$$

It is the heat required to change the state of unit mass

Latent heat of fusion

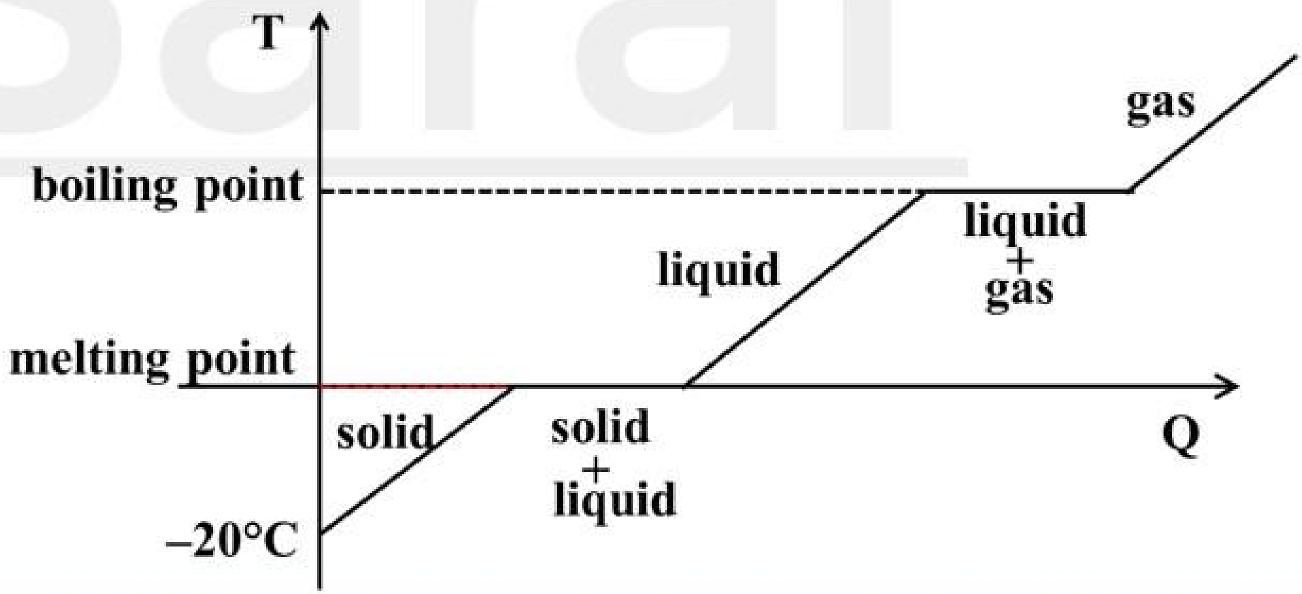
Latent heat of fusion for ice = 80 cal/g



Latent heat of vaporization

Latent heat of vaporization for water = 540 cal/g

Graph Between T v/s Heat Supplied



Elasticity

When a body is subjected to deforming forces, a restoring force is developed in the body. This restoring force per unit area is known as Stress.

$$\text{Stress} = \frac{\text{Restoring force}}{\text{Area of cross section}} = \frac{F}{A}$$

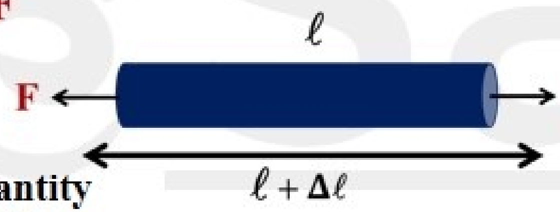
SI Unit : N/m^2 Dimension : $M^1 L^{-1} T^{-2}$

Longitudinal Stress (Normal Stress)

Longitudinal Stress (Tensile Stress)

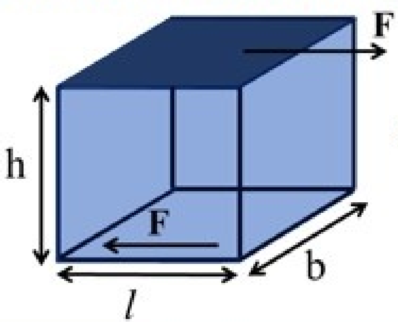
Longitudinal Stress (Compressive Stress)

$$\text{Longitudinal Strain} = \frac{\Delta\ell}{\ell}$$

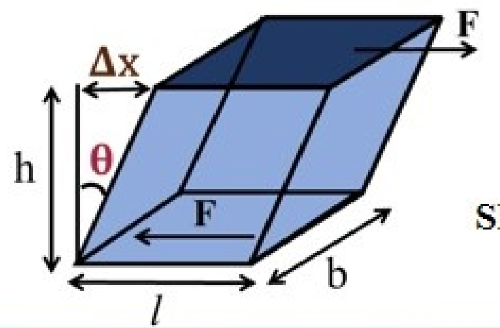


Unitless and dimensionless quantity

Tangential Stress or Shear Stress



$$\text{Shear Stress} = \frac{F}{\text{Area}} = \frac{F}{l \times b}$$



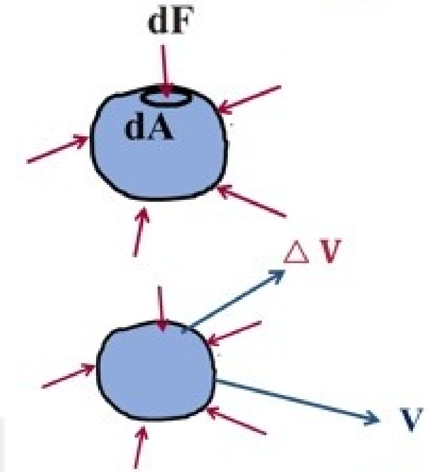
$$\text{Shear Strain} = \frac{\Delta x}{h} = \tan\theta \approx \theta \text{ (for small } \theta\text{)}$$

The property of body by virtue of which it tends to regain its original size & shape when the applied forces are removed, is known as Elasticity & the deformation caused is known as Elastic deformation.

If we apply a force to a lump or mud or putty, they have no cross tendency to regain their previous shape & they get permanently deformed. Such substances are caused Plastic & the property is called Plasticity.

Volumetric Stress

$$\text{Volumetric Stress} = \frac{dF}{dA}$$



$$\text{Volumetric Strain} = \frac{\triangle V}{V}$$

Hooke's Law

For small deformation, Stress and Strain are proportional to each other.

stress $\propto$ strain

stress $= k$ strain

Elasticity

Young's Modulus

Young's Modulus is a property of material of the body.

$$Y = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$$

$$\boxed{Y = \frac{F}{A}\frac{\ell}{\Delta\ell}}$$

Poisson's Ratio (σ)

σ deals with corresponding change in diameter with change in length

$$\sigma = -\frac{\Delta d / d}{\Delta \ell / \ell}$$

The stress at which material breaks is called **Breaking Stress.**

$$\boxed{B.S. = \frac{\text{Force}}{\text{Area}}}$$

property of a material

Force that a material can bear ∝ Area of cross-section

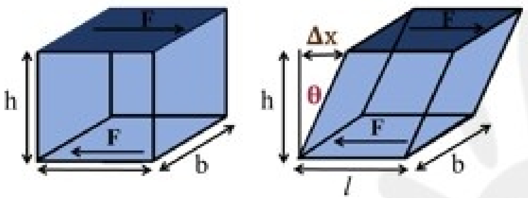
Elastic Potential Energy

$$U = \frac{1}{2}\frac{AY}{\ell}(\Delta\ell)^2$$

Energy Density

$$\text{Energy Density} = \frac{1}{2} \times \text{Stress} \times \text{Strain}$$

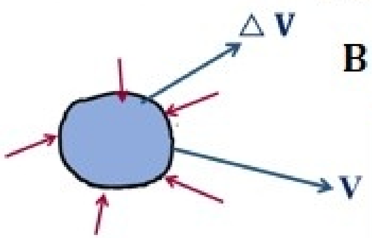
Shear Modulus / Modulus of rigidity / Torsional Modulus [G or η]



$$G = \frac{\text{Shear Stress}}{\text{Shear Strain}}$$

$$\boxed{G = \frac{F}{A\theta}}$$

Bulk Modulus (B)

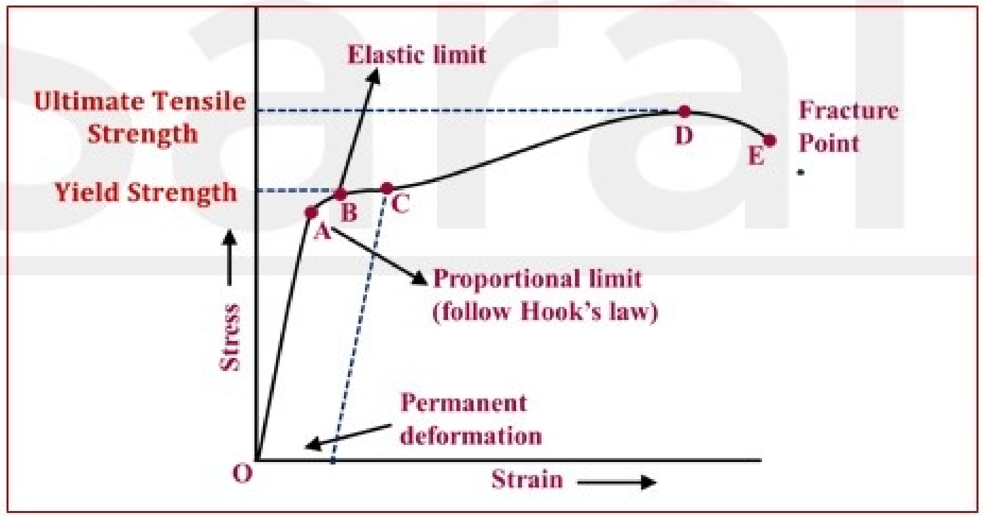


$$B = \frac{\text{Volumetric stress}}{\text{Volumetric strain}}$$

$$\boxed{B = -\frac{\Delta P}{\frac{\Delta V}{V}}}$$

ΔP is excess (additional) pressure which caused ΔV.

Stress-Strain Curve

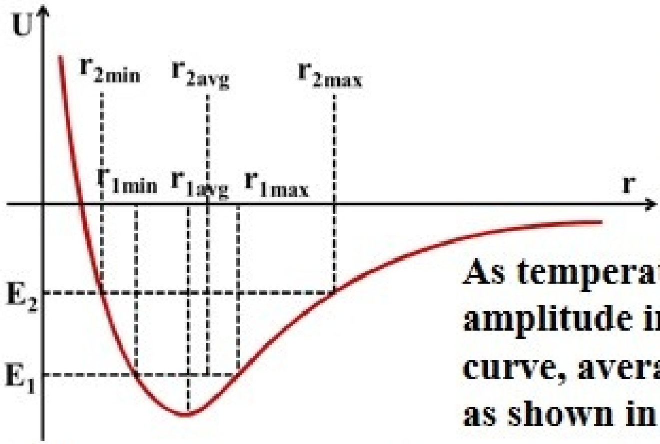


- If the ultimate strength and fracture points D and E are close then the material is said to be Brittle.
- If they are far apart then the material is said to be Ductile.

$$\boxed{\text{Compressibility} = \frac{1}{\text{Bulk modulus}} = -\frac{\Delta V / V}{\Delta P}}$$

Thermal Expansion

Potential Energy v/s Distance Curve



$T_2 > T_1$

$E_2 > E_1$

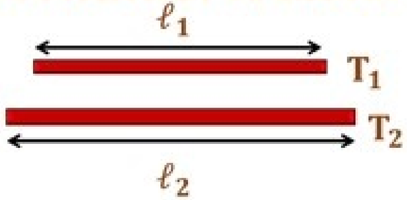
$r_{2avg} > r_{1avg}$

(due to asymmetric nature of curve)

As temperature increases, relative vibrational amplitude increases. Due to asymmetric nature of curve, average distance between the atoms increases as shown in the curve.

On increasing temperature, most of the substances expand. This expansion in dimension of body due to increase in temperature is called **Thermal expansion.**

Coefficient of Linear Expansion (α)

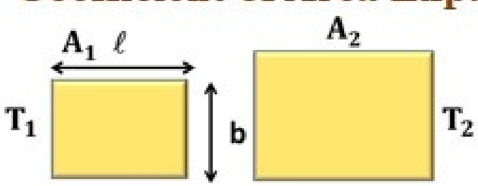


$$\frac{\Delta \ell}{\ell_1} = \alpha \Delta T$$

α = Coefficient of linear expansion

$$\ell_2 = \ell_1[1 + \alpha(T_2 - T_1)]$$

Coefficient of Area Expansion (β)

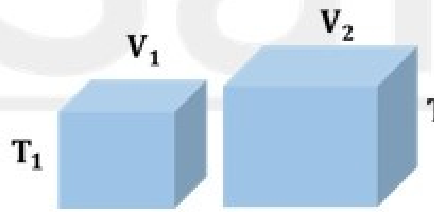


$$\frac{\Delta A}{A_1} = \beta \Delta T$$

β = Coefficient of Area expansion

$$A_2 = A_1(1 + \beta \Delta T)$$

Coefficient of Volume Expansion (γ)



$$\frac{\Delta V}{V_1} = \gamma \Delta T$$

γ = Coefficient of volume expansion

$$V_2 = V_1(1 + \gamma \Delta T)$$

[Unit of α, β, γ is /°C or $°C^{-1}$ or /K or K^{-1}]

Calculating Fractional Change or Percentage Change

$Z = k A^x B^y$ (k: Constant)

$$\frac{dZ}{Z} = x\frac{dA}{A} + y\frac{dB}{B}$$

For small change

$$\frac{\Delta Z}{Z} \times 100 = x\frac{\Delta A}{A} \times 100 + y\frac{\Delta B}{B} \times 100$$

(%change in Z) = x (%change in A) + y (% change in B)

Time period of a Simple Pendulum

$$T = 2\pi\sqrt{\frac{\ell}{g}} \qquad \frac{\Delta T}{T} = \frac{1}{2}\alpha \Delta\theta$$

$$\text{Time lost or gain in a day} = \frac{\Delta T}{T} \times 24 \times 3600$$

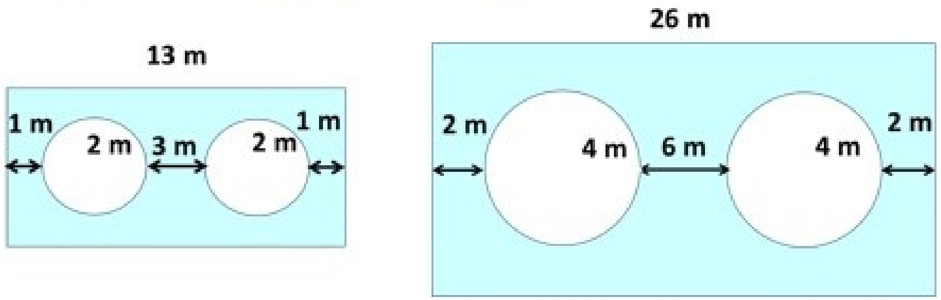
When θ ↑ , ℓ ↑, T↑
clock will run slow & time will be lost.

When θ ↓ , ℓ ↓, T ↓
clock will run fast & time will be gained.

Thermal Expansion

Isotropic Expansion

In this expansion, percentage change in linear dimension at any point and in any direction is same for same change in temperature.



This expansion is similar to uniform photographic enlargement.

Relationship between α, β, & γ

For Isotropic Expansion

$$\beta = 2\alpha \qquad \gamma = 3\alpha$$

The expansion which is not Isotropic is called <u>Anisotropic Expansion</u>.

Relationship between α, β, & γ

For Anisotropic Expansion

$A = \ell \times b \qquad V = \ell \times b \times h$

$\beta = \alpha_\ell + \alpha_b \qquad \gamma = \alpha_\ell + \alpha_b + \alpha_h$

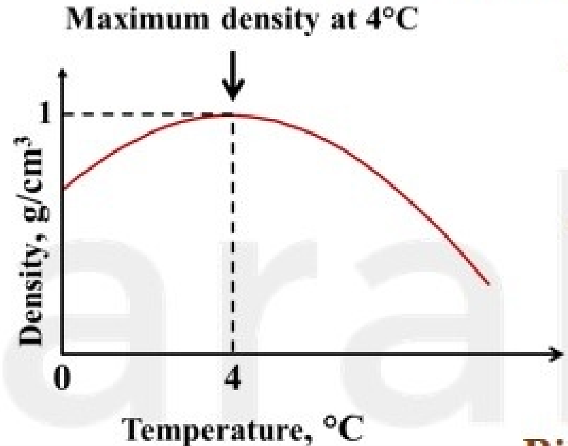
Variation of Density With Temperature

$$\rho = \frac{\text{Mass}}{\text{Volume}}$$

At temperature $T_0 + \Delta T$, $\rho = \dfrac{M}{V_0(1+\gamma\Delta T)}$

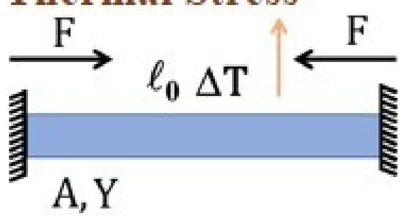
$$\rho = \frac{\rho_0}{1+\gamma\Delta T} \approx \rho_0(1-\gamma\Delta T)$$

Anomalous Expansion of Water



- If water at 0 °C is heated, it's volume decreases until the temperature reaches 4 °C.
- Above 4 °C water behaves normally and it's volume increases as temperature increases.

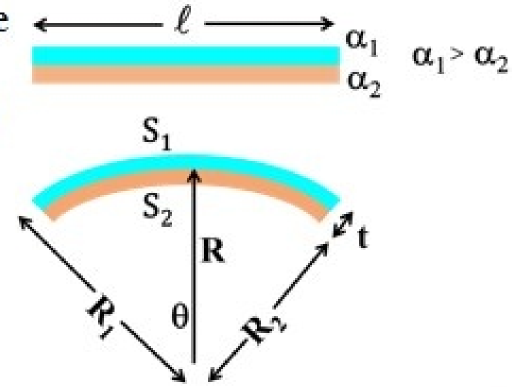
Thermal Stress



$$\text{Stress} = \frac{F}{A} = Y\,\alpha\,\Delta T$$

Bimetallic Strip

A bimetallic strip is made from two thin strips of metal that have different coefficients of linear expansion.



$$R = \frac{t}{(\alpha_1-\alpha_2)\Delta T}$$

eSaral

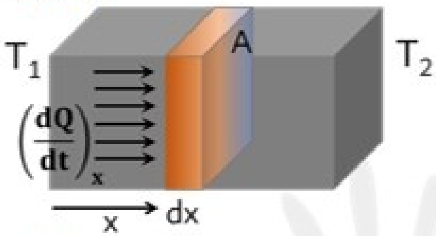
Heat Transfer

Mode	Medium	Bulk of Medium
1. Conduction	Required	Not transferred
2. Convection	Required	Transferred
3. Radiation	Not required	

Conduction

FOURIER'S LAW

$T_1 > T_2$



$$\left(\frac{dQ}{dt}\right)_x = -KA\left(\frac{dT}{dx}\right)$$

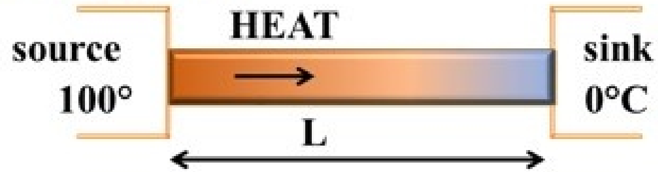
K = Thermal Conductivity of the substance (property of substance)

$\left(\frac{dQ}{dt}\right)_x$ is the amount of heat flow per unit time through a given cross–section area A. It is known as 'Heat Current'.

$\left(\frac{dT}{dx}\right)$ is the temperature gradient at the place where $\left(\frac{dQ}{dt}\right)$ is measured.

–ve sign indicates that heat flows in the direction of decreasing temperature.

Heat Flow In A Uniform Rod In Steady State



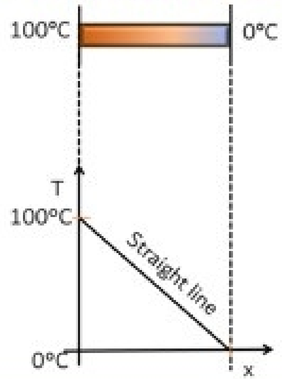
Rate of heat flow from each cross-section of the rod will be same.

$$\left(\frac{dQ}{dt}\right) = KA\left(\frac{dT}{dx}\right) = KA\left(\frac{\Delta T}{\Delta x}\right)$$

For a uniform rod, K and A are same for each element

$$i_H = KA\left(\frac{\Delta T}{\Delta x}\right)$$

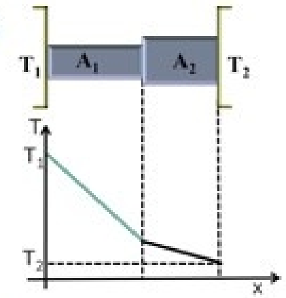
Variation of Temperature



$$i_H = KA\left(\frac{dT}{dx}\right)$$

$\left|\frac{dT}{dx}\right|$ is same

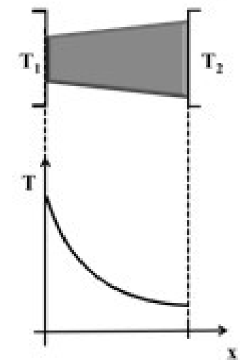
K is same & A different.



$$i_H = KA\left(\frac{dT}{dx}\right)$$

$$\left|\frac{dT}{dx}\right| \propto \frac{1}{A}$$

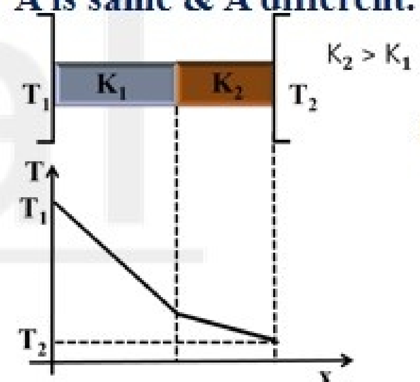
K is same & A different.



$$i_H = KA\left(\frac{dT}{dx}\right)$$

$$\left|\frac{dT}{dx}\right| \propto \frac{1}{A}$$

A is same & A different.



$$i_H = KA\left(\frac{dT}{dx}\right)$$

$$\left|\frac{dT}{dx}\right| \propto \frac{1}{A}$$

$$i_H = \frac{KA(T_1 - T2)}{L}$$

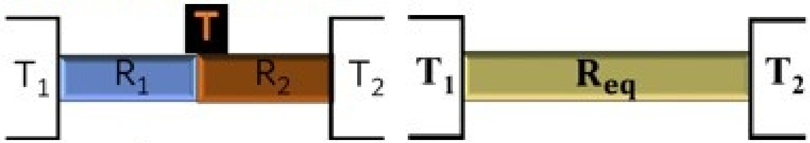
Thermal Resistance $R = \frac{L}{KA}$

$$i_H = \frac{(T_1 - T2)}{R}$$

Heat Transfer

Equivalent Resistance

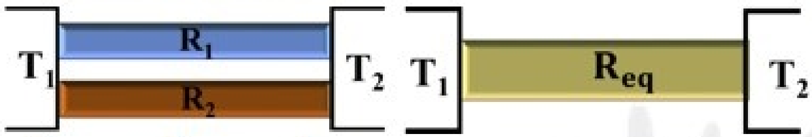
Series Combination



$$R_1 = \frac{\ell_1}{K_1A_1} \qquad R_2 = \frac{\ell_2}{K_2A_2}$$

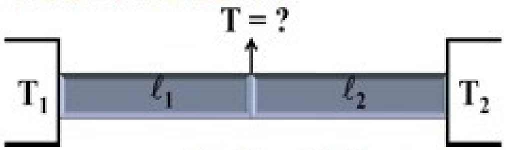
$$R_{eq} = R_1 + R_2$$

Parallel Combination



$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

Junction Rule



$$\frac{T_1 - T}{R_1} = \frac{T - T_2}{R_2} \qquad i_1 + i_2 = 0$$

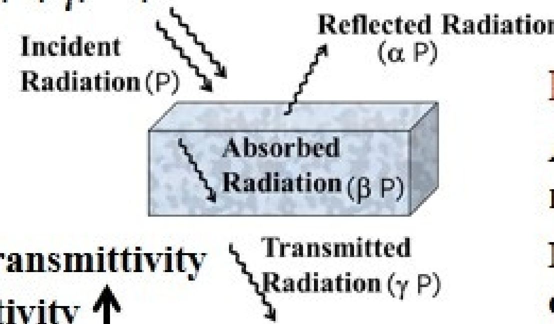
$$\sum \text{Outgoing} = 0 \quad \frac{T - T_1}{R_1} + \frac{T - T_2}{R_2} = 0$$

Radiation

Energy as electromagnetic waves.

Radiation by virtue of its temperature is called

$$\alpha P + \beta P + \gamma P = P$$



Transmittivity

Reflectivity

$\alpha + \beta + \gamma = 1$

Absorptivity

Opaque body : $\gamma = 0$ $\quad \alpha + \beta = 1$

Black body : $\alpha = 0, \ \beta = 1, \ \gamma = 0$

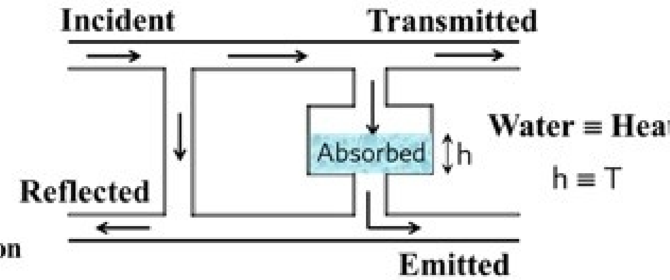
$T_b > T_s$: Rate of emission > Rate of absorbing $\Rightarrow T_b \downarrow$

$T_b < T_s$: Rate of emission < Rate of absorbing $\Rightarrow T_b \uparrow$

$T_b = T_s$: Rate of emission = Rate of absorbing $\Rightarrow T_b$ no change

Emitted Radiation

Due to its own temperature a body also emits radiation (caused by thermal vibrations of atoms, molecules & dipoles) which is known as Emitted Radiation.

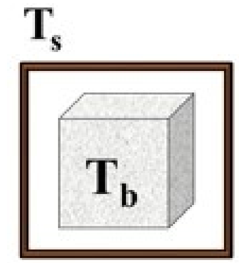


Prevost Theory

All bodies absorb as well as emit radiation at all temperatures.

Rate of emission of radiation depends on nature & temperature of the surface of the body.

Rate of absorbing radiation depends on nature of surface of the body & temperature of the surrounding.



Absorptive Power

Absorptive power of a body is defined as the fraction of incident radiation absorbed by the surface.

$$a = \frac{\text{Absorbed radiation}}{\text{Incident radiation}} = \beta(\text{Unitless})$$

$$0 \le a \le 1$$

For a Black body : $a = 1$

Heat Transfer

Emissive Power

$E = \frac{\Delta Q}{A\,\Delta t}$ Unit : W/m^2

Amount of radiation energy emitted by a surface per unit time per unit area is called its Emissive Power.

Laws of Radiation

STEFAN-BOLTZMANN LAW

The emissive power from a black body surface is directly proportional to the fourth power of it's absolute Temperature (i.e. in Kelvin). $E \propto T^4$ $E = \sigma T^4$

σ = Stefan-Boltzmann constant

$= 5.67\times10^{-8}\ Wm^{-2}K^{-4}$

$P = A\sigma T^4$ P – Power emitted

A – Surface Area of the body

T – Temperature in Kelvin

Emissivity (e)

The Emissivity of the surface of a material is its effectiveness in emitting energy as thermal radiation. It is the property of surface of the material.

$0 \le e \le 1$ $e = 1$ for a Black body

Black Body	General
$E = \sigma T^4$	$E = e\sigma T^4$
$P = A\sigma T^4$	$P = eA\sigma T^4$

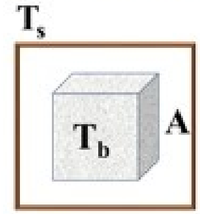
Kirchhoff's Law of Thermal Radiation

$\boxed{e = a}$ Emissivity of a body is equal to the Absorptivity of the body at a given temperature.

Net Rate of Heat Loss Through Radiation

Rate of heat emission $R_e = eA\sigma T_b^4$

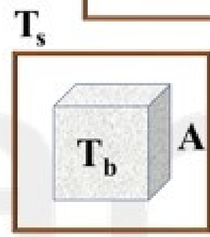
Rate of heat absorption $R_a = eA\sigma T_s^4$



Net rate of heat loss

$$\frac{dQ}{dt} = eA\sigma(T_b^4 - T_s^4)$$

Rate of Cooling



$$\frac{dQ}{dt} = eA\sigma(T_b^4 - T_s^4)$$

$$\frac{dQ}{dt} = -ms\frac{dT_b}{dt}$$

$$\frac{dT_b}{dt} = -\frac{eA\sigma}{ms}(T_b^4 - T_s^4)$$

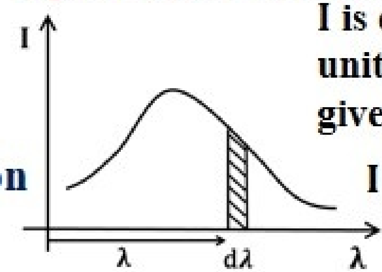
Rate of cooling i.e. Rate of Loss of Temperature

Newton's Law of Cooling

$$\frac{dQ}{dt} = k(T_b - T_s) = -ms\frac{dT_b}{dt}$$

$$\frac{dT_b}{dt} = -\frac{k}{ms}(T_b - T_s)$$

Spectral Emissive Power

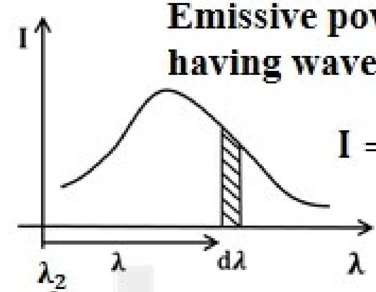


I is emissive power per unit wavelength near a given wavelength (λ).

$I = \frac{dE}{d\lambda} \Rightarrow I\,d\lambda = dE$

$\downarrow$

Emissive power of radiation having wavelength from λ to $\lambda + d\lambda$

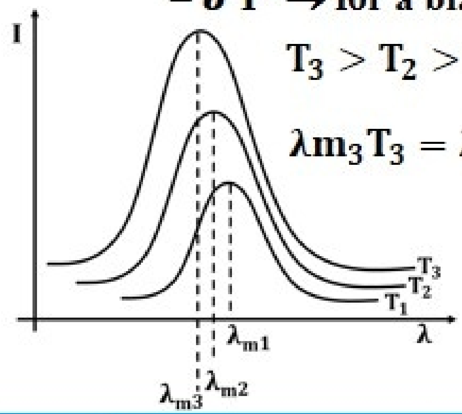


$I = \frac{dE}{d\lambda} \Rightarrow I\,d\lambda = dE$

$\int_{\lambda_1}^{\lambda_2} I\,d\lambda$ = emissive power of radiation having wavelength from $\boldsymbol{\lambda_1}$ to $\boldsymbol{\lambda_2}$

$\int_0^{\infty} I\,d\lambda$ = total emissive power

Total area of graph = Total emissive power $= \boldsymbol{\sigma} T^4 \rightarrow$ for a black body



$T_3 > T_2 > T_1$ Wein's constant

$\uparrow$

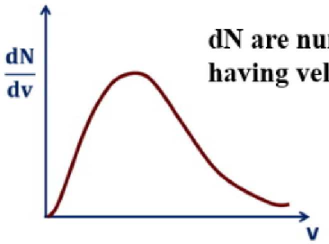
$\lambda m_3 T_3 = \lambda m_2 T_2 = \lambda m_1 T_1 = b$

$b = 0.0029$ m-K for a black body

$\lambda_m T = \text{constant}$

Kinetic Theory of Gases

Maxwell Distribution Function



dN are number of molecules having velocity from v to v+dv.

Average Speed

$$V_{avg} = \sqrt{\frac{8RT}{\pi M}}$$

↓ average speed

$$V_{avg} = \frac{(|v_1| + |v_2| + \cdots)}{N}$$

Average velocity $= 0$

Root Mean Square Speed

$$V_{rms} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3RT}{M}}$$

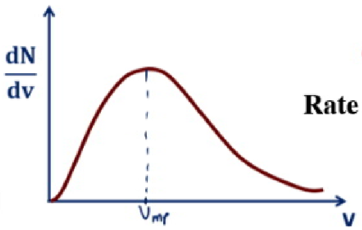
V_{rms} ↓ Root mean square speed; M ↓ Molecular mass

$$V_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + \ldots\ldots + v_N^2}{N}}$$

Most Probable Speed

$$V_{mp} = \sqrt{\frac{2RT}{M}}$$

↓ most probable speed



Law of Equipartition of Energy

Each degree of freedom contributes $\frac{1}{2}kT$ of energy per molecule.

k is Boltzmann constant

T = Temperature of gas

$k = 1.38 \times 10^{-23}$ J/K

$$k = \frac{R}{N_A}$$

R → Universal Gas Constant; N_A → Avogadro number; k ↓ Boltzmann constant

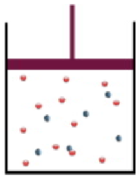
Internal energy of 'n' moles of gas $U = \frac{f}{2}nRT$

Translational kinetic energy of 'n' moles $= \frac{3}{2}nRT = \frac{3}{2}PV$

Dalton's Law of Partial Pressure

The total pressure of the mixture of ideal gases is the sum of the partial pressures.

$$P = P_1 + P_2 + \ldots..$$



$V \propto \frac{1}{P}$ (Boyle's law) $V \propto T$ (Charles's law) $V \propto n$ (Avogadro's law)

Ideal Gas Equation

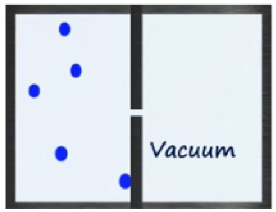
$$V \propto \frac{nT}{P} \qquad V = \frac{nRT}{P} \qquad PV = nRT$$

R is universal gas constant $= 8.314 = \frac{25}{3} \frac{J}{Mol - K}$

Graham's Law of Diffusion

Rate of diffusion $\propto \frac{1}{\sqrt{\text{Molecular Mass}}}$

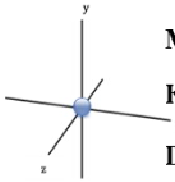
$$\frac{r_1}{r_2} = \frac{\sqrt{M_2}}{\sqrt{M_1}}$$



Degree of Freedom (f)

It refers to the minimum numbers of independent means by which a molecule can possess energy.

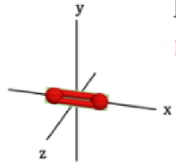
Atomicity of gas	f
Monoatomic	3
Diatomic and other linear molecules	5
Non-linear	6



Monoatomic Gas

$$KE = \frac{1}{2}mv_x^2 + \frac{1}{2}mv_y^2 + \frac{1}{2}mv_z^2$$

DOF (f) = 3



Diatomic Gas

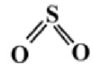
DOF = 3 (Translational) + 2 (Rotational) = 5

Triatomic Gas

Linear (eg. CO_2): O = C = O

DOF = 3 (Translational) + 2 (Rotational) = 5

Non-linear (eg. SO_2)

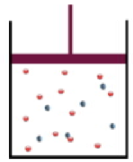


DOF = 3 (Translational) + 3 (Rotational) = 6

Equivalent DOF (feq)

$$n_1 f_1 RT + n_2 f_2 RT = (n_1 + n_2) f_{eq} RT$$

$$f_{eq} = \frac{n_1 f_1 + n_2 f_2}{n_1 + n_2}$$



Mean Free Path

The average distance between two successive collisions is called mean free path.

$$\text{mean free path 'l'} = \frac{1}{\sqrt{2}n\pi d^2}$$

n ← no of mol/vol; d → diameter of molecule

Thermodynamics

Molar Heat Capacity of Gas (C)

$C = \frac{\Delta Q}{n\Delta T}$ It depends on process

$C_P - C_V = R$ Mayor's equation

$\frac{C_P}{C_V} = \gamma$ Adiabatic constant

$C_V = \frac{f}{2}R$ $C_P = \frac{f}{2}R + R$ $\frac{C_P}{C_V} = \gamma$

Atomicity of gas	f	C_v	C_P	γ
Monoatomic	3	$\frac{3}{2}R$	$\frac{5}{2}R$	$\frac{5}{3} = 1.67$
Diatomic or Triatomic linear	5	$\frac{5}{2}R$	$\frac{7}{2}R$	$\frac{7}{5} = 1.40$
Polyatomic or Triatomic Non-linear	6	$\frac{6}{2}R = 3R$	$\frac{8}{2}R = 4R$	$\frac{4}{3} = 1.33$

$f = \frac{2}{\gamma - 1}$ $C_V = \frac{R}{\gamma - 1}$ $C_P = \frac{R\gamma}{\gamma - 1}$

First Law of thermodynamics

$\Delta Q = \Delta U + W$

Based on Conservation of Energy

	Definition	+ve	-ve
ΔQ	Heat supplied to gas	If heat is supplied to gas	Heat is taken from gas
ΔU	Change in U	If U increases	If U decreases
W	Work done by gas	If V increases	If V decreases

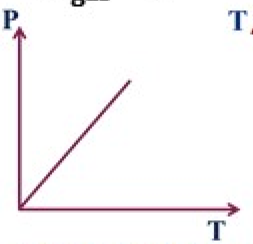
Thermodynamic Processes

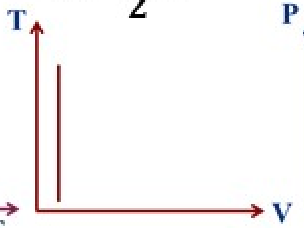
1. Isochoric Process

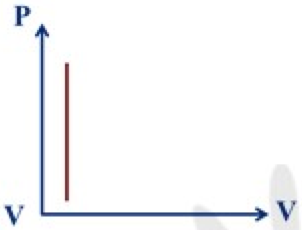
$V = \text{Constant}$ $\Delta Q = n\, C_V\, \Delta T$

Bulk modulus is not defined

$W_{gas} = 0$ $C_V = \frac{f}{2}R$





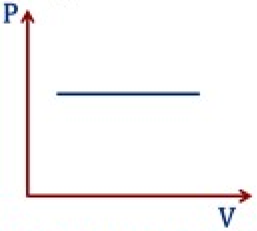


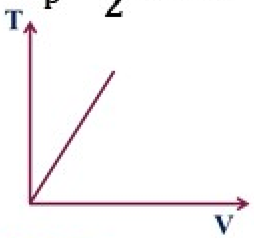
2. Isobaric Process

$P = \text{constant}$ $\Delta Q = n\, C_P\, \Delta T$

Bulk modulus B = 0

$W_{gas} = n R \Delta T$ $C_P = \frac{f}{2}R + R$





3. Isothermal Process

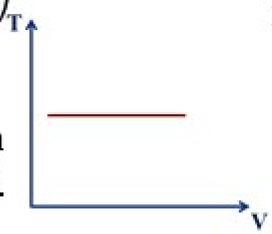
$T = \text{constant}$

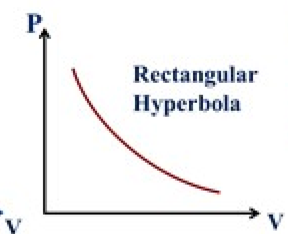
C of this process is not defined

$W_{gas} = n R T \ell n\left(\frac{V_f}{V_i}\right)$

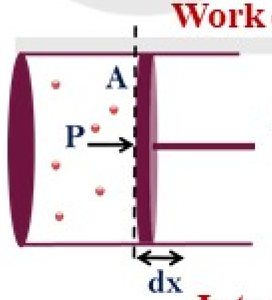
Bulk modulus B = P

Two isotherms for a gas do not intersect.

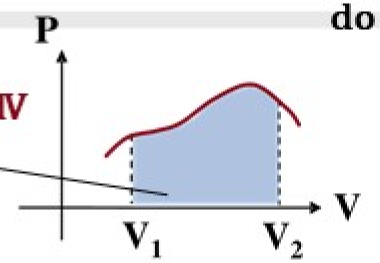




Work done by Gas



$W_{gas} = \int P\, dV$



Internal Energy

It is sum of molecular kinetic and potential energies in the reference relative to which COM (Centre of Mass) of the system is at rest.

$U = \frac{f}{2}nRT$

$\Delta U = U_f - U_i = \frac{f}{2}nR\Delta T$

$= \frac{f}{2}(P_f V_f - P_i V_i)$

4. Adiabatic Process

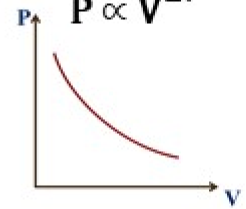
$dQ = 0 \Rightarrow \Delta Q = 0$ $PV^{\gamma} = \text{Constant}$ $B = \gamma P$ $C = 0$

$W = -\Delta U$

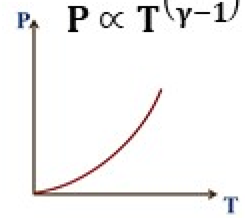
$W_{gas} = -\frac{f}{2}nR\Delta T = \frac{nR\Delta T}{1-\gamma} = \frac{P_f V_f - P_i V_i}{1-\gamma}$

Two adiabatic curves for a particular gas do not intersect.

$P \propto V^{-\gamma}$



$P \propto T^{\left(\frac{\gamma}{\gamma-1}\right)}$



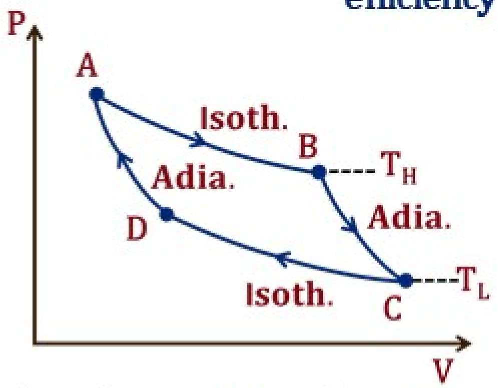
5. Polytropic Process

$PV^x = \text{Constant}$ $W_{gas} = \frac{nR\Delta T}{1-x}$

Slope of P–V curve $= -\frac{xP}{V}$ $B = -xP$

$C = R\left(\frac{1}{\gamma-1} - \frac{1}{x-1}\right)$

Carnot cycle



efficiency $'\eta' = \dfrac{W_{net}}{Q_{in}}$

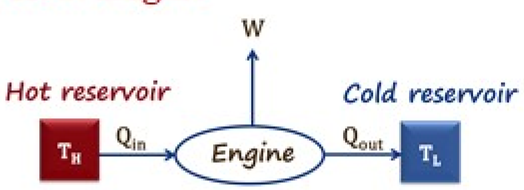
$$= \frac{|Q_{in}| - |Q_{out}|}{|Q_{in}|}$$

$$= 1 - \frac{|Q_{out}|}{|Q_{in}|}$$

$$\eta = 1 - \frac{T_L}{T_H}$$

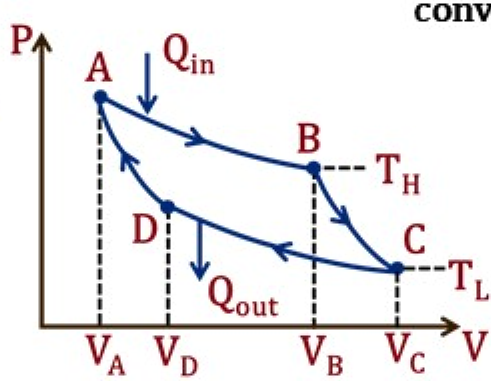
	W	ΔU	ΔQ
AB	$nRT_H \ell n\left(\frac{V_B}{V_A}\right)$	0	$nRT_H \ell n\left(\frac{V_B}{V_A}\right)$
BC	$\frac{nR(T_L - T_H)}{1-\gamma}$	$-\frac{nR(T_L - T_H)}{1-\gamma}$	0
CD	$nRT_L \ell n\left(\frac{V_D}{V_C}\right)$	0	
DA	$\frac{nR(T_H - T_L)}{1-\gamma}$	$-\frac{nR(T_H - T_L)}{1-\gamma}$	0

Heat Engine

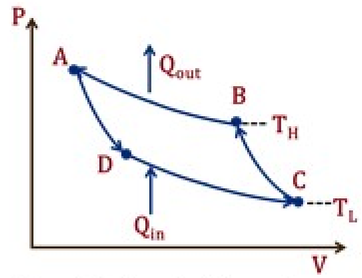


$W = |Q_{in}| - |Q_{out}|$

Efficiency $= \dfrac{W_{net}}{Q_{in}} = 1 - \dfrac{|Q_{out}|}{|Q_{in}|}$



Reverse Carnot



Coefficient of performance (α)

$$\alpha = \frac{|Q_{in}|}{|Q_{out}| - |Q_{in}|} = \frac{T_L}{T_H - T_L}$$

Thermodynamics

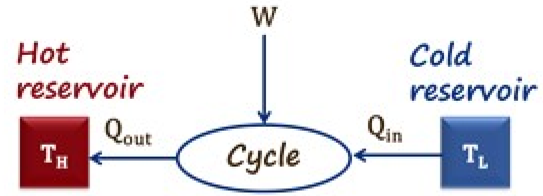
Second Law of Thermodynamics

No process is possible whose sole result is the absorption of heat from a reservoir and the complete conversion of heat to work.

Efficiency $= \dfrac{W_{net}}{Q_{in}} = 1 - \dfrac{|Q_{out}|}{|Q_{in}|}$

$$\eta = 1 - \frac{T_L}{T_H}$$

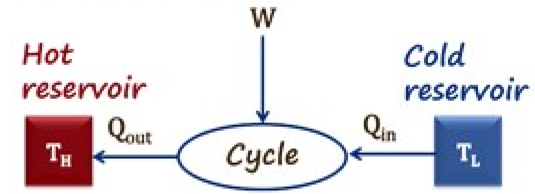
Heat Pump



Coefficient of performance (α)

$$= \frac{Q_{out}}{W}$$

Refrigerator



Coefficient of performance (α) $= \dfrac{Q_{in}}{W}$

$$= \frac{|Q_{in}|}{|Q_{out}| - |Q_{in}|}$$

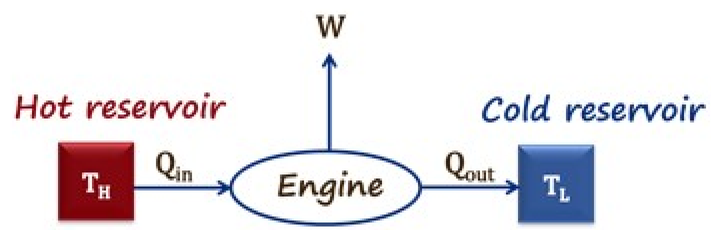
Entropy

Related to disorder in system.

$$dS = \frac{dQ}{T}$$

For adiabatic process $\Delta S = 0$

$$\Delta S = \int dS = \int \frac{dQ}{T}$$



Efficiency of Cyclic process must be less than 1.

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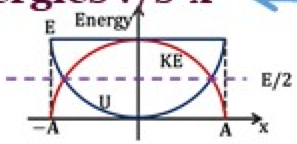
SIMPLE HARMONIC MOTION

Energies v/s 'x'

$U = \frac{1}{2}kx^2$

$KE = \frac{1}{2}k(A^2 - x^2)$

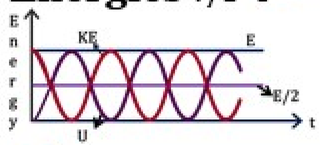
$KE + U = E = \frac{1}{2}kA^2$



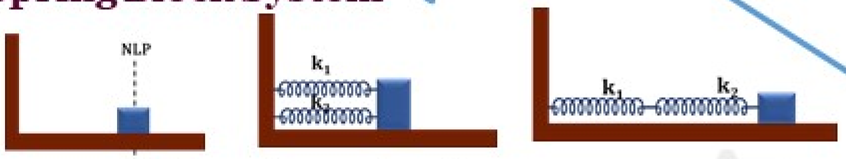
Energies v/s 't'

$U = \frac{1}{2}kA^2\sin^2(\omega t + \phi)$

$KE = \frac{1}{2}kA^2\cos^2(\omega t + \phi)$



Spring Block System



$T = 2\pi\sqrt{\frac{m}{k}}$

$k_{eq} = k_1 + k_2$

$T = 2\pi\sqrt{\frac{m}{k_{eq}}}$

$T = 2\pi\sqrt{\frac{m}{k_{eq}}} = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$

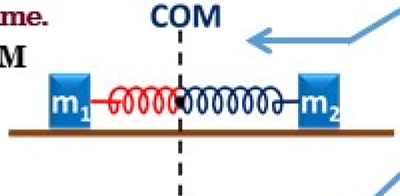
$k_{eq} = \frac{k_1 k_2}{k_1 + k_2}$

In a spring block system, if an additional constant force (along line of SHM) is applied on the block then time period remains same.

Two Blocks SHM

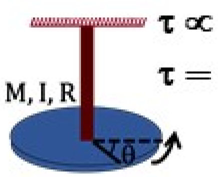
$\mu = \frac{m_1 m_2}{m_1 + m_2}$

Reduced Mass



$T_1 = 2\pi\sqrt{\frac{m_1 m_2}{k(m_1 + m_2)}} = 2\pi\sqrt{\frac{\mu}{k}}$

Torsional Pendulum

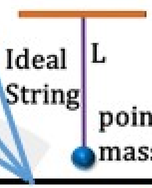


$\tau \propto -\theta$

$\tau = -C\theta$ (C= Torsional constant)

$T = 2\pi\sqrt{\frac{I}{C}}$

Simple Pendulum

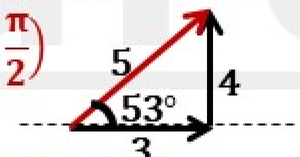


$T = 2\pi\sqrt{\frac{\mu}{k}}$

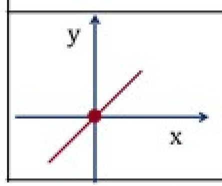
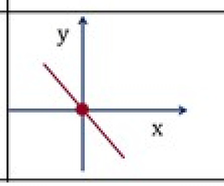
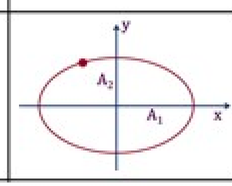
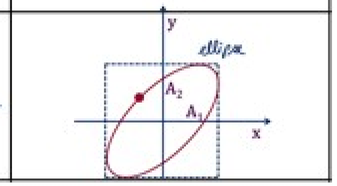
Composition of Two SHM's of same ω along mutually perpendicular directions

$x = 3\sin(\omega t) + 4\sin\left(\omega t + \frac{\pi}{2}\right)$

$x = 5\sin(\omega t + 53°)$



$\vec{x} = A_1\sin(\omega t)\hat{i}$ $\quad$ $\vec{y} = A_2\sin(\omega t + \phi)\hat{j}$

When $\phi = 0$	When $\phi = \pi$	When $\phi = \frac{\pi}{2}$	When $\phi \neq 0, \frac{\pi}{2}, \pi$
$y = \frac{A_2}{A_1}x$	$y = -\frac{A_2}{A_1}x$	$\frac{x^2}{A_1^2} + \frac{y^2}{A_2^2} = 1$	
			

(1) Periodic
(2) Oscillatory
(3) $|F| \propto |x|$

$F_{net} = -kx$

$a = -\omega^2 x$

$\omega = \sqrt{\frac{k}{m}}$

$x = A\sin(\omega t + \phi)$

Time Period & Frequency

$T = \frac{2\pi}{\omega}$ $\quad$ $f = \frac{1}{T} = \frac{\omega}{2\pi}$

Velocity Analysis

$x = A\sin(\omega t + \phi)$

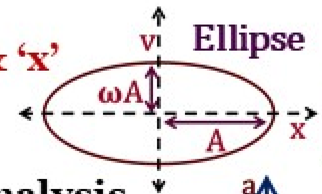
$v = \frac{dx}{dt} = A\omega\cos(\omega t + \phi)$

$v = \omega\sqrt{(A^2 - x^2)}$

Max velocity $v_{max} = \omega A$ at $x = 0$ (Mean Position)

Min velocity $v_{min} = 0$ at $x = \pm A$ (Extreme Position)

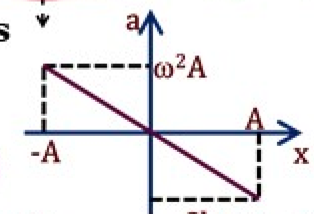
Graph b/w 'v' & 'x'



$\frac{x^2}{A^2} + \frac{v^2}{(\omega A)^2} = 1$

Acceleration Analysis

$a = -\omega^2 x$



$x = 2 + 1\sin(\omega t)$

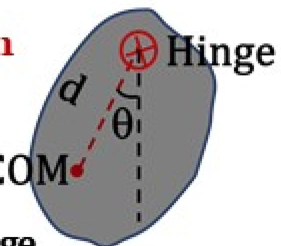
Position

eq$^{m.}$ position

Amplitude

Physical Pendulum

$T = 2\pi\sqrt{\frac{I}{mgd}}$



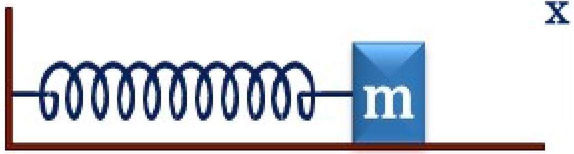
I : MOI of system about hinge.

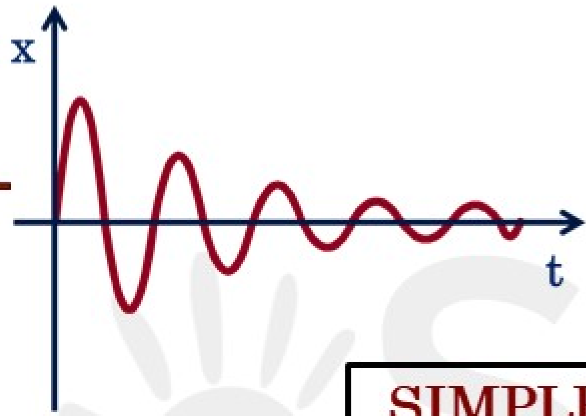
m : Mass of system

d : distance between COM and hinge.

SIMPLE HARMONIC MOTION

Damped SHM





$$F_{drag} = -bv$$

$$F = -kx - bv$$

$$x = Ae^{-bt/2m}\sin(\omega' t + \phi)$$

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

$$E = \frac{1}{2}kA^2 e^{-bt/m}$$

Forced Oscillations and Resonance

$$F_{driving} = F_0 \sin(\omega_d t)$$

$$ma = -kx - bv + F_0 \sin(\omega_d t)$$

$$x = A\sin(\omega_d t + \phi)$$

$$A = \frac{F_0/m}{\sqrt{(\omega^2 - \omega_d^2)^2 + (b\omega_d/m)^2}}$$

If small damping, (b very small)

$$A = \frac{F_0/m}{(\omega^2 - \omega_d^2)}$$

$$\omega_d \approx \omega \Rightarrow \text{RESONANCE}$$

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Wave on String

Power Transmission in Travelling Wave on String

$$P = -T\left(\frac{\partial y}{\partial t}\right)\left(\frac{\partial y}{\partial x}\right)$$ valid for all waves on string

$$\langle P \rangle = \frac{1}{2}\underbrace{\sqrt{T\mu}}_{\text{Property of medium}}\ \overbrace{A^2\omega^2}^{\text{Property of Source}}$$

General Equation of Standing Wave

$$y = 2A\sin(kx+\phi_1)\sin(\omega t+\phi_2)$$

Amplitude of component wave = A

Coefficient of x = k

Wavelength of component waves $= \frac{2\pi}{k}$

Loop length $= \frac{\lambda}{2} = \frac{\pi}{k}$

Velocity of Wave on String

$$v_w = \sqrt{\frac{T}{\mu}}$$

μ = mass per unit length

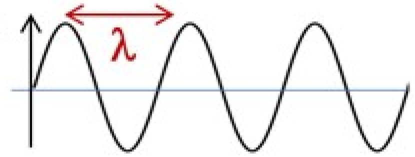
$$v_w = \sqrt{\frac{Stress}{\rho}}$$

Wave Function $y = f(x, t)$

$$\underset{\text{Velocity of wave}}{v_w} = -\frac{\text{Coefficient of } t}{\text{Coefficient of } x}$$

$y = A\sin(kx-\omega t)$	$y = A\sin(kx+\omega t)$
$v_w = -\frac{-\omega}{k} = \frac{\omega}{k}$	$v_w = -\frac{\omega}{k} = -\frac{\omega}{k}$
Wave travelling in + dir.	Wave travelling in − dir.

Sinusoidal Wave Equation



$$y = A\sin(\omega t - kx + \phi)$$

$\lambda = \frac{2\pi}{k}$ $\quad T = \frac{2\pi}{\omega}$ $\quad$ Angular wave number, $k = \frac{2\pi}{\lambda}$

Velocity & Acceleration of Particle

$$v_P = \frac{\partial y}{\partial t} = \omega\sqrt{A^2 - y^2}$$

$y = A\sin(\omega t - kx)$ → Displacement

$$a_P = \frac{\partial v_P}{\partial t} = -\omega^2 y$$

Relation Between Particle Velocity & Wave Velocity

$$v_P = -v_w\left(\frac{\partial y}{\partial x}\right)$$ → Slope of the string at point x.

WAVES

Mechanical Waves

Medium is required

Waves on String, Sound Wave

Non-Mechanical Waves

Medium is not required

Light Wave

Transverse Wave

Constituents of the medium oscillates perpendicular to the direction of wave propagation.

Waves on String

Longitudinal Wave

Constituents of the medium oscillates along the direction of wave propagation.

Sound Wave

Wave on String

Interference of Waves

Constructive Interference

Destructive Interference

Standing Waves

$$y_{res} = 2A\sin(kx)\cos(\omega t)$$

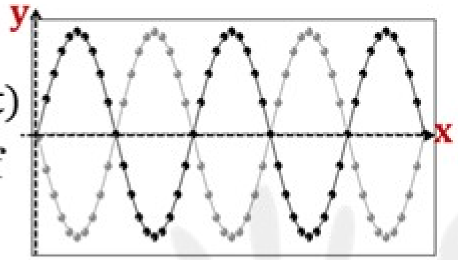
For $x = x_0$ Equation of Motion

$$y = \underbrace{2A\sin(kx_0)}_{A'}\cos(\omega t)$$

$$y = A'\cos(\omega t)$$

$\downarrow$

SHM



$$y_{res} = 2A\sin(kx)\cos(\omega t)$$

$$v_P = \frac{\partial y}{\partial t} = \omega\sqrt{A^2 - y^2}$$

$$a_p = \frac{\partial^2 y}{\partial t^2} = -\omega^2 y$$

For Nodes - Amplitude = 0

$$\sin(kx + \phi_1) = 0$$

$$kx + \phi_1 = n\pi$$

For Antinodes - Amplitude = max

$$\sin(kx + \phi_1) = \pm 1$$

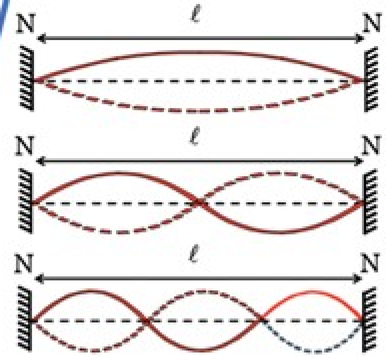
$$(kx + \phi_1) = \left(n + \frac{1}{2}\right)\pi$$

Energy in Standing Wave

For Node	For Anitnode
$KE = 0$	$PE = 0$
$\frac{\partial y}{\partial t} = 0$	$\frac{\partial y}{\partial x} = 0$
$P_{Node} = 0$	$P_{Antinode} = 0$

Power transmission through node & antinode is zero. So, energy of one loop (even half loop) remains conserved.

Normal Modes String Fixed at Both Ends



$\frac{\lambda}{2} = \ell$ $f_0 = \frac{v}{2\ell}$ Fundamental Frequency

$\frac{2\lambda}{2} = \ell$ $f = \frac{2v}{2\ell}$ 2nd Harmonic 1st Overtone

$\frac{3\lambda}{2} = \ell$ $f = \frac{3v}{2\ell}$ 3rd Harmonic 2nd Overtone

$\frac{n\lambda}{2} = \ell$ $f = \frac{nv}{2\ell}$ n th Harmonic (n–1)th Overtone

(1) $\frac{n\lambda}{2} = \ell$

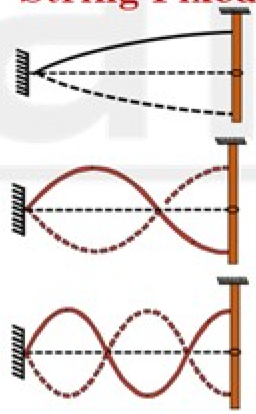
(2) $f = \frac{nv}{2\ell}$

(3) $f = \frac{n}{2\ell}\sqrt{\frac{T}{\mu}}$

(4) $f = n f_0$

For fundamental frequency, n = 1

String Fixed at One End & Free at Other



$\frac{\lambda}{4} = \ell$ $f = \frac{v}{4\ell}$ Fundamental Frequency

$\frac{3\lambda}{4} = \ell$ $f = \frac{3v}{4\ell}$ 3rd Harmonic 1st Overtone

$\frac{5\lambda}{4} = \ell$ $f = \frac{5v}{4\ell}$ 5th Harmonic 2nd Overtone

$\frac{(2n+1)\lambda}{4} = \ell$ $f = \frac{(2n+1)v}{4\ell}$ (2n +1)Harmonic nth Overtone

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Sound Waves

Variation of Excess Pressure in Gas Due to Propagation of Longitudinal Wave

$p = -B\left(\frac{\partial s}{\partial x}\right)$

p → Excess pressure

$s = s_0 \sin(\omega t - kx)$

$p = p_0 \cos(\omega t - kx)$

$|p_0| = |Bks_0|$

Amplitude of excess pressure

Power Transmission in Sound Waves

$\text{Power} = -BA\left(\frac{\partial s}{\partial x}\right)\left(\frac{\partial s}{\partial t}\right)$

$< I > = \frac{1}{2}\sqrt{B\rho}\, s_0^2\omega^2$

$\sqrt{B\rho}$ → Property of medium

$s_0^2\omega^2$ → Property of source

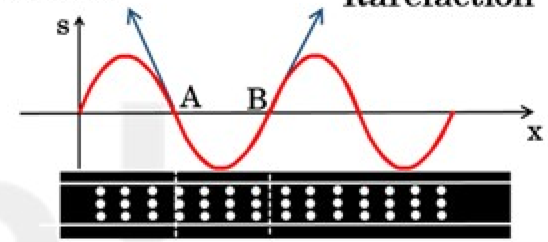
Equation of Sound Wave

Sinusoidal Wave Equation $s = s_0 \sin(\pm\omega t \pm kx + \phi)$

s → displacement of particle present at x in x direction

Max Compressive Stress
Max. Density, Pressure
Compression

Max Tensile Stress
Min. Density, Pressure
Rarefaction

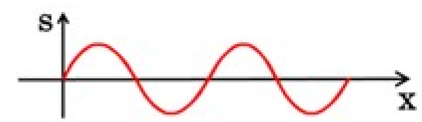


$s = s_0 \sin(\omega t - kx + \phi)$

$v_w = -\frac{\text{Coefficient of t}}{\text{Coefficient of x}} = \frac{\omega}{k}$

$v_P = -v_w\left(\frac{\partial s}{\partial x}\right)$

v_P → Velocity of particle



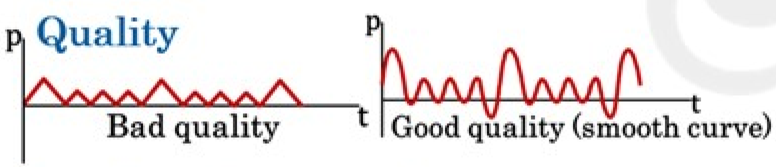
Characteristics of Sound

Pitch (Dominant Frequency)

Low Frequency → Low Pitch

High Frequency → High Pitch

Quality



Loudness (Intensity)

$\left(\frac{I}{I_0}\right)$

Sound level (SL) $= 10\log_{10}\left(\frac{I}{I_0}\right)$

→ measured in decibel (dB)

$I_0 = 10^{-12}\,\frac{\text{Watt}}{m^2}$

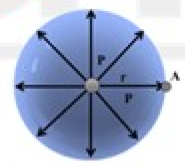
I_0 → Minimum audible intensity

Variation of Intensity

Point source

$I_A = \frac{P}{4\pi r^2}$

$I \propto \frac{1}{r^2}$

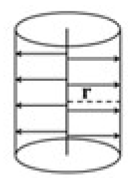


Line source

Power per unit length = P

$I = \frac{P\ell}{2\pi r\ell}$

$I \propto \frac{1}{r}$



Velocity of Sound Wave in Gas

$v_w = \sqrt{\frac{\gamma P}{\rho}}$ $\quad v_w = \sqrt{\frac{\gamma RT}{M_0}}$ (T is in Kelvin)

On ↑ P if T is constant v_w will not change

At same T if humidity ↑, v_w increases ($\because M_0\downarrow$)

Velocity of sound wave in a thin solid rod $= \sqrt{\frac{Y}{\rho}}$

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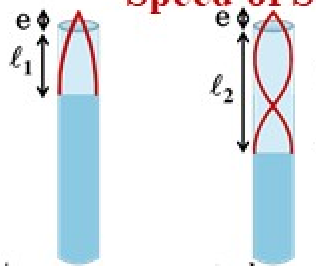
Sound Waves

Resonance Tube Method to Calculate Speed of Sound in Air



$$v = f\lambda$$

$$v = 2f(\ell_2 - \ell_1)$$

$$e = 0.3\,d$$

diameter of tube

Beat frequency

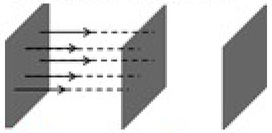
$$\text{Beat frequency} = \Delta f = f_1 - f_2$$

Wavefront

Point source — Cylindrical wavefronts

Line source — Spherical wavefronts

Planar wavefronts



Change in wavelength due to motion of a sound source

$$\lambda' = \frac{v_w \pm v_s}{f}$$

Change in frequency observed due to motion of both source & observer

$$f' = \left(\frac{v_w + v_O}{v_w - v_S}\right) f$$

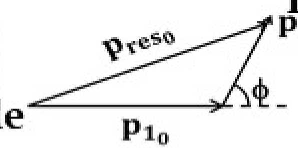
If wind is blowing, take its effect also

Interference of Sound Waves

$$p_1 = p_{1_0}\sin(kx - \omega t)$$

$$p_2 = p_{2_0}\sin(kx - \omega t + \phi)$$

By Superposition Principle



$$(p_{res_0})^2 = p_{1_0}^2 + p_{2_0}^2 + 2\,p_{1_0}p_{2_0}\cos\phi$$

$$I_{res} = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\phi \qquad (I \propto p_o^2)$$

Constructive Interference

$$(p_{res_0})_{max} = p_{1o} + p_{2o}$$

$$(I_{res})_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$$

$$\Delta x = 0, \lambda, 2\lambda, 3\lambda \ldots = n\lambda$$

Constructive Interference

Destructive Interference

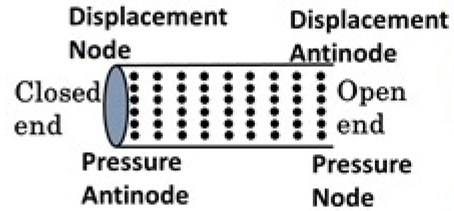
Destructive Interference

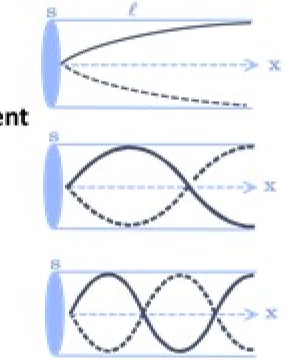
$$(p_{res_0})_{min} = |p_{1o} + p_{2o}|$$

$$(I_{res})_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$$

$$\Delta x = 0.5\lambda, 1.5\lambda, 2.5\lambda \ldots = \left(n + \frac{1}{2}\right)\lambda$$

Standing Wave Inside an Organ Pipe Closed at One End





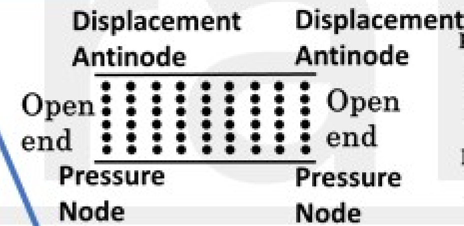
$\frac{\lambda}{4} = \ell$, $f = \frac{1v}{4\ell}$ — Fundamental frequency

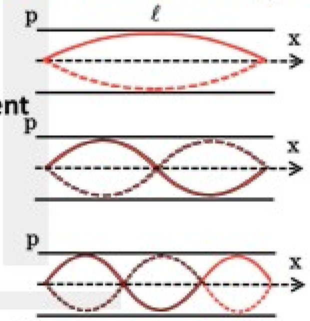
$\frac{3\lambda}{4} = \ell$, $f = \frac{3v}{4\ell}$ — 3rd Harmonic, 1st overtone

$\frac{5\lambda}{4} = \ell$, $f = \frac{5v}{4\ell}$ — 5th Harmonic, 2nd overtone

$\frac{(2n+1)\lambda}{4} = \ell$, $f = \frac{(2n+1)v}{4\ell}$ — $(2n+1)^{th}$ Harmonic, n^{th} overtone

Standing Wave Inside an Organ Pipe Open at Both End





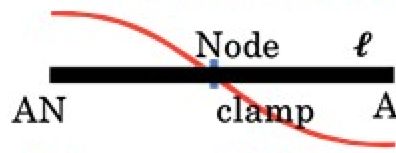
$\frac{\lambda}{2} = \ell$, $f_o = \frac{1v}{2\ell}$ — Fundamental frequency

$\frac{2\lambda}{2} = \ell$, $f = \frac{2v}{2\ell}$ — 2nd Harmonic, 1st overtone

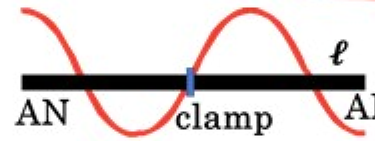
$\frac{3\lambda}{2} = \ell$, $f = \frac{3v}{2\ell}$ — 3rd Harmonic, 2nd overtone

$\frac{n\lambda}{2} = \ell$, $f = \frac{nv}{2\ell}$ — n^{th} Harmonic, $(n-1)^{th}$ overtone

Standing Wave in Solids



$\frac{\lambda}{2} = \ell$, $f = \frac{1}{2\ell}\sqrt{\frac{Y}{\rho}}$ — fundamental mode



$\frac{3\lambda}{2} = \ell$, $f = \frac{3}{2\ell}\sqrt{\frac{Y}{\rho}}$ — 1st overtone, 3rd harmonic

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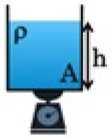
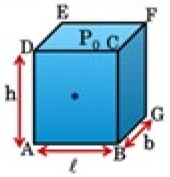
Fluid Mechanics- Statics & Dynamics

Force By Liquid

Reading $= mg = \rho a g h$



Force Applied by Liquid on Vertical Surface



$$F = \left(P_0 + \frac{\rho g h}{2}\right) h\ell$$

Buoyant Force

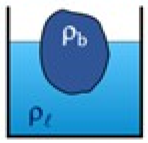
Buoyant force is equal to weight of the fluid displaced.

Buoyant force depends on g_{eff}

Buoyant force acts opposite to g_{eff}

$BF = \rho_f g h A$

Archimedes Principle



Loss of weight of a body submerged (partially or completely) in a fluid is equal to the weight of the fluid displaced.

Weight of fluid displaced $= \rho_\ell V_S g = BF$

If $\rho_b < \rho_\ell$; then body will float

If $\rho_b = \rho_\ell$; body will just float with fully submerged

If $\rho_b > \rho_\ell$; then body will sink

Law of Floatation

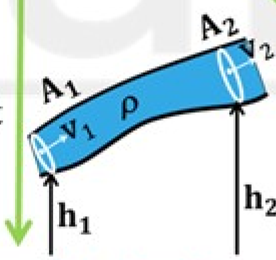
For Floating : $\dfrac{V_S}{V_b} = \dfrac{\rho_b}{\rho_\ell}$

Fraction of vol. submerged = ratio of density of body and fluid

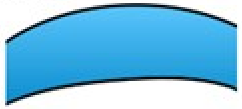
Bernoulli's Principle

$$P + \rho g h + \frac{1}{2}\rho v^2 = \text{Constant}$$

Based on law of conservation of energy



Conditions of Ideal Fluid Flow



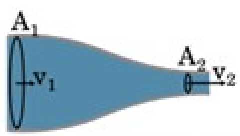
Fluid is Incompressible and Non- Viscous).

Flow is Irrotational & Steady

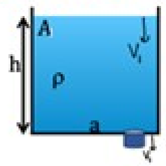
Volume flow rate.	Mass flow rate
Volume flowing through per unit time.	Mass flowing through per unit time.
$\dfrac{dV}{dt} = Av$	$\dfrac{dm}{dt} = \rho A v$

Equation of Continuity

$A_1 v_1 = A_2 v_2$



Speed of Efflux: Torricelli's Law



$v = \sqrt{2gh}$

Pressure

The value of this force acting on unit area at any point is called Pressure at that point.

$$\text{Pressure} = \frac{dF_1}{dA_1} \longrightarrow \text{Scalar Quantity}$$

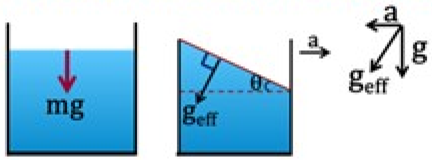
SI unit is N/m^2 = Pascal (Pa)

$1 \text{ bar} = 10^5 \text{ Pa}$

$1 \text{ atm} = 101325 \text{ Pa} \approx 10^5 \text{Pa}$

$1 \text{ atm} = 1.01325 \text{ bar} \approx 1 \text{ bar}$

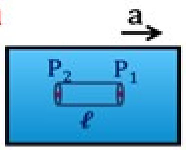
Ideal fluid cannot sustain shear stress



$\tan\theta = \dfrac{a}{g}$

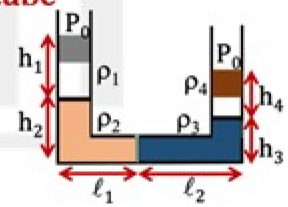
Variation of Pressure with Depth

$P_2 = P_1 + \rho_\ell g h_1$

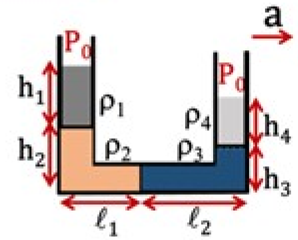


$P_2 = P_1 + \rho a \ell$

U-shaped tube

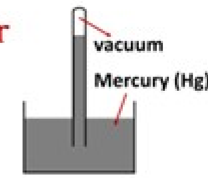


$P_0 + \rho_1 g h_1 + \rho_2 g h_2 - \rho_3 g h_3 - \rho_4 g h_4 = P_0$



$P_0 + \rho_1 g h_1 + \rho_2 g h_2 - \rho_2 a \ell_1 - \rho_3 a \ell_2 - \rho_3 g h_3 - \rho_4 g h_4 = P_0$

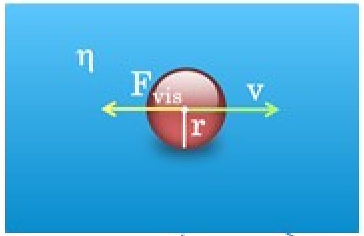
Barometer



Pascal's Law

Whenever external pressure is given at any part of fluid it is transmitted undiminished and equally in all direction

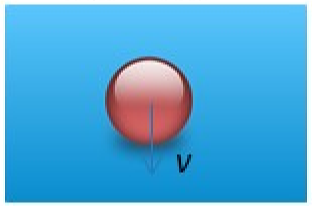
Stokes' Law

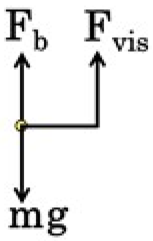


$$F_{vis} = 6\pi\eta r v$$

$\vec{F}_{vis}$ is opp. to $\vec{v}$

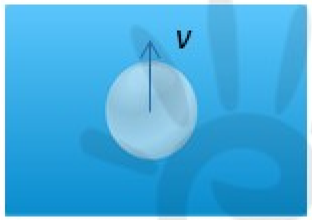
$\rho_b > \rho_f$

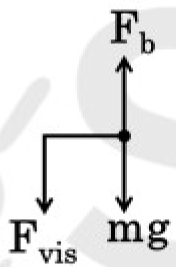




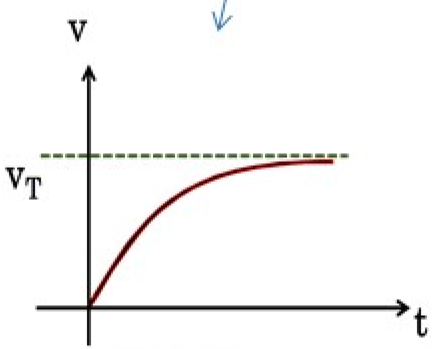
$$V_T = \frac{2}{9}\frac{(\rho_b - \rho_f)r^2 g}{\eta}$$

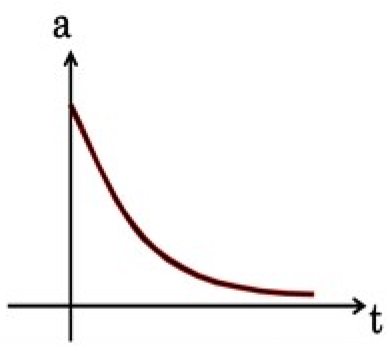
$\rho_b < \rho_f$



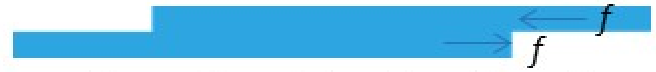


$$V_T = \frac{2}{9}\frac{(\rho_f - \rho_b)r^2 g}{\eta}$$

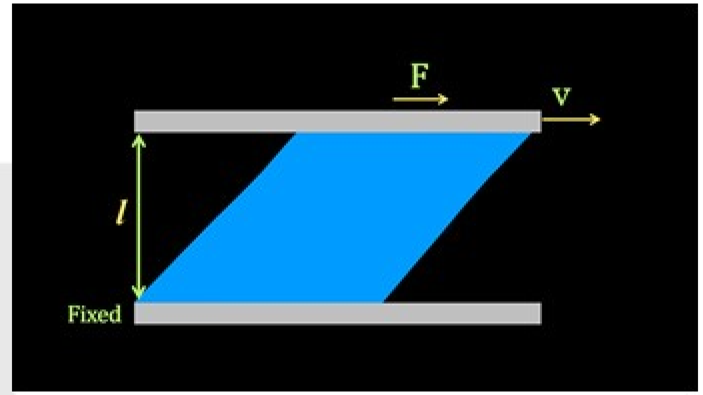




Viscosity



Viscosity exists when there is relative motion between layers of the liquid.



Reynolds Number (R_e)

$$R_e = \frac{\rho v d}{\eta}$$

If $R_e < 1000$	Flow is steady
If $1000 < R_e < 2000$	Flow is unstable
If $R_e > 2000$	Flow is turbulent

Largest velocity till which flow is steady is called Critical Velocity.

$$F = \eta A \left(\frac{v}{\ell}\right)$$

$\left(\frac{v}{\ell}\right)$ → Velocity gradient

η ↓ Coefficient of viscosity

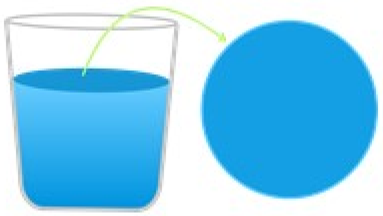
SI unit of η is Poiseuille (Pl)

CGS unit is Poise

1 Poise = 0.1 Poiseuille

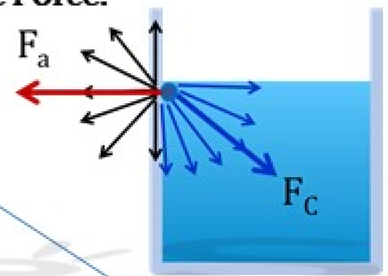
Surface tension

$U_{surface} > U_{inside}$



$$\text{Surface Tension (S)} = \frac{\text{Force}}{\text{Length}} = \frac{F}{L}$$

Force between different types of molecules is called Adhesive Force.



Force between same types of molecules is called Cohesive Force.

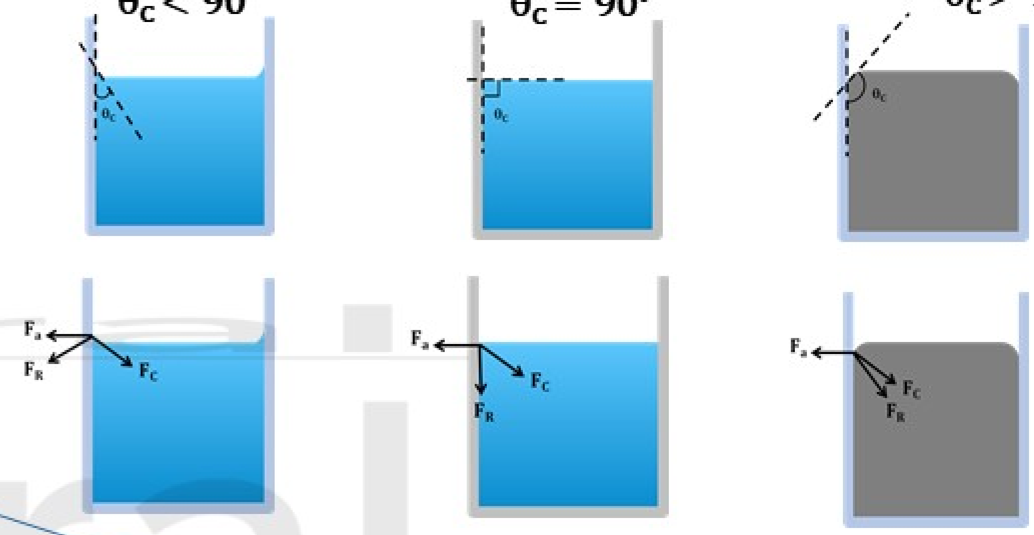
Surface Tension

Contact Angle (θc)

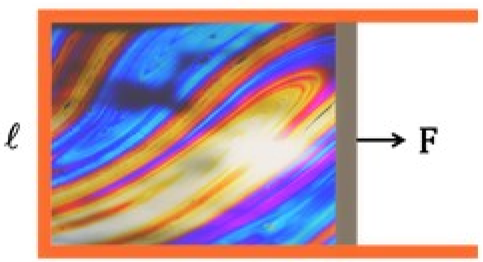
Glass Vessel – Water $\theta_c < 90^\circ$

Silver Vessel – Water $\theta_c = 90^\circ$

Glass Vessel – Mercury $\theta_c > 90^\circ$



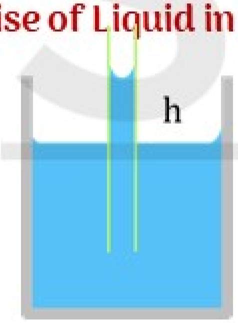
Surface Energy



$$\text{Surface Energy} = \text{Surface Tension} \times \text{Surface Area}$$

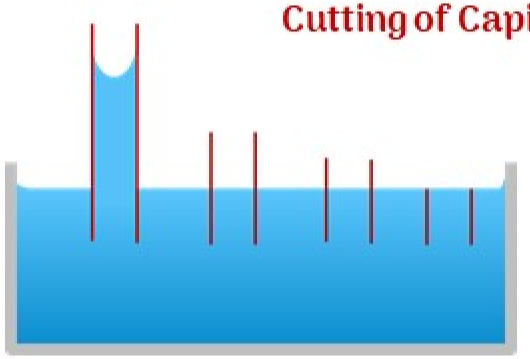
$$U = S \times (\ell d) \times 2$$

Rise of Liquid in Capillary Tube



$$h = \frac{2\,S}{\rho\, gR} = \frac{2\,S}{\rho\, gr}\, cos\theta_c$$

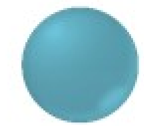
Cutting of Capillary Tube



$$h = \frac{2\,S}{\rho\, gR} = \frac{2\,S}{\rho\, gr}\, cos\theta_c$$

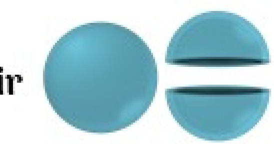
Excess Pressure

Inside a Drop in Air



$\frac{2S}{R}$

Thin Soap Bubble in Air



$\frac{4S}{R}$

Air Bubble in Water

$\frac{2S}{R}$

Electrostatics

1. Electric charge properties:

- Similar charges repel each other and Opposite charges attract each other
- Charge adds like real numbers
- Charges of isolated system remains conserved
- Charge is quantised
- Charge is invariant from frame of reference

2. Coulomb's Law

$$F = \frac{1}{4\pi \epsilon_0} \times \frac{q_1 q_2}{r^2}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \; \frac{C^2}{N - m^2}$$

3. Coulomb's Law in Vector form

$$\vec{F}_{21} = \frac{1}{4\pi \epsilon_0} \frac{q_1 q_2}{r_{21}^2} \hat{r}_{21}$$

4. Electric Field due to point charge

$$E = \frac{kQ}{r^2}$$

- For positive charge field is radially outwards.
- For negative charge field is radially inwards.

5. EFI due to system of point charges

$$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

6. EFI on perpendicular bisector of line joining two point charges

$$E_{net} = \frac{k\,2q\,x}{(x^2 + R^2)^{\frac{3}{2}}}$$

18. EFI due to thin spherical shell

$$r < R,\; E = 0 \qquad r > R, \qquad E = \frac{kQ}{r^2}$$

19. EFI due to uniformly charged solid sphere

$$r < R,\; E = \frac{\rho r}{3\epsilon_0} \qquad r = R \;\; E = \frac{\rho R}{3\epsilon_o} \qquad r > R \;\; E = \frac{kQ}{r^2}$$

20. EFI due to infinite line charge $\quad E = \frac{k\lambda}{r}$

21. EFI due to ∞ long uniformly charged cylindrical shell

$$r < R,\; E = 0 \qquad r > R \;\; E = \frac{\sigma R}{r\epsilon_o}$$

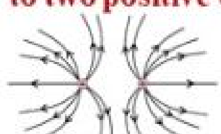
22. EFI due to ∞ long uniformly charged solid cylinder

$$r \le R \;\; E = \frac{\sigma R}{r\epsilon_o} \qquad r > R \;\; E = \frac{\sigma R^2}{2r\epsilon_o}$$

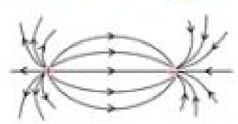
23. EFI due to ∞ long uniformly charged sheet

$$E = \frac{\sigma}{2\epsilon_o}$$

13. Electric field lines due to two positive charges



13. Electric field lines due to One positive and one negative charge



7. EFI on the axis of uniformly charged ring

$$E = \frac{k\,Q\,x}{(x^2 + R^2)^{\frac{3}{2}}}$$

8. EFI due to uniformly charged thin straight wire

$$E_\perp = \frac{k\lambda}{r}(\sin\theta_1 + \sin\theta_2) \qquad E_{||} = \frac{k\lambda}{r}(\cos\theta_1 - \cos\theta_2)$$

9. EFI due to uniformly charged infinite long wire

$$E_\perp = \frac{k\lambda}{r}(\sin\theta_1 + \sin\theta_2) \qquad E_{||} = \frac{k\lambda}{r}(\cos\theta_1 - \cos\theta_2)$$

10. EFI due to uniformly charged semi infinite long wire

$$E_\perp = \frac{k\lambda}{r}(\sin\theta_1 + \sin\theta_2) \qquad E_{||} = \frac{k\lambda}{r}(\cos\theta_1 - \cos\theta_2)$$

11. EFI due to uniformly charged arc at centre of curvature

$$E = \frac{2k\lambda}{R} \sin\left(\frac{\alpha}{2}\right)$$

12. EFI on axis of uniformly charged disc

$$E = \frac{\sigma}{2\epsilon_0}[1 - \cos\theta\,]$$

17. Gauss Law

$$\oint \vec{E}.\,\vec{dA} = \frac{\sum q_{inside}}{\epsilon_0}$$

14. Motion of charged particle in electric field $\quad \vec{F} = q\,\vec{E}$

15. Electric Flux $\quad \phi = \vec{E} \cdot \vec{A} \qquad \phi = E\,A\cos\theta$

16. Net flux linked with closed Surface is-

$$\phi = \oint \vec{E} \cdot d\vec{A}$$

1. **Potential energy of a system of two point charges** $U = \dfrac{kq_1q_2}{r}$

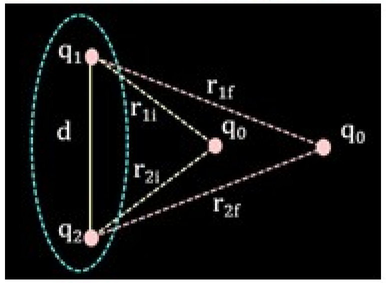
2. **Work done by electrostatic force**

$$W = -\Delta U \qquad W = \frac{kq_1q_2}{r_i} - \frac{kq_1q_2}{r_f}$$

Work done by electrostatic force in a closed loop = 0

3. **Work energy theorem** $W_{net} = KE_f - KE_i$

4. **Interaction potential energy and self potential energy**



$$U_f = \underbrace{\frac{kq_1q_2}{d}}_{\text{Self PE}} + \underbrace{\frac{kq_1q_0}{r_{1f}} + \frac{kq_2q_0}{r_{2f}}}_{\text{Interaction PE}}$$

5. **Electric potential from EFI** $V_f - V_i = -\int_{r_i}^{r_f} \vec{E}.\,d\vec{r}$

6. **EFI from EP** $E_r = -\dfrac{dV}{dr}$

7. **Electric potential due to**

a. **Uniformly charged thin spherical shell**

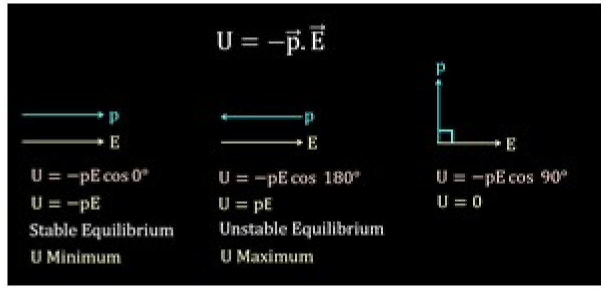
$$r < R \quad V_0 = \frac{kQ}{R}, \qquad r > R \quad V_0 = \frac{kQ}{r},$$

b. **Uniformly charged solid sphere**

$$r < R \quad V_r = \frac{\rho}{6\epsilon_o}\left(3R^2 - r^2\right)$$

$$r = R \quad = \frac{\rho R^2}{3\epsilon_o} \qquad r > R \quad V_r = \frac{kQ}{r},$$

14 **Electric potential energy of electric diapole of Electric field.**



Electric Potential

$$V_A = \frac{U_{int}}{q_0}$$

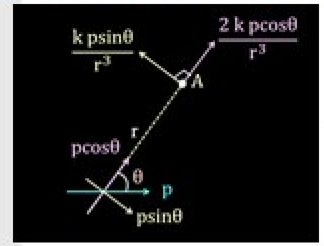
Due to point charge	$V_A = \dfrac{kQ}{r}$
Due to system of charges	$V_A = \dfrac{kQ_1}{r_1} + \dfrac{kQ_2}{r_2} + \dfrac{kQ_3}{r_3}$
At the center of uniformly charged ring	$V_0 = \dfrac{kQ}{R}$
At the center of thin charged spherical shell	$V_0 = \dfrac{kQ}{R}$
On the axis of uniformly charged ring	$V_A = \dfrac{kQ}{\sqrt{x^2 + R^2}}$

8. **Equipotential Surface- same potential at each and every point**

- Electric field is perpendicular to the Equipotential surface.
- Net work done by electrostatic force on moving a charged particle in a path having starting and end point on same ES is always zero.

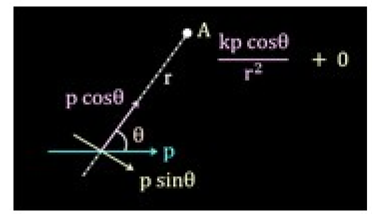
9. **Electric Dipole moment** $p = |\vec{p}| = q\,d$

10. **EFI at general point**



$$|E_A| = \frac{kp}{r^3}\sqrt{3\cos^2\theta + 1}$$

11. **Electric potential at general point**



$$V_A = \frac{k\,\vec{p}\,.\hat{r}}{r^2}$$

$$V_A = \frac{k\,\vec{p}\,.\vec{r}}{r^3}$$

12. Torque of uniform $\vec{E}$ tries to align $\vec{p}$ in direction of $\vec{E}$ through smaller angle. $\vec{\tau} = \vec{p} \times \vec{E}$

13. Torque of uniform $\vec{E}$ tries to align $\vec{p}$ in direction of $\vec{E}$ through smaller angle. $\vec{p}_{eq} = \sum q_i\vec{r}_i$

Electric Conductors

Having unlimited supply of free charge is called Conductor

- Electric Field is 0 inside a conductor in electrostatic steady state.
- Charge density inside a conductor is zero.
- Conductor is an equipotential body.

If a conductor is connected with earth through a conducting wire (earthing), potential of the conductor becomes zero

1. Induced charges in case of concentric spherical cylinder $\dfrac{Q_{inc}}{\varepsilon_0}$

2. Electric potential due to spherical conductor

$$V_{in} = \frac{kQ}{R} = V_{surface}$$

$$V_{outside} = \frac{kQ}{r}$$

3. EFI on the surface of the conductor

$$\frac{\sigma}{\varepsilon_0}$$

4. Conductors with cavities

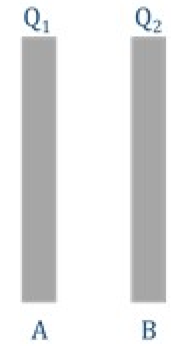
No charge inside cavity

1) EFI inside the cavity is zero.
2) There will be no induced charges on surface of cavity.
3) Potential inside the cavity is same as potential of the conductor.

5. Faraday Cage

Electrostatic shielding, faraday cage is an enclosure to block electromagnetic fields

6. Parallel conducting plate



On facing surfaces, equal and opposite charges appear.

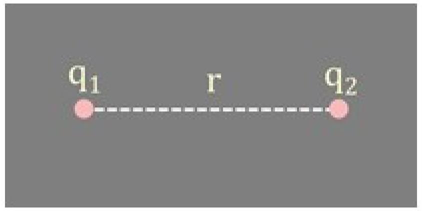
On outer most surfaces, $\dfrac{\sum Q_i}{2}$ charge appear.

7. Electrostatic pressure $P = \dfrac{\sigma^2}{2\epsilon_0}$

8. Energy Density $= \dfrac{1}{2}\epsilon_0 E^2$

Electric Field Intensity

9. Effect of medium on net electrostatic force



$$F_{net} = \frac{1}{4\pi \in} \times \frac{q_1 q_2}{r^2}$$

Permittivity of Medium

$$\frac{F_0}{F_{net}} = \frac{\in}{\in_0} = \in_r$$

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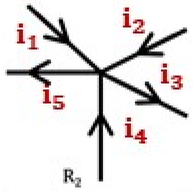
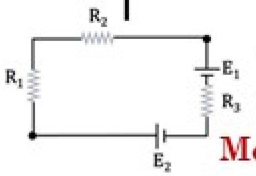
Electric Current



Kirchhoff's Current Law (KCL)
At Junction $\sum$ Current Entering $= 0$
Current Entering = Current Leaving



Kirchhoff's Voltage Law (KVL)
The algebraic sum of changes in potential around any closed loop is zero.

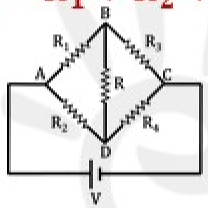
Mesh Analysis 1. Assume current using KCL
2. In loops apply KVL

Nodal Analysis 1. Assume potential in a circuit (at junction) and try to assign minimum variables.
2. Apply KCL at junction.

Parallel Combination (V is same), $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots$

Series Combination (i is same), $R_{eq} = R_1 + R_2 + \cdots$

Wheatstone Bridge $\boxed{\frac{R_1}{R_2} = \frac{R_3}{R_4}}$



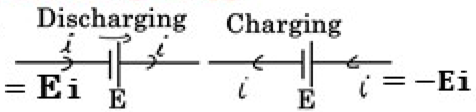
Internal Resistance and Terminal Voltage of a Battery
$V_A - iR + E = V_B$

Series Combination of Batteries, $E_{eq} = E_1 + E_2 + \cdots$

Parallel Combination of Batteries $\frac{E_{eq}}{R_{eq}} = \frac{E_1}{R_1} + \frac{E_2}{R_2} + \cdots$

Power Delivered Through a Device $P = Vi$

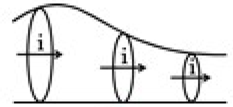
Power Delivered by Battery



Maximum Power Transmission Theorem Power consumed by load resistance will be maximum when its value is equal to internal resistance of battery.

Power Consumed by a Resistor $P_{loss} = Vi = i^2 R = \frac{V^2}{R}$

Steady Current I is same through all cross sections



Current density $= \frac{\text{Amount of Current}}{\text{Cross sectional area } (\perp \text{ to current})} \Rightarrow j = \frac{i}{A_\perp}$

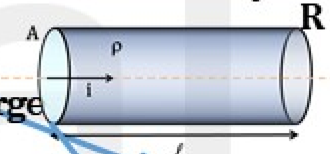
Drift velocity $\vec{v}_d = -\left(\frac{e\vec{E}}{m}\right)\tau$ where τ is avg. time between collisions

OHM's law (Microscopic) $\vec{E} = \rho\,\vec{j}$ where $\rho = \frac{m_e}{ne^2\tau}$ **n, τ, and ρ are properties of material**

Conductivity $= \frac{1}{\text{Resistivity}}$ $\sigma = \frac{1}{\rho}$ $\vec{j} = \sigma\vec{E}$

Resistivity (or Specific Resistance) of a material $\rho = \frac{m_e}{ne^2\tau}$ $\rho_T = \rho_0(1 + \alpha\Delta T)$
Temperature coefficient of resistivity

Resistance of the body $R = \frac{\rho\ell}{A}$ ℓ = Length of wire along i
A = Cross section area $\perp$ to i
ρ = Resistivity of the material
ρ is a property of material R is a property of body.



Conductance $= \frac{1}{\text{Resistance}}$ **Unit is mho (Ω^{-1})**

Dependency of R on Temperature (T)
$R_2 = R_1(1 + \alpha(T_2 - T_1))$ α is Temperature coefficient of Resistance

SI unit - Ampere (A) → $\frac{\text{Coulomb}}{\text{Second}}$

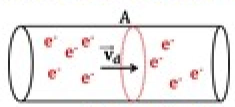
Rate of flow of charge

$i = \vec{j}.\vec{A}$

Instantaneous $i = \frac{dQ}{dt}$

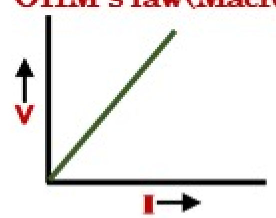
Average $\langle i \rangle = \frac{\Delta Q}{\Delta t}$

By convention, direction of flow of positive charge is taken as direction of flow of current.



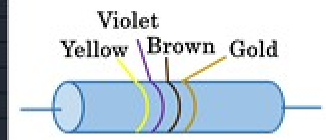
$i = neAv_d$

OHM's law (Macroscopic) $V = iR$
Current
Resistance
Potential difference



Heat loss by resistor in time 't' $= i^2Rt = \frac{V^2t}{R}$

Colour	Number	Multiplier	Tolerance (%)
Black	0	1	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Gray	8	10^8	
White	9	10^9	
Gold		10^{-1}	5
Silver		10^{-2}	10
No colour			20



Ideal battery

Resistor

Rheostat is Variable Resistance

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CAPACITOR

Capacity of a Capacitor, $C = \dfrac{Q}{V}$

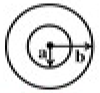
where C is a +ve constant for a conductor and is called capacity (capacitance) of capacitor.

SI unit of Capacity is Farad

Capacity of a Capacitor depends on
1) Shape and size of the plates
2) Distance between plates
3) Medium between the plates

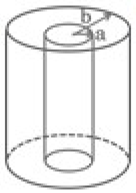
Parallel Plate Capacitor $C = \dfrac{A\varepsilon_0}{d}$

Spherical Capacitor $C = \dfrac{4\pi\varepsilon_0 ab}{b-a}$



Cylindrical Capacitor

$$C = \frac{2\pi\varepsilon_0 l}{\ln\left(\frac{b}{a}\right)}$$



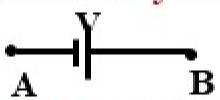
Series Combination (charge is same), $\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \cdots$

Parallel Combination (V is same), $C_{eq} = C_1 + C_2 + \cdots$

Mesh Analysis
- Assume charge using Junction Law.
- Apply KVL

Nodal Analysis
- Assume potential in a circuit.
- Apply Junction Law

Work Done By Battery

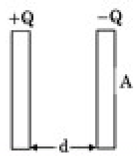


$W = QV$

Energy Stored in a Capacitor $= \dfrac{1}{2}CV^2 = \dfrac{Q^2}{2C} = \dfrac{1}{2}QV$

Electrostatic Force b/w the Plates of a Parallel Plate Capacitor

$$F = \frac{Q^2}{2A\epsilon_0}$$



Bound Charges (Induced Charges) on Dielectric $q = Q\left(1 - \dfrac{1}{K}\right)$

Heat Loss $= \dfrac{Q_0^2}{2C}$

Conductor

$Q \rightarrow$ Charge on conductor

$V \rightarrow$ Potential of conductor

Capacitor

$Q \rightarrow$ Charge on positive plate

$V \rightarrow$ Potential difference between plates

1 Farad is a very BIG unit. $\mu F(10^{-6}F)$, $nF(10^{-9}F)$, $pF(10^{-12}F)$

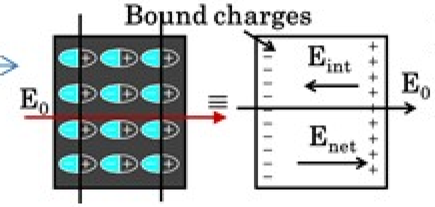
Heat Generated in Circuit $H = \sum W_b - \sum \Delta E$

$\Delta E = (E_f - E_i)$

$\sum W_b$ = Work done by all batteries

$\sum \Delta E$ = Energy change in all capacitors

Dielectric
- Polar (Eg. H_2O): have dipoles even in absence of external $\vec{E}$
- Non-Polar (Eg. Graphite): do not have dipoles in absence of external $\vec{E}$



$E_{net} = E_0 - E_{int}$

$E_{net} = \dfrac{E_0}{K}$ [K is dielectric constant]

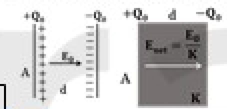
$1 \le K \le \infty$

Vacuum   Conductors

Dielectric Inside Capacitor $E_{net} = \dfrac{E_0}{K}$ $C = KC_0$

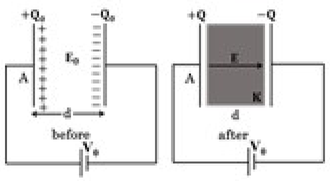
Effect on Parameters due to Introduction of Dielectric Slab (Completely Filled)

1. Battery is not connected



Before	After
Q_0	Q_0
C_0	KC_0
E_0	E_0/K
V_0	V_0/K
U_0	U_0/K

2. Battery is connected

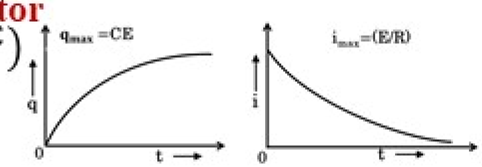


Before	After
Q_0	KQ_0
C_0	KC_0
E_0	E_0
V_0	V_0
U_0	U_0K

Charging of Capacitor

$q = CE\left(1 - e^{-t/RC}\right)$

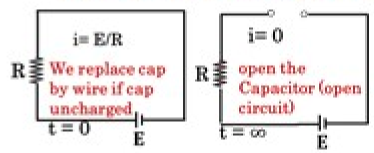
$i = \dfrac{E}{R}\left(e^{-t/RC}\right)$



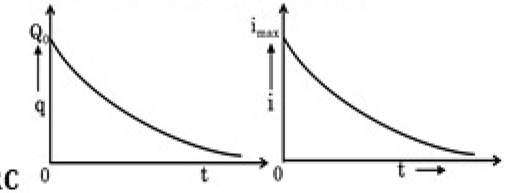
Time Constant (τ) $\tau = RC$

After 1τ time capacitor gets 63% charged

Calculation of current at $t = 0$ and $t = \infty$



Discharging of Capacitor



In 1τ approx capacitor 63% discharged

$q = Q_0 e^{-t/RC}$

$i = i_{max.}\, e^{-t/RC}$

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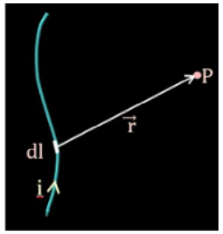
Magnetic Effect of Currents

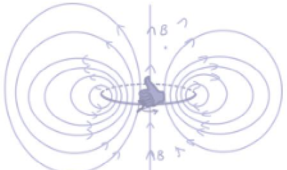
Biot Savart Law



$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{\ell}\times\vec{r}}{r^3}$$

SI unit of magnetic field ($\vec{B}$) is Tesla (T)

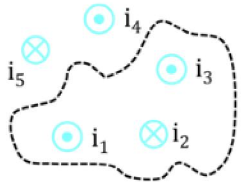
MF Lines due to current carrying loop



Torroid

$B = \mu_0 n i$

Ampere's Circuital Law



$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 \sum i$$

Magnetic Force on a Moving Charge Particle

$$\vec{F}_m = q(\vec{v}\times\vec{B})$$

Motion of particle in plane ⊥ to $\vec{B}$

$$F = q v B = \frac{mv^2}{R}$$

Radius of circular path

If $\vec{v}$ is not ⊥ to $\vec{B}$

Helical path

$$R = \frac{mv_{\perp}}{qB} = \frac{mv\sin\theta}{qB}$$

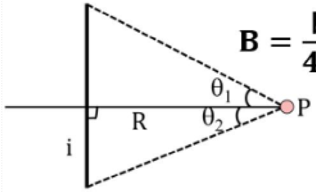
Pitch of helix $T = \frac{2\pi m}{qB}$

Hold the wire with right hand with thumb pointing along current then direction of curling of fingers will give direction of MFI.

Motion of Charged Particle in Presence of Both $\vec{E}$ and $\vec{B}$

Lorentz force $\vec{F} = q\vec{E} + q(\vec{v}\times\vec{B})$

Magnetic field intensity due to straight current carrying wire



$$B = \frac{\mu_0 i}{4\pi R}(\sin\theta_1 + \sin\theta_2)$$

MFI due to semi infinite wire $B = \frac{\mu_0 i}{4\pi R}$

MFI due to infinite wire $B = \frac{\mu_0 i}{2\pi R}$

Solenoid

$$B = \frac{\mu_0 n i}{2}(\sin\theta_1 + \sin\theta_2)$$

Magnetic Moment

$$\vec{\mu} = N i \vec{A}$$

Torque on a Current Carrying Loop

$$\vec{\tau} = \vec{\mu}\times\vec{B}$$

Magnetic Moment due to Rotation of Charge

$$\frac{\mu}{L} = \frac{Q}{2m}$$

Magnetic field lines

- In case of straight current carrying wire magnetic field lines are circular with their centres on the wire.
- Tangent to the magnetic field line at a given point represents the direction of net magnetic field at that point.
- Magnetic field lines form continuous closed loops.

Magnetic Force on a Current Carrying Wire Kept in Magnetic Field

$$\vec{dF} = i\,\vec{d\ell}\times\vec{B}$$

Magnetic Force Between Two Current Carrying Wires

$$\frac{F}{\ell} = \frac{\mu_0\, i_1 i_2}{2\pi d}$$

Same direction currents attract each other and opposite direction currents repel each other.

When wires are perpendicular to each other.

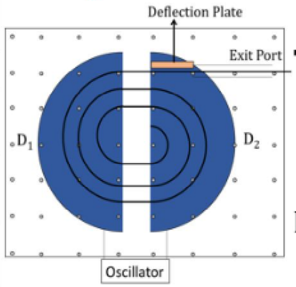
$$F = \frac{\mu_0 i_1 i_2}{2\pi}\ell n\frac{r_2}{r_1}$$

MFI due to

Magnetic field	
Center of the current carryng loop	$\frac{\mu_0 i}{2R}$
Center of the current carrying circular loop	$\frac{\mu_0 i}{2R}\frac{\alpha}{2\pi}$
Axis of current carrying loop	$B = \frac{\mu_0 i}{2R}(\sin^3\theta)$

Wire		$B = \frac{\mu_0 i}{2\pi R}$
Current Carrying ∞ Long Hollow Cylinder	r<R	$B = 0$
	r>R	$B = \frac{\mu_0 i}{2\pi r}$
Current Carrying ∞ Long Solid Cylinder	r<R	$B = \frac{\mu_0 i}{2}r$
	r>R	$B = \frac{\mu_0 i}{2\pi r}$

Cyclotron



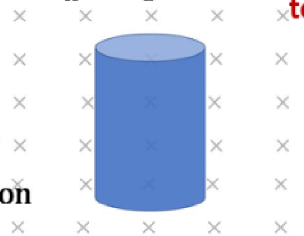
$$T = \frac{2\pi m}{qB}$$

$$f_c = \frac{qB}{2\pi m} = f_{osc}$$

Resonance condition

Hall Effect

$$\Delta V_H = v_d B d$$



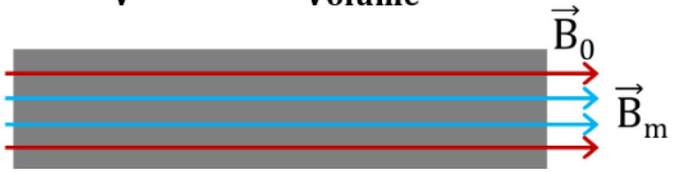
Moving Coil Galvanometer

$$\tau = NIAB \qquad \theta = \frac{NIAB}{k}$$

Magnetism and Matter

Magnetisation

$$\vec{M} = \frac{\vec{m}_{net}}{V} = \frac{\text{Net Magnetic Moment}}{\text{Volume}}$$



$$\vec{B} = \vec{B}_0 + \vec{B}_m$$

$\vec{B}_0 \downarrow \mu_0 \vec{H}$, $\vec{B}_m \downarrow \mu_0 \vec{M}$

$$\vec{B} = \mu_0 (\vec{H} + \vec{M})$$

$$\vec{B} = \mu_0 (1 + \chi)\vec{H}$$

$$\vec{M} = \vec{H}\chi$$

$$\vec{B} = \mu_0 \mu_r \vec{H}$$

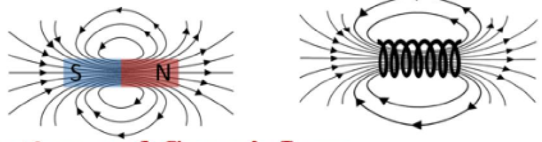
$$\vec{B} = \mu \vec{H}$$

Magnets

Same poles repel

Opposite poles attract each other

The magnetic field lines of a magnet and Solenoid form continuous closed loops.



Magnetism and Gauss's Law

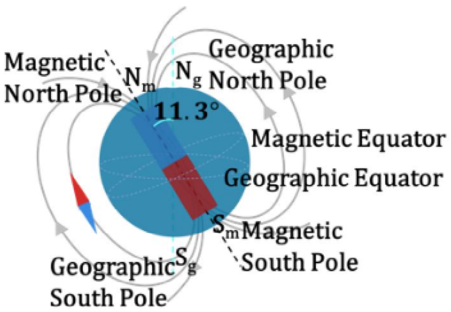
Net magnetic flux through any closed surface $\left(\oint \vec{B} \cdot \vec{dA}\right)$ is always zero.

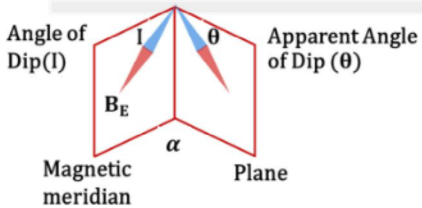
Dipole in Uniform Magnetic Field

$$T = 2\pi\sqrt{\frac{I}{mB}}$$

Electrostatic Analog	
$\vec{B}$	$\vec{E}$
$\vec{m}$	$\vec{p}$
μ_0	$\frac{1}{\varepsilon_0}$
$\vec{\tau} = \vec{m} \times \vec{B}$	$\vec{\tau} = \vec{p} \times \vec{E}$
$U = -\vec{m}.\vec{B}$	$U = -\vec{p}.\vec{E}$
$\vec{B}_{axial} = \frac{\mu_0}{4\pi}\frac{2\,\vec{m}}{r^3}$	$\vec{E}_{axial} = \frac{1}{4\pi\varepsilon_0}\frac{2\,\vec{p}}{r^3}$
$\vec{B}_{eq} = -\frac{\mu_0}{4\pi}\frac{\vec{m}}{r^3}$	$\vec{E}_{eq} = -\frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{r^3}$

Earth's Magnetic Field

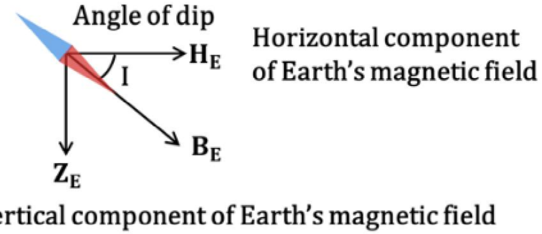




$$B_E \cos I = H_E$$

$$B_E \sin I = Z_E$$

$$\tan I = \frac{Z_E}{H_E}$$



Magnetism and Matter

Paramagnetic
Magnetisation of paramagnetic material is inversely proportional to the absolute temperature.

$$\chi = C\frac{\mu_0}{T}$$

Curie's Law-

Diamagnetic
Superconductors cooled to very low temperatures show both perfect conductivity and perfect diamagnetism.

$\chi = -1$

$\mu_r = 0$

Meissner Effect.

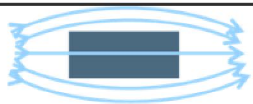
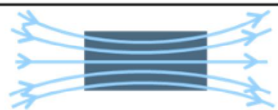
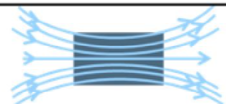
Ferromagnetic
At high enough temperature, a ferromagnet becomes a paramagnet. This temperature of transition from ferromagnetic to paramagnetic is called Curie Temperature T_C.

$Fe \rightarrow 1043\ K$ $\quad$ $Ni \rightarrow 631\ K$

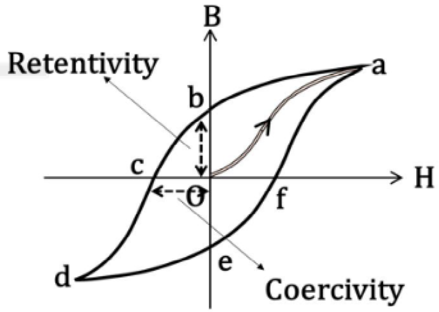
Above Curie Temperature in paramagnetic phase,

$$\chi \propto \frac{1}{T - T_C}$$

Classification of Magnetic Materials

Properties	Diamagnetic	Paramagnetic	Ferromagnetic
χ	$-1 \le \chi < 0$	$0 < \chi < k$	$\chi >> 1$
μ_r	$0 \le \mu_r < 1$	$1 < \mu_r < 1 + k$	$\mu_r >> 1$
μ	$\mu < \mu_0$	$\mu > \mu_0$	$\mu >> \mu_0$
Magnetisation	Weak Magnetisation in opposite direction	Weak Magnetisation in Same direction	Strong Magnetisation in Same direction
Movement in magnetic field	(Weak tendency) From strong to weak magnetic field	(Weak tendency) From weak to strong magnetic field	(Strong tendency) From weak to strong magnetic field
Magnet	Weak Repulsion	Weak Attraction	Strong Attraction
E.g	Bi, Au, Pb, Si, H_2O, NaCl, N_2 (STP)	Al, Na, Ca, O_2(STP)	Fe, Co, Ni,Gd
Magnetic Field Lines			

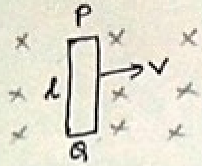
Permanent Magnets	Electromagnets
High Retentivity	Low Retentivity
High Coercivity	Low Coercivity



Hysteresis Loss
It is the energy lost in form of heat during a complete cycle of magnetization and demagnetization.

Electromagnetic Induction

Motional EMF



$V_P - V_Q = Blv$

Force, $F_{ext} = \frac{B^2 l^2 v}{R}$

Power, $P_{ext} = F_{ext} V = \frac{B^2 l^2 v^2}{R} = P_{loss}$

Magnetic Energy Density

$= \frac{B^2}{2\mu_0}$

Energy per unit Volume

Magnetic Flux

$\phi = \int \vec{B} \cdot d\vec{A}$ [SI → Wb or Tm^2]

$\vec{B}$ is uniform ⇒ $\phi = \vec{B} \cdot \vec{A} = BA\cos\theta$

Faraday Law of Induction

$E = \left|\frac{d\phi}{dt}\right|$

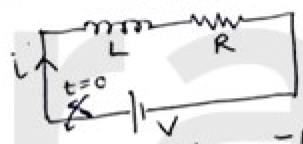
E → Emf Induced

For N turns, $E = N\left|\frac{d\phi}{dt}\right|$

Lenz's Law

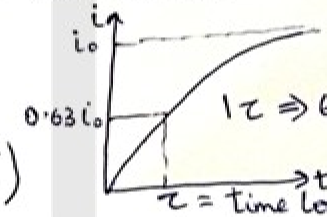
Induced emf opposes Cause of Generation

Growth of Current in LR Circuit

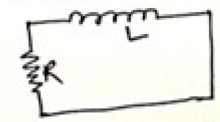


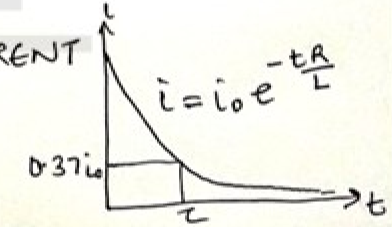
$i = i_0\left(1 - e^{-\frac{tR}{L}}\right)$

$1\tau = L/R$



Decay of Current





Energy stored in Inductor

L, i

Energy stored in Inductor $= \frac{1}{2} L i^2$

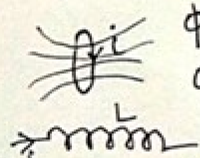
Induced Electric Field

$\oint \vec{E} \cdot d\vec{l} = -\frac{d\phi_B}{dt}$

→ Induced by change in Magnetic field

→ Nature - Non-electrostatic and Non-conservative

Self Inductance



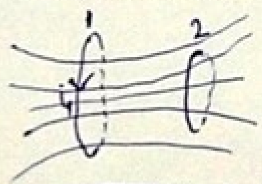
$\phi \propto i$

$\phi = Li$

L → Self Inductance of coil

Self Induction $E = -\frac{d\phi}{dt} = -L\frac{di}{dt}$

Mutual Inductance



$\phi_{21} \propto i_1$

$\phi_{21} = M_{21} i_1$

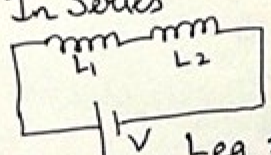
M_{21} → Mutual Inductance of ② w.r.t. ①

$\boxed{M_{21} = M_{12}}$

Theorem of Reciprocity

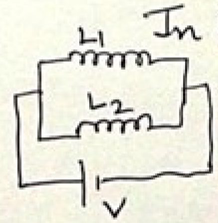
Inductors

In Series



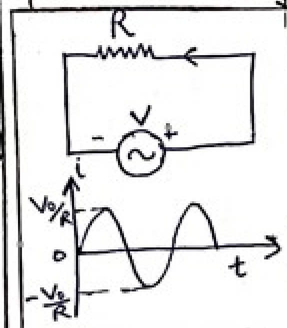
$L_{eq} = L_1 + L_2$

In Parallel



$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$

ALTERNATING CURRENT



$V = V_0 \sin\omega t$

$i = \frac{V_0}{R}\sin\omega t$

$i = i_0 \sin\omega t$

$V_{rms} = \frac{V_0}{\sqrt{2}}$

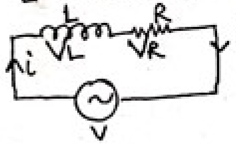
220V is rms Value of Voltage in India.

$f = 50Hz$; $V_{rms} = \frac{V_0}{\sqrt{2}} \Rightarrow V_0 = 220\sqrt{2} \cong 311\,V$

AVERAGE VALUE OF A FUNCTION $\langle y \rangle = \frac{\int y\,dt}{\int dt}$

RMS (Root Mean Square) Value of A FUNCTION $\Rightarrow y_{rms} = \sqrt{\langle y^2 \rangle}$

L-R CIRCUIT



$i = i_0 \sin\omega t$

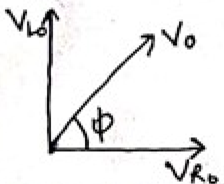
$V = V_R + V_L$

$= V_{R_0}\sin\omega t + V_{L_0}\sin\left(\omega t + \frac{\pi}{2}\right)$

$= V_0 \sin(\omega t + \phi)$

$V_{R_0} = i_0 R$

$V_{L_0} = i_0 X_L$

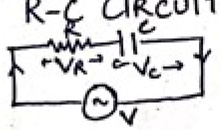


$V_0 = \sqrt{V_{L_0}^2 + V_{R_0}^2}$

$= i_0\sqrt{X_L^2 + R^2} = i_0 Z$

$Z = \sqrt{X_L^2 + R^2}$ $\tan\phi = \frac{X_L}{R}$

R-C CIRCUIT

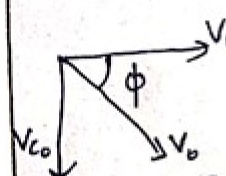


$i = i_0 \sin\omega t$

$V = V_R + V_C$

$= V_{R_0}\sin\omega t + V_{C_0}\sin\left(\omega t - \frac{\pi}{2}\right)$

$= V_0 \sin(\omega t - \phi)$

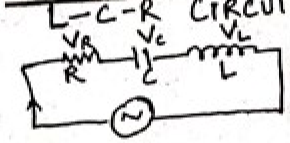


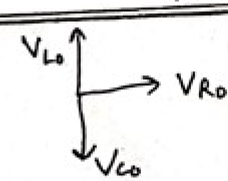
$V_0 = \sqrt{V_{C_0}^2 + V_{R_0}^2}$

$= i_0\sqrt{X_C^2 + R^2} = i_0 Z$

$Z = \sqrt{X_C^2 + R^2}$ $\tan\phi = \frac{X_C}{R}$

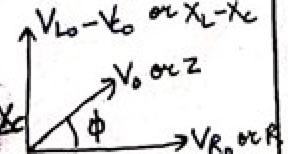
L-C-R CIRCUIT





$V = V_{R_0}\sin\omega t + V_{C_0}\sin\left(\omega t - \frac{\pi}{2}\right) + V_{L_0}\sin\left(\omega t + \frac{\pi}{2}\right)$

$V_0 = i_0\sqrt{(X_L - X_C)^2 + R^2}$

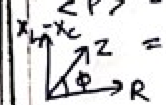


$Z = \sqrt{(X_L - X_C)^2 + R^2}$ $\tan\phi = \frac{X_L - X_C}{R}$

$V_{rms} = I_{rms} Z$

POWER DELIVERED BY SOURCE

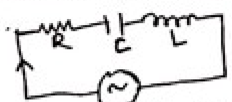
$\langle P \rangle = i_{rms} V_{rms} \cos\phi$ → Power Factor



$= \frac{i_0}{\sqrt{2}} \frac{V_0}{\sqrt{2}} \cos\phi$

$\cos\phi = \frac{R}{Z}$ $0 \le \cos\phi \le 1$

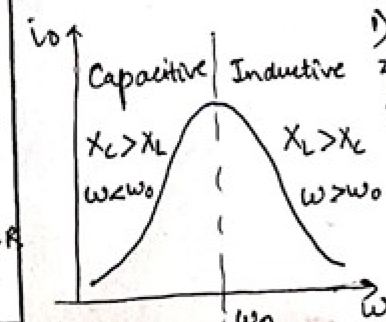
RESONANCE IN LCR CIRCUIT



$V = V_0 \sin(\omega t + \phi)$

Resonant frequency $f_0 = \frac{1}{2\pi\sqrt{LC}}$

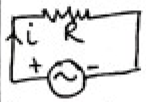
$i_0 = \frac{V_0}{Z} = \frac{V_0}{\sqrt{(X_L - X_C)^2 + R^2}}$



At Resonance

1) i_0 is Max
2) $X_L = X_C$
3) $Z = R$
4) $\cos\phi = 1$
5) $\omega_0 = \frac{1}{\sqrt{LC}}$

PURE RESISTIVE CIRCUIT



$V = V_0 \sin\omega t$

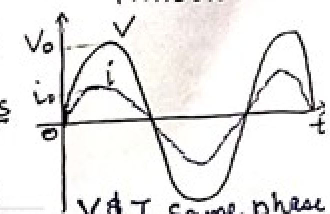
$i = i_0 \sin\omega t$

$i_0 = \frac{V_0}{R}$ $i_{rms} = \frac{V_{rms}}{R}$

$i_{rms} = \frac{i_0}{\sqrt{2}}$ $V_{rms} = \frac{V_0}{\sqrt{2}}$

$\langle P \rangle = i_{rms}^2 R = \frac{V_{rms}^2}{R}$

PHASOR



V & I Same phase

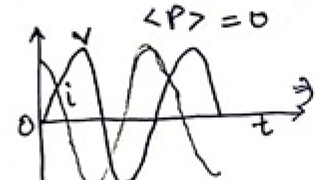
PURE CAPACITIVE CIRCUIT

$V = V_0 \sin\omega t$

$i = i_0 \sin\left(\omega t + \frac{\pi}{2}\right)$ $\Rightarrow i_0 = \frac{V_0}{X_C}$

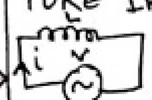
where $i_0 = CV_0\omega = \frac{V_0}{1/\omega C}$ → X_C → unit Ω

$\langle P \rangle = 0$



⇒ Current leads Voltage by phase $\frac{\pi}{2}$

PURE INDUCTIVE CIRCUIT

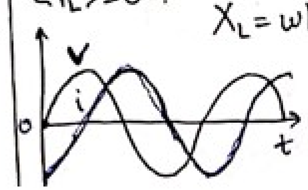


$V = V_0 \sin\omega t$ $i_0 = \frac{V_0}{X_L}$

$i = i_0 \sin\left(\omega t - \frac{\pi}{2}\right)$ $i_{rms} = \frac{V_{rms}}{X_L}$

$\langle P_L \rangle = 0$

$X_L = \omega L \Rightarrow$ Inductive Reactance



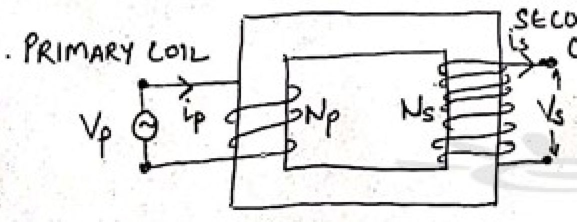
⇒ Voltage leads current by phase $\pi/2$

TRANSFORMERS

→ Use to change (i.e. transform) an alternating voltage from one value to another.

→ Use principle of Mutual Induction

$V_p = -N_p \frac{d\phi}{dt}$ $\qquad V_s = -N_s \frac{d\phi}{dt}$



$\boxed{\frac{V_s}{V_p} = \frac{N_s}{N_p}}$

$\Rightarrow V_s = \left(\frac{N_s}{N_p}\right)V_p$

$V_p i_p = V_s i_s$

$\frac{i_s}{i_p} = \frac{V_p}{V_s} = \frac{N_p}{N_s}$

If $N_s > N_p$: STEP UP TRANSFORMER

If $N_p > N_s$: STEP DOWN TRANSFORMER

ASSUMPTION: NO LOSS IN Flux & Energy (Ideal Transformer ∴ P same)

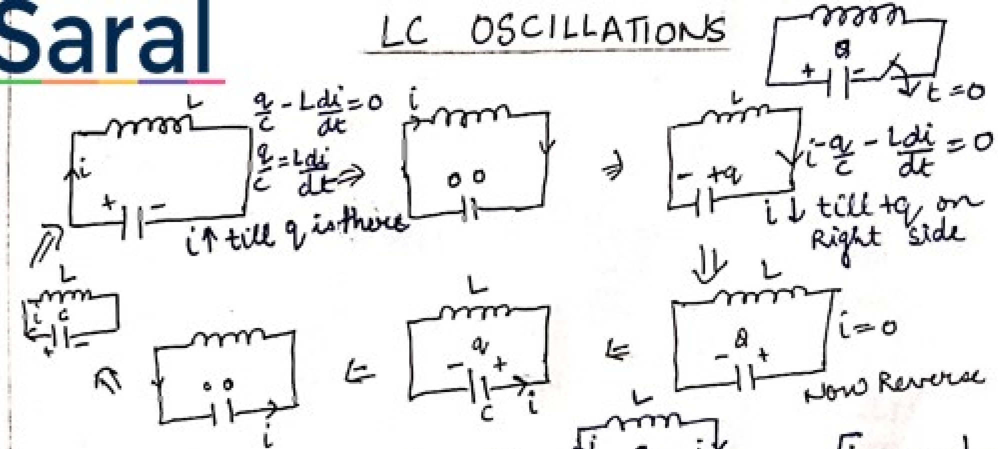
IN ACTUAL Transformer, some Energy Losses Takes Place

CAUSE	RECTIFICATION
1. Flux Linkage (Due to poor design & air gaps in core)	1. By winding primary & secondary coil one over the other.
2. Resistance of the windings: Causes Heat Loss	2. In High current, low voltage windings, these are minimized using thick wire
3. Eddy currents	3. Can be Reduced Using Laminated core
4. Hysteresis Loss	4. Min. Using Magnetic Material having low Hysterisis Loss.

ELECTRICAL TRANSMISSION

POWER Plants Generates Electricity → Transformers Step Up Voltage for Transmission → Transmission Line Carries Electricity over Long Distance → Step down Transformer → Distribution Line Carries Electricity to House

LC OSCILLATIONS

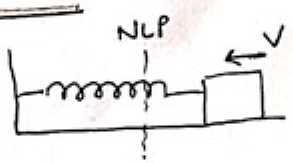


$-\frac{q}{C} - L\frac{di}{dt} = 0$ & $i = \frac{dq}{dt}$

$\omega = \sqrt{\frac{1}{LC}} = \frac{1}{\sqrt{LC}}$

$L\frac{di}{dt} = -\frac{q}{C} \Rightarrow L\frac{d^2q}{dt^2} = -\frac{q}{C} \Rightarrow \boxed{\frac{d^2q}{dt^2} = \frac{-q}{LC}}$

LC and SPRING BLOCK



LC	Spring block
Q	x
i	V
$\frac{1}{C}$	K
L	m

Potential Energy $= \frac{Q^2}{2C}$

Magnetic Energy $= \frac{1}{2}Li^2$

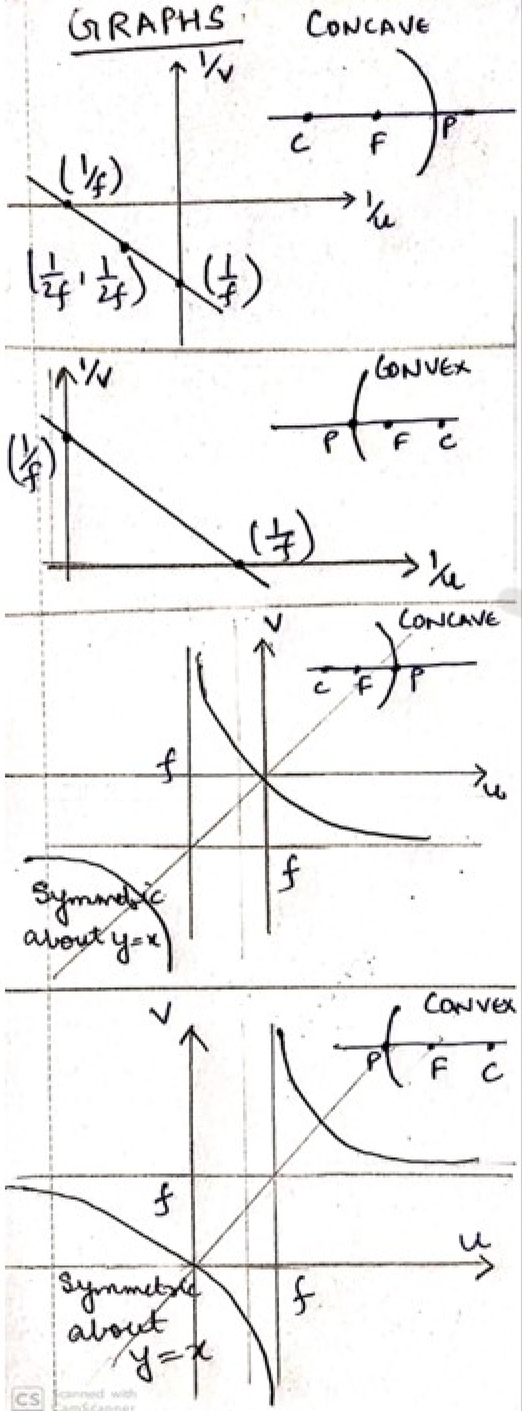
$\frac{Q^2}{2C} = \frac{1}{2}Li_0^2 = \frac{Q^2}{2C} + \frac{1}{2}Li^2$

Potential Energy $\frac{1}{2}Kx^2$

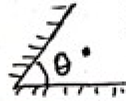
Kinetic Energy $\frac{1}{2}mv^2$

$\frac{1}{2}KA^2 = \frac{1}{2}mV_{max}^2 = \frac{1}{2}Kx^2 + \frac{1}{2}mv^2$

GRAPHS



MIRRORS AT AN ANGLE

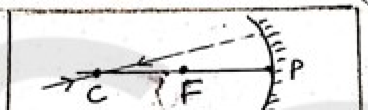


$N = 360/\theta$

CONDITION	No. OF IMAGES
If N = EVEN Integer	N − 1
If N = ODD Integer 1. Object on ∠ Bisector 2. If Not	 N − 1 N
If N ≠ Integer	Count Manually

RAY TRACING DAIGRAM

A ray ∥ to principal axis after reflection passes/appear to pass through focus.



A ray passing through centre of curvature after reflection retraces its path.

A ray ∥ to principal axis after reflection passes through focus

A ray passing through centre of curvature after reflection retraces its path

MAGNIFICATION

$m = -\frac{v}{u} = \frac{f}{f-u}$

LINEAR Magnification

$m = \frac{\text{ht. of Image}}{\text{ht. of Object}} = \frac{h_i}{h_o}$

(h_i & h_o with Sign.

$|m| > 1 \rightarrow$ Enlarged

$|m| < 1 \rightarrow$ Diminished

$m < 0 \rightarrow$ Inverted; $m > 0 \rightarrow$ Erect

NET MAGNIFICATION

$m_{net} = \frac{h_{I\text{-}final}}{h_{object}}$

$m_{net} = m_1 \times m_2 \times m_3 \times -$

$m_{net} = \frac{h_{I_1}}{h_o} \times \frac{h_{I_2}}{h_{I_1}} \times \frac{h_{I_3}}{h_{I_2}} \times -$

$= \frac{h_{I\text{-}final}}{h_o}$

Min. size of a plane mirror required to see the full image of an observer is half the size of that observer

REFLECTION

FOCAL LENGTH

$\boxed{f = R/2}$

Distance of focus from Pole

MIRROR Equation

$\boxed{\frac{1}{v} + \frac{1}{u} = \frac{1}{f}}$

$u \rightarrow$ Coordinate of Object

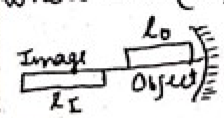
$v \rightarrow$ Coordinate of Image

$f \rightarrow$ focus Coord^n

LONGITUDINAL Magnification

$LM = -m^2 = \frac{l_I}{l_o}$

where m is (TM)



LAWS OF REFLECTION

1. $[\hat{e}\ \hat{r}\ \hat{n}] = 0$

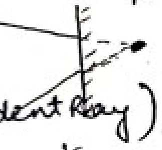
   $[\hat{e} \times \hat{r}] \cdot \hat{n} = 0$

2. $\angle i = \angle r$

OBJECT

→ Real (Intersection of Diverging Rays)

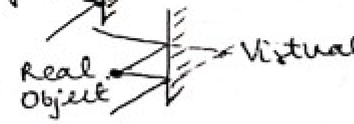
→ Virtual (Intersection of Converging Incident Ray)



IMAGE

→ Real

→ Virtual



PROPERTIES OF IMAGE FORMED BY PLANE MIRROR

1. It forms (i) Real Image of Virtual Object
   (ii) Virtual Image of Real Object
2. Distance of Object from mirror = Distance of image from the mirror
3. Size of Image = Size of Object ($h_i = h_o$)
4. Image formed is SIDE-WAYS INVERTED.

VELOCITY OF Image

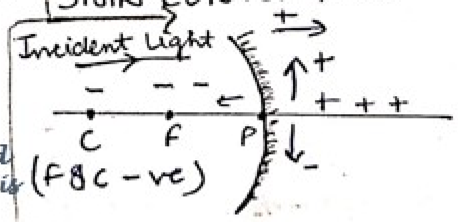
$(\vec{V}_o)_{\parallel} = (\vec{V}_I)_{\parallel}$

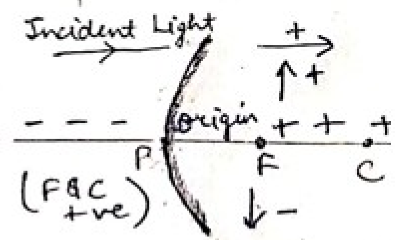
$(\vec{V}_{o/m})_{\perp} = -(\vec{V}_{I/m})_{\perp}$

VELOCITY OF Image of Moving Object (Sph. Mirror)

$\boxed{\vec{V}_I = -m^2 \vec{V}_o}$

SIGN-CONVENTION



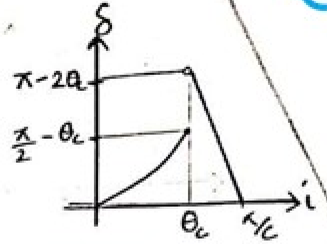


REFRACTION

Total Internal Reflection

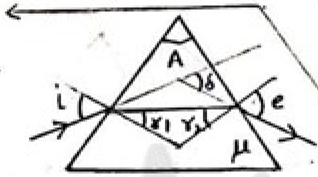
$\sin\theta_c = \frac{1}{\mu}$

Angle of Deviation



PRISM

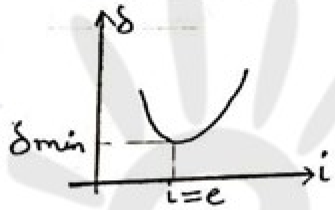
$A = r_1 + r_2$



Angle of Deviation

$\delta = i + e - A$

GRAPH b/w ∠ of Deviation & ∠ of Incidence



$$\mu = \frac{\sin\left(\frac{\delta_{min}+A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

Maximum ∠ of Deviation

$\delta_{max} = i_g + 90 - A$

Thin Prism

$\delta = (\mu - 1)A$

Refraction from Curved Surface

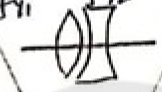
$$\frac{\mu_r}{v} - \frac{\mu_i}{u} = \frac{\mu_r - \mu_i}{R}$$

Transverse Magnification

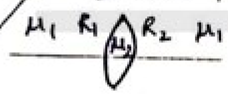
$$m = \frac{h_I}{h_o} = \frac{v/\mu_r}{u/\mu_i}$$

Achromatic Combination

$$\frac{\omega_1}{f_{Y_1}} + \frac{\omega_2}{f_{Y_2}} = 0$$



LENS



→ Lens Maker's formula

$$\frac{1}{f} = \left(\mu_{2/1} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

→ Lens Formula $= \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

→ Longitudinal Magnification in Lens ⇒ $LM = m^2$

→ Velocity of Image ← $\vec{V}_I = m^2 \vec{V}_o$

→ Power of Lens $= P = 1/f$

REFRACTIVE INDEX OF MEDIUM

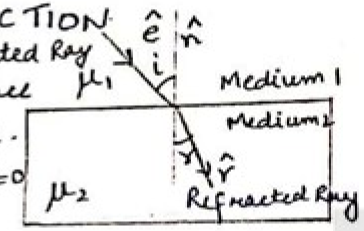
$\mu = \frac{C}{V_m}$; C → Speed of Light in Vaccum ; V_m → Speed of Light in Medium

$$V_m = f\lambda_m \Rightarrow \frac{V_1}{V_2} = \frac{\lambda_1}{\lambda_2} = \frac{\mu_2}{\mu_1} \quad [\text{frequency same}]$$

LAWS OF REFRACTION

1) Incident Ray, Refracted Ray and normal to Surface always lie in same plane.

$[\hat{e}\ \hat{n}\ \hat{r}] = 0 \Rightarrow (\hat{e} \times \hat{n}) \cdot \hat{r} = 0$



2. SNELL'S LAW

$\mu_1 \sin i = \mu_2 \sin r$

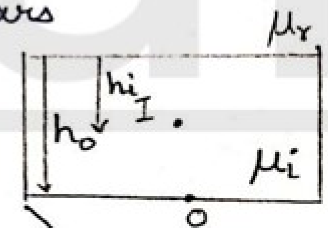
Atmospheric Refraction

1. Early Sunrise & Delayed Sunset
2. Apparent flattening of Sun at Sunrise and Sunset
3. Twinkling of Stars

APPARENT DEPTH

$$h_I = \frac{h_o}{\mu_{i/r}}$$

$$\text{Shift} = h_o\left(1 - \frac{1}{\mu_{i/r}}\right)$$



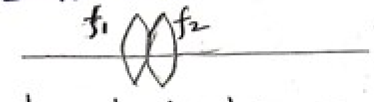
Velocity of Image formed due to Refraction from Plane Surface

$$\vec{V}_I = \frac{\vec{V}_o}{\mu_{i/r}}$$

→ Refraction through Parallel Slab

$$\text{SHIFT} = t\left(1 - \frac{1}{\mu_{s/m}}\right)$$

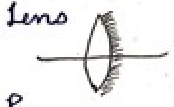
Lens kept Very Close to Each Other



$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + \cdots$$

$P_{eq} = P_1 + P_2 + P_3 + \cdots$ (with sign)

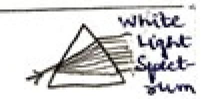
Silvering of Lens



$P_{eq} = P_L + P_m + P_L$

$$P_{eq} = \frac{1}{f_L} - \frac{1}{f_m} + \frac{1}{f_L} = -\frac{1}{f_{eq}}$$

Dispersion



Cauchy's eq^n

$$\mu = A + \frac{B}{\lambda^2}$$

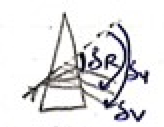
$\lambda\uparrow \quad \mu\downarrow$

V I B G Y O R

Most Deviation — Least Deviation

THIN PRISM

$$\mu_y \approx \frac{\mu_V + \mu_R}{2}$$



Angular Dispersion $(\theta) = \delta_V - \delta_R = (\mu_V - \mu_R)A$

Avg. Deviation $\delta_y = (\mu_y - 1)A$

Dispersive Power $\omega = \frac{\theta}{\delta_y} = \frac{(\mu_V - \mu_R)A}{(\mu_y - 1)A}$

θ is property of Prism

ω is property of Material

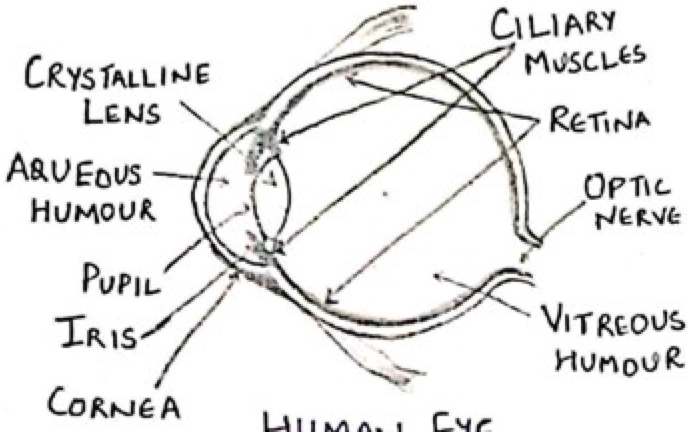
Dispersion but not Avg Deviation

$\delta_{net} = 0$

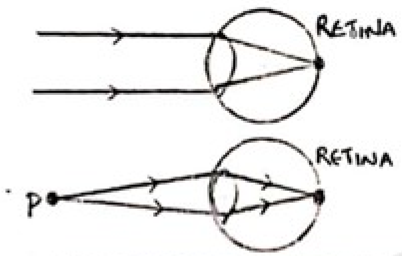
Avg Deviation but Not Dispersion

$\theta_{net} = 0$

OPTICAL INSTRUMENTS

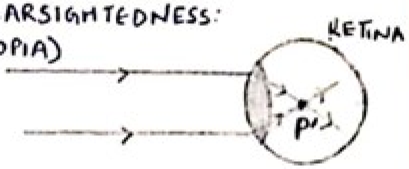
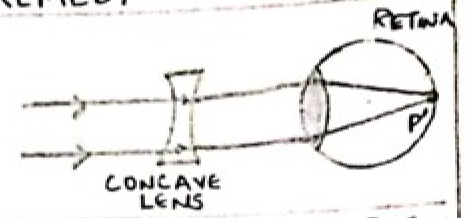
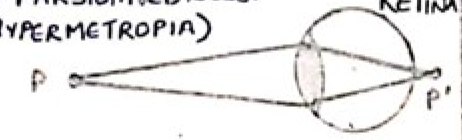
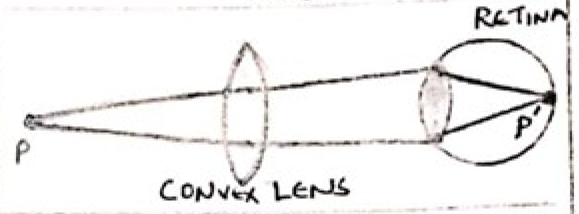
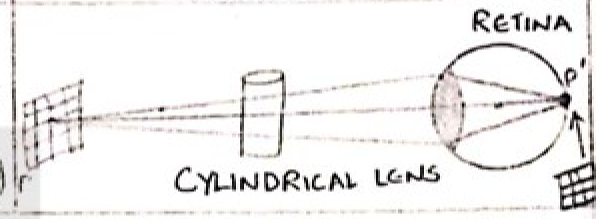


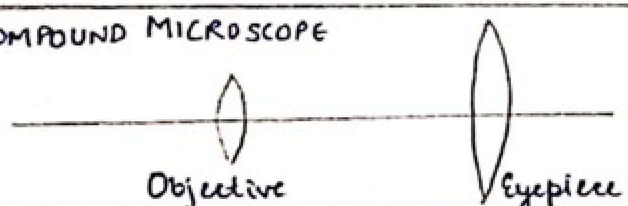
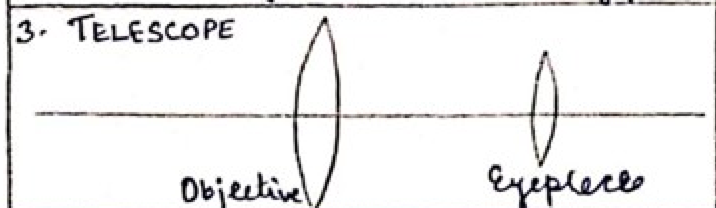
HUMAN EYE



Accommodation
- Far point at ∞
- Near point at 25cm

Least Distance for Distinct Vision

DEFECTS OF EYES	REMEDY
1. NEARSIGHTEDNESS: (MYOPIA) 	
2. FARSIGHTEDNESS: (HYPERMETROPIA) 	
3. ASTIGMATISM: 	
4. PRESBYOPIA: In old age, Near point increases to as much 200Cm.	

INSTRUMENT	IMAGE AT ∞ (Normal Adjustment)	IMAGE at Least Distance of Distinct Vision
1. SIMPLE MICROSCOPE 	Angular Magnification $= \frac{D}{f}$	Angular Magnification $= \frac{D}{f}+1$
2. COMPOUND MICROSCOPE 	Total Magnification, $M = m_o \times m_e$ $= \frac{L}{f_o}\frac{D}{f_e}$	Total Magnification $= \left\lvert\frac{v_o}{u_o}\right\rvert\left(\frac{D}{f_e}+1\right)$
3. TELESCOPE 	Magnifying Power $= \beta/\alpha = \left\lvert\frac{f_o}{u_e}\right\rvert$ $= \frac{f_o}{f_e}$ In this Case, Length of Telescope is f_o+f_e	Magnifying Power $= f_o\left(\frac{1}{f_e}+\frac{1}{D}\right)$

eSaral

WAVE OPTICS

Constructive Interference

Destructive Interference

- Two sources are said to be Coherent if they produce waves having constant (with respect to time) phase difference.
- For Incoherent sources, phase difference varies with time.
- Two independent ordinary sources (like lamps) are incoherent sources.

Constructive Interference

$\cos\phi = 1$

$\phi = 2n\pi$

$A_{max} = (A_1 + A_2)$

$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

$I_{max} = 4I_0$ (If same source of I0)

$\Delta x = 0, \lambda, 2\lambda, 3\lambda... = n\lambda$

Destructive Interference

$\cos\phi = -1$

$\phi = (2n+1)\pi$

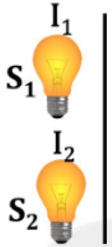
$A_{min} = (A_1 - A_2)$

$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$

$I_{min} = 0$ (If same source of I0)

$\Delta x = 0.5\lambda, 1.5\lambda, 2.5\lambda... = \left(n + \frac{1}{2}\right)\lambda$

Incoherent Sources



$I_{res} = I_1 + I_2$

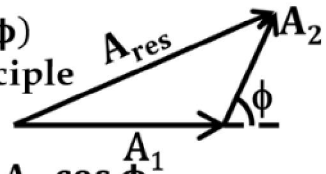
Interference of Waves

$y_1 = A_1 \sin(kx - \omega t)$

$y_2 = A_2 \sin(kx - \omega t + \phi)$

By Superposition Principle

$y_{res} = y_1 + y_2$



$(A_{res})^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi$

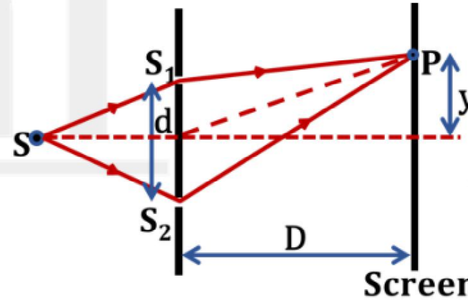
$I \propto A^2$

$I_{res} = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\phi$

If $I_1 = I_2 = I0$, then

$I_{res} = 4I_0\cos^2\left(\frac{\phi}{2}\right)$

Young's Double Slit Experiment (YDSE)



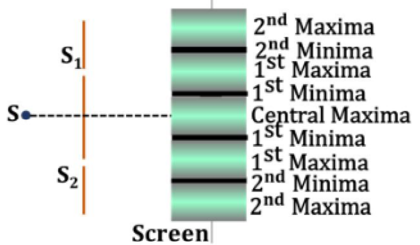
$\Delta x = \frac{yd}{D}$

d is distance between slits S_1 and S_2

D is distance between slit and screen

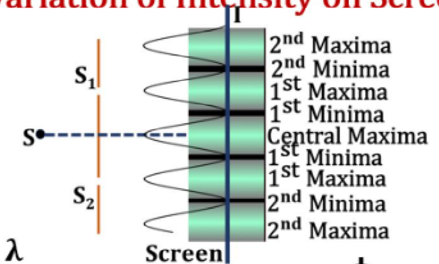
y	0	$\frac{\lambda D}{2d}$	$\frac{\lambda D}{d}$	$\frac{3\lambda D}{2d}$	$\frac{2\lambda D}{2d}$
Max /Mina	Central Maxima	1st Minima	1st Maxima	2nd Minima	2nd Maxima

Fringe Width β'



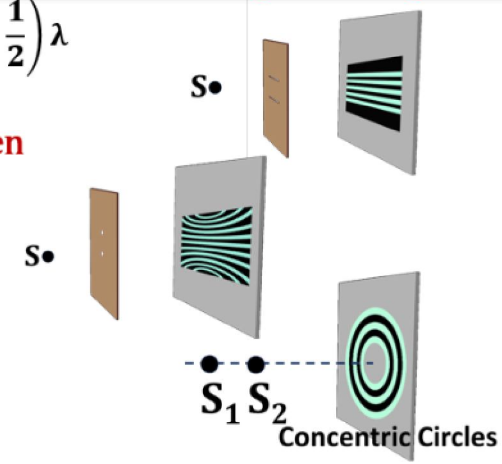
$'\beta' = \frac{\lambda D}{d}$ Angular Fringe Width $\theta = \frac{\beta}{D} = \frac{\lambda}{d}$

Variation of Intensity on Screen



$I_{res} = 4I_0\cos^2\frac{\phi}{2}$

Shape of Fringes



WAVE OPTICS

$$2\mu t = n\lambda \longrightarrow \text{Des}$$

Thin Film Interference

a) Interference in Reflection

$$\Delta x_{opt} = \left(n + \frac{1}{2}\right)\lambda_{air} \longrightarrow \text{Cons}$$

$$2\mu t = \left(n + \frac{1}{2}\right)\lambda_{air} \longrightarrow \text{Cons t}$$

$$2\mu t = n\lambda_{air} \longrightarrow \text{Dest}$$

b) Interference in Transmission

$$(\Delta x)_{opt.} = 2\mu t$$

$$2\mu t = \left(n + \frac{1}{2}\right)\lambda_{air} \longrightarrow \text{Dest}$$

$$2\mu t = n\lambda_{air} \longrightarrow \text{Cons}$$

$$(\Delta x)_{opt} = 2\mu t$$

$$2\mu t = \left(n + \frac{1}{2}\right)\lambda \longrightarrow \text{Cons}$$

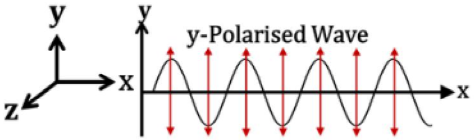
$$2\mu t = n\lambda \longrightarrow \text{Des}$$

$$(\Delta x)_{opt.} = 2\mu t$$

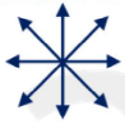
$$2\mu t = \left(n + \frac{1}{2}\right)\lambda_{air} \longrightarrow \text{Cons}$$

$$2\mu t = n\lambda_{air} \longrightarrow \text{Dest}$$

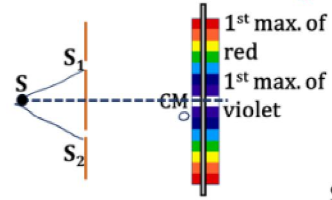
Polarisation



Unpolarised Wave



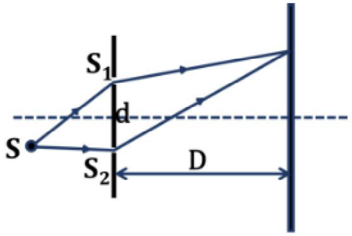
YDSE with white light



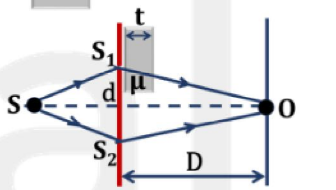
$$\beta = \frac{\lambda D}{d}$$

Variations in YDSE

In this case CM shifts but fringe width remains same.



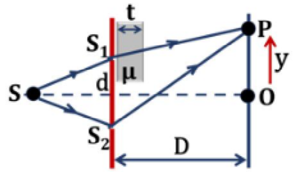
Optical path $= \mu t$



Phase difference at O

$$\Delta x = (\mu - 1)t$$

$$\beta = \frac{\lambda_{air} D}{d} \quad \text{No change}$$



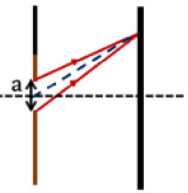
$$(\Delta x) = (\mu - 1)t - \left(\frac{yd}{D}\right)$$

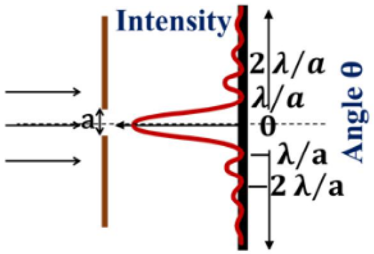
$$\beta = \frac{\lambda_{air} D}{d} \quad \text{No change}$$

Diffraction Through Single Slit

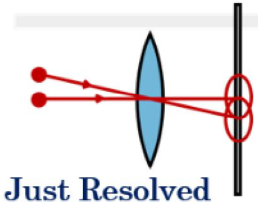
$a\theta = n\lambda$ Minima

$a\theta = \left(n + \frac{1}{2}\right)\lambda$ Maxima





Resolution of Optical Instruments



Just Resolved

(Resolved : seen as separated)

$$\Delta\theta_{min} = \frac{1.22\lambda}{D}$$

Resolving Power of Telescope

$$\text{R.P of Telescope} = \frac{1}{\Delta\theta_{min}} = \frac{D}{1.22\lambda}$$

D = Diameter of Objective Lens.

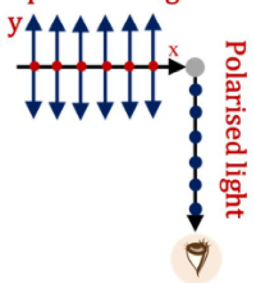
Resolving Power of Microscope

$$\text{R.P of Microscope} = \frac{1}{d_{min}}$$

$$d_{min} = \frac{1.22\lambda}{2\tan\beta} = \frac{1.22\lambda}{2\sin\beta}$$

$$d_{min} = \frac{1.22\,\lambda}{2\mu\sin\beta} \qquad \tan\beta = \frac{D}{2u}$$

Polarisation by Scattering



Polarisation by Reflection

$$\tan(i_B) = \mu$$

Brewster's Law

$$i_B = \tan^{-1}\mu$$

$$i_B + r = 90°$$

ELECTROMAGNETIC WAVES

Maxwell's Equation

1) $\oint \vec{E}\cdot\overrightarrow{dA} = \dfrac{Q}{\epsilon_0}$ Gauss's Law for electricity

2) $\oint \vec{B}\cdot\overrightarrow{dA} = 0$ Gauss's Law for magnetism

3) $\oint \vec{E}\cdot\overrightarrow{d\ell} = -\dfrac{d\phi_B}{dt}$ Faraday's Law

4) $\oint \vec{B}\cdot\overrightarrow{d\ell} = \mu_0 i_c + \mu_0\epsilon_0 \dfrac{d\phi_E}{dt}$ Ampere-Maxwell Law

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$v = \frac{1}{\sqrt{\mu\, \varepsilon}}$$

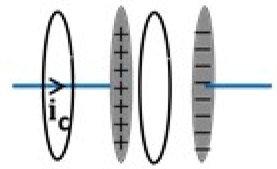
μ_0 - Permeability of free space (vacuum)

ε_0 - Permittivity of free space (vacuum)

μ - Permeability of medium

ε - Permittivity of medium

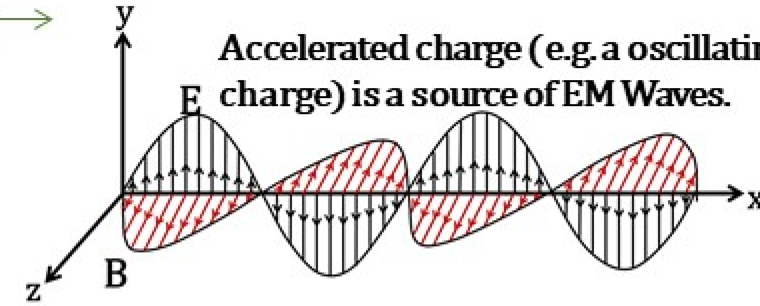
Displacement Current



Ampere-Maxwell Law

$$\oint \vec{B}\cdot\overrightarrow{d\ell} = \mu_0\,(i_c + i_d)$$

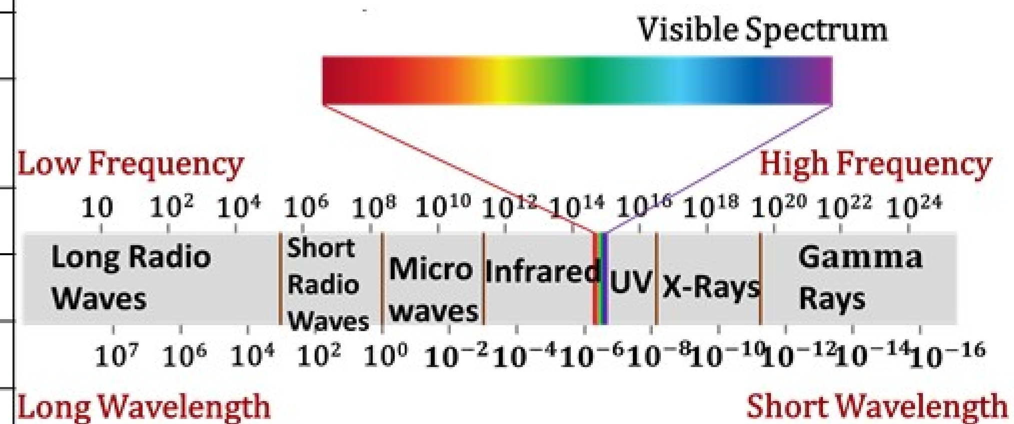
$$\oint \vec{B}\cdot\overrightarrow{d\ell} = \mu_0 i_c + \mu_0\epsilon_0 \frac{d\phi_E}{dt}$$



Accelerated charge (e.g. a oscillating charge) is a source of EM Waves.

- In EM waves both electric and magnetic fields vary with time and space. $\boxed{E = Bc}$ $\boxed{\hat{E} \times \hat{B} = \hat{v}}$
- $\vec{E}$ and $\vec{B}$ are perpendicular to each other and are also perpendicular to direction of propagation of wave.
- Electromagnetic waves can be polarized.
- Energy is equally divided in electric and magnetic field.
- Energy Density $= \dfrac{1}{2}E_0 E_{rms}^2 = \dfrac{B_{rms}^2}{2\mu_0}$
- Total Energy Density $= E_0 E_{rms}^2 = \dfrac{B_{rms}^2}{\mu_0}$

Type	Wavelength Range	Uses
Radio Waves	> 0.1 m	Radio and television communication
Microwave	0.1 m to 1 mm	Microwave Oven, Radar System
Infrared	1mm to 700nm	Remote Switches and Household electronic devices
Visible Rays	700 nm to 400nm	To see objects
Ultraviolet	400 nm to 1nm	Eye surgery, Water purifier
X-rays	1nm to 10^{-3} nm	Medical diagnosis
Gamma rays	< 10^{-3} nm	Medical treatment(to destroy cancer cells)



Atomic Structure

Rutherford's Gold Foil Experiment

1. Most of the α-particles (nearly 99.9%) went straight without suffering any deflection.
2. A few of them got deflected.

$1 eV = 1.6 \times 10^{-19}$ J

Value of charge on electron $= 1.602 \times 10^{-19}$ C

The distance of closest approach for α-particle is $r = \dfrac{2KZe^2}{K.E._{\alpha}}$

Bohr's Postulate

Electrons can revolve only in those orbits whose angular momentum (mvr) is integral multiple of $\dfrac{h}{2\pi}$ $\quad mvr = \dfrac{nh}{2\pi}$

$$\frac{k\,(Ze)\,e}{r^2} = \frac{mv^2}{r}$$

Radius of Bohr orbit:

$$r = \frac{n^2h^2}{4\pi^2 mKZe^2} \qquad r_n = 0.529\frac{n^2}{Z}\,\text{Å}$$

Bohr Radius $'a0' = 0.529$ Å

Energy Of Electrons

$$E = -KE = \frac{PE}{2}$$

$$E = Rhc\,\frac{Z^2}{n^2}$$

R is 'Rydberg Constant' $\approx 1.1 \times 10^7\ m^{-1}$

$$E = -21.8 \times 10^{-19} \times \frac{Z^2}{n^2}\ J$$

Velocity of electron:

$$v = \frac{2\pi KZe^2}{nh}$$

$$v = 2.2 \times 10^6\,\frac{Z}{n}\ m/s$$

Energy During Transitions

$$\Delta E = E_{\text{final state}} - E_{\text{initial state}}$$

$$\Delta E = 13.6\left[\frac{1}{n_i^2} - \frac{1}{n_f^2}\right]Z^2 \quad \text{(in eV)}$$

$$E = -13.6 \times \frac{Z^2}{n^2}\ eV$$

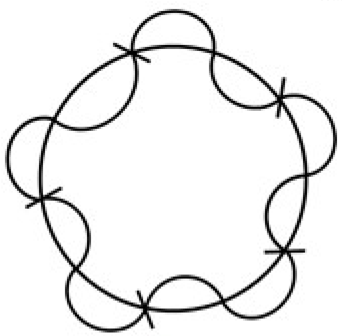
$$\frac{1}{\lambda} = R\left[\frac{1}{n_i^2} - \frac{1}{n_f^2}\right]Z^2$$

-0.85 eV — n = 4, 3rd excited state

-1.51 eV — n = 3, 2nd excited state

-3.4 eV — n = 2, 1st excited state

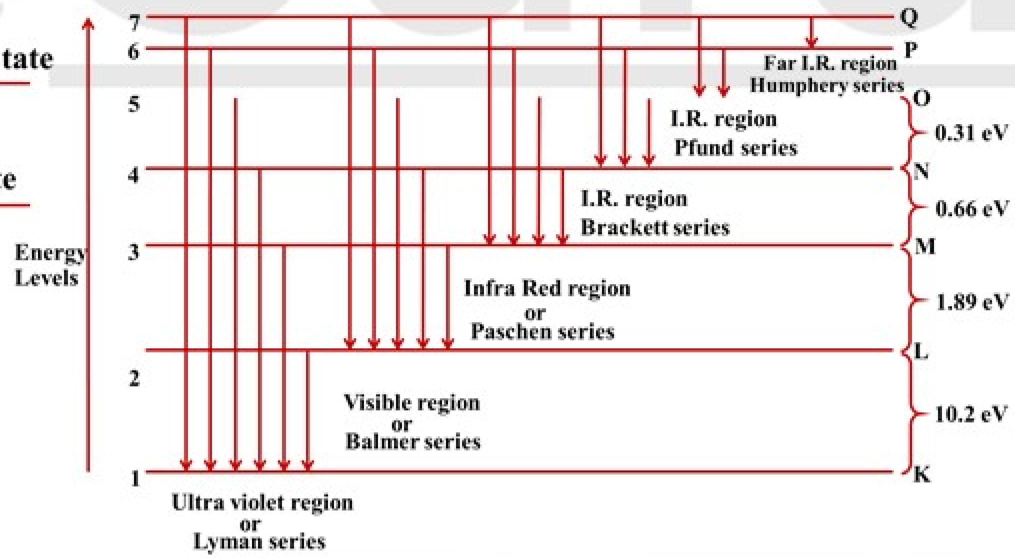
-13.6 eV — n = 1, ground state

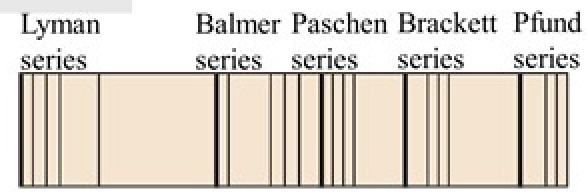


For nth Shell

$2\pi r = n\lambda$

Hydrogen Line Spectrum





Total no. of emission lines between n_2 & n_1 ($n_2 > n_1$) are

$$\frac{(n_2 - n_1 + 1)\,(n_2 - n_1)}{2}$$

Photoelectric Effect

De-Broglie wavelength

$\lambda = \frac{h}{p}$ $\quad \lambda = \frac{h}{\sqrt{2m\,K.E.}}$

For electron

$\left(\lambda = \sqrt{\frac{150}{V}}\ \text{Å}\right)$

Photon

Particle of light is called as 'photon'. It is packet of energy.

$E = \frac{hc}{\lambda} = h\nu = \frac{1242}{\lambda\ (\text{in nm})}$ in eV

Momentum of photon $P = \frac{h}{\lambda} = \frac{E}{c}$

Planck's constant 'h' $= 6.626 \times 10^{-34}$ Js

$1\text{eV} = 1.6 \times 10^{-19}$ J

Threshold wavelength

Maximum wavelength for which e^- just comes out is called threshold wavelength (λ_0).

$\frac{hc}{\lambda_0} = \phi$ Photoelectric effect will take place for $\lambda \le \lambda_0$

$KE_{max} = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$

Threshold frequency

Minimum frequency for which electron just comes out is called threshold frequency (ν_0).

$h\nu_0 = \phi$

Photoelectric effect takes place for $\nu \ge \nu_0$.

$KE_{max} = h\nu - \phi = h\nu - h\nu_0$

$KE_{max} = h(\nu - \nu_0)$

Photoelectric Effect

$0 \le KE \le h\nu - \phi$

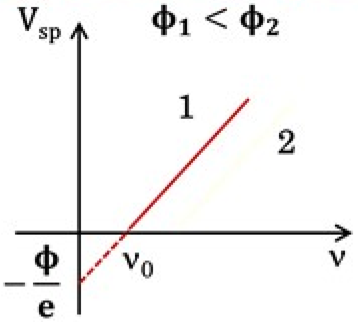
$(KE)_{max} = h\nu - \phi$ Einstein's Equation

$(KE)_{max}$: Value between 0 to KE_{max}.

$h\nu$: E_{photon}

ϕ: Work Function → Minimum energy required to exit an electron from surface of the substance is called work function (F) of the substance.

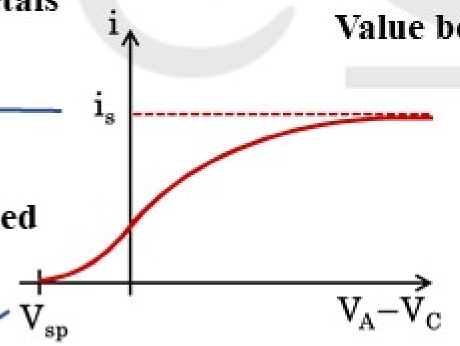
Graph between V_{sp} and n



$eV_{sp} = h\nu - \phi$

$V_{sp} = \left[\frac{h}{e}\right]\nu - \left[\frac{\phi}{e}\right]$

slope $= \frac{h}{e}$ independent of metals



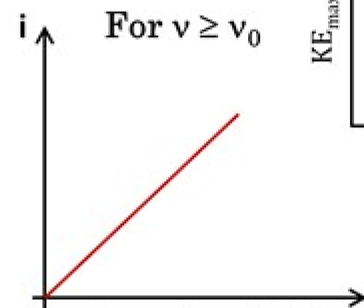
Saturation Current i_s

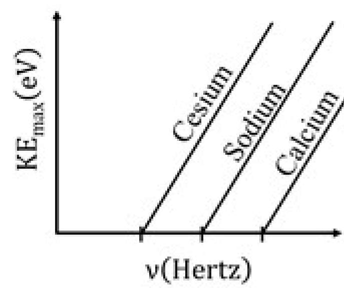
The maximum value of photoelectric current is called saturation current.

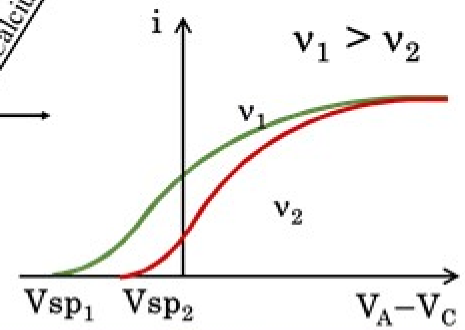
Stopping Potential V_{sp}

The value of $V_C - V_A$ at which photo current just stops is called Stopping Potential (V_{sp}).

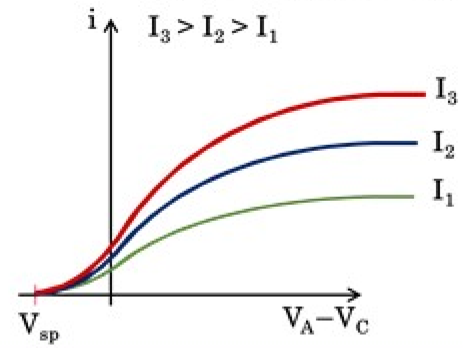
$eV_{sp} = KE_{max}$





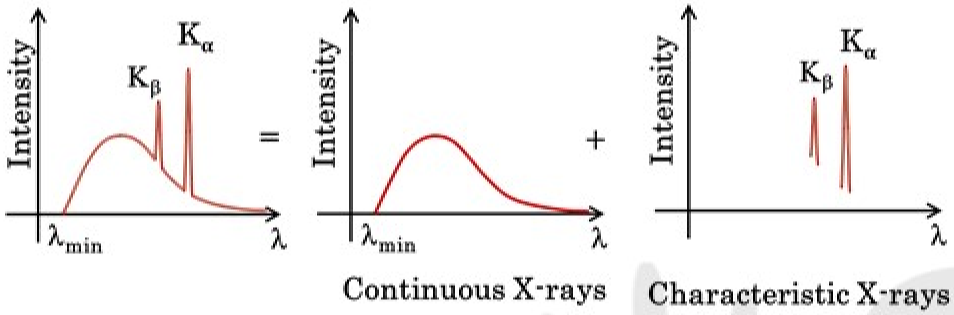


Same metal, given freq.



X-Rays

X-Rays Spectrum



Continuous X-rays

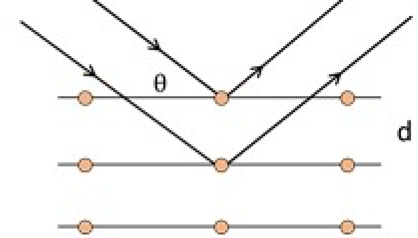
Continuous X-rays are emitted due to high retardation of electrons coming from cathode

Minimum wavelength below which no X-rays is emitted is called Cutoff wavelength or Threshold wavelength.

$$\lambda_{min} = \frac{hc}{eV_0}$$

λ_{min} is independent of filament voltage or filament temperature.

Bragg's Law

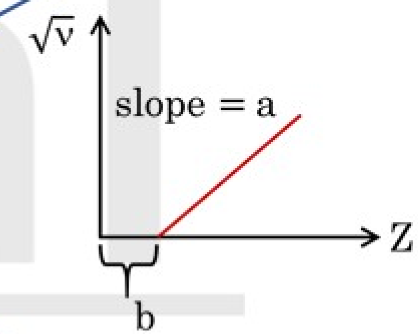


For Constructive Interference

$$2d\sin\theta = n\lambda$$

θ = bragg's angle

Moseley's Law



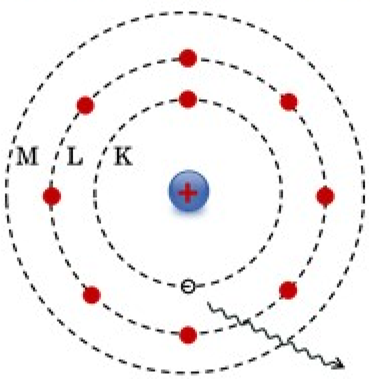
$$\sqrt{\nu} = a\,(Z - b)$$

$$\sqrt{\nu} = \sqrt{\frac{3}{4}Rc}\,(Z - 1)$$

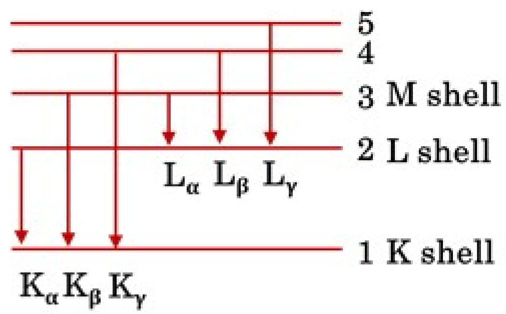
For K_α : b=1

Shielding Effect

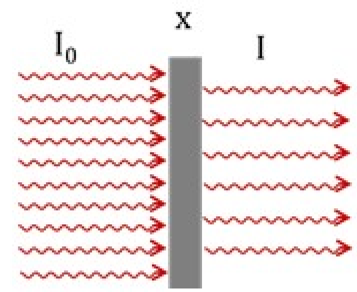
Characteristic X-ray



These are emitted when electron from cathode, knocks out an inner electron from target material and then it's position is filled by higher energy level electron.



Absorption of X-rays



$$I = I_0 e^{-\mu x}$$

eSaral

Nuclear Physics

Radius of a Nucleus

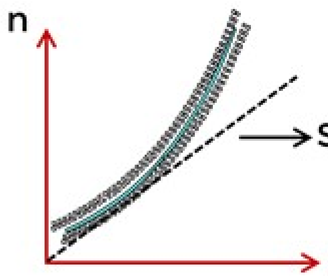
$R = R_0 A^{1/3}$

Density of Nucleus (ρ) =

$$\frac{\text{Mass}}{\text{Volume}} = \frac{\text{Mass of 1 Nucleon} \times A}{\frac{4}{3}\pi R^3 = \frac{4}{3}\pi R_0^3 A}$$

Nuclear Force Theory

Nuclear force is a force which holds the Nucleons together.



For atomic number < 20, most stable Nuclei have n : p ratio nearly 1 : 1

For n/p ratio > 1.52, Nucleus is unstable.

For atomic number > 83, there are no stable nuclei.

Mass Defect

The difference (Δm) between mass of constituent nucleons and the nucleus is called mass defect of nucleus.

$$\underbrace{{}^{m}p}_{m_1} + \underbrace{{}^{n}n}_{m_2} \longrightarrow \underbrace{{}^{m+n}_{\quad m}X}_{m_3} + \underbrace{\text{Energy}}_{\text{(BE of nucleus)}}$$

mass defect $= \Delta m = (m_1 + m_2) - m_3$

$BE = (\Delta m)c^2$

Q-value

$$\underset{m_A}{A} + \underset{m_B}{B} \longrightarrow \underset{m_C}{C} + \underset{m_D}{D} + \text{Energy}$$

Reactants $\longrightarrow$ Products + Q-Value

If BE products > BE reactants then energy will be released

Q value = |BE products − BE reactants|

$$\text{Q-value} = [(m_A + m_B) - (m_C + m_D)]c^2$$

$$\text{Q-value} = (KE_C + KE_D) - (KE_A + KE_B)$$

Representation of atom ${}_zX^A$

A = Mass number   z = Atomic number

Atomic mass unit(amu)

$$1 \text{ amu (u)} = \frac{1}{12}\text{ (Mass of Carbon} - 12 \text{ atom)}$$

$$1 \text{ amu} \equiv 931.5 \text{ MeV}$$

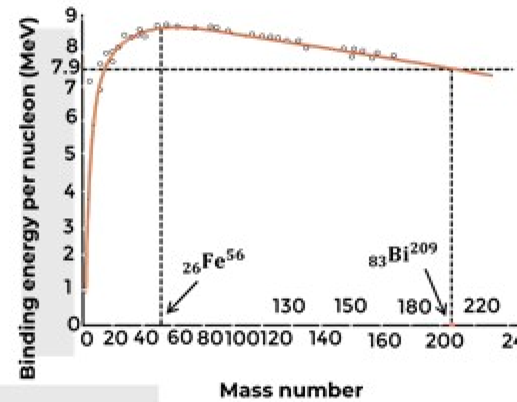
Binding Energy of Nucleus

Binding energy of nucleus is energy released when constituent nucleon are bought from infinity to form nucleus.

$$2p + 2n \longrightarrow {}^4_2He + \text{Energy}$$

Binding energy of nucleus $= \Delta mc^2$

$$\text{B.E. per nucleon} = \frac{\text{B.E.}}{\text{No. of nucleons}}$$



Nuclear binding energy is maximum for mass number 50–60.

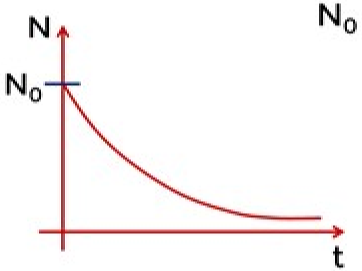
Mass and Energy

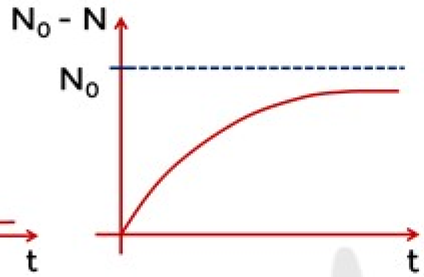
Mass m of a particle is equivalent to energy given by $E = mc^2$.
It is also kown as rest mass energy.

Law of Radioactivity

$$\frac{dN}{dt} = -\lambda N$$

$$N = N_0 e^{-\lambda t}$$





Number of nuclei decayed $= N_0 - N$

$= N_0(1 - e^{-\lambda t})$

Half Life ($T_{1/2}$)

$$T_{1/2} = \frac{\ln 2}{\lambda}$$

$$N = \frac{N_0}{2^{\frac{t}{t_{1/2}}}}$$

Mean or Average Life (T_a)

$$T_{avg} = \frac{\text{Sum of ages of all nuclei}}{\text{Number of nuclei}}$$

$$T_a = \frac{1}{\lambda}$$

α- decay

$$\overset{m_x}{{}^{A}_{Z}X} \longrightarrow \overset{m_y}{{}^{A-4}_{Z-2}Y} + + \overset{m_{He}}{{}^{4}_{2}He} + \gamma - \text{rays}$$

Q-value $= (m_x - m_y - m_Z)\ c^2$

Q-value = KE of products + Energy of γ-rays

Spontaneous Decay

$A \xrightarrow{\lambda_1} B \quad A \xrightarrow{\lambda_2} C \quad \frac{dN_A}{dt} = -(\lambda_1 + \lambda_2)N_A$

$$N_A = N_0 e^{-(\lambda_1+\lambda_2)t}$$

$\lambda_1 \longrightarrow N_B \quad \frac{\lambda_1}{\lambda_1+\lambda_2} N_0(1 - e^{-(\lambda_1+\lambda_2)t})$

$\lambda_2 \longrightarrow N_C \quad \frac{\lambda_2}{\lambda_1+\lambda_2} N_0(1 - e^{-(\lambda_1+\lambda_2)t})$

Spontaneous decay of unstable nuclei is called radioactivity.

Radioactivity

γ – Decay

$${}^{A}_{Z}X \longrightarrow {}^{A}_{Z}X + \gamma - \text{rays}$$

K-capture

$${}^{A}_{Z}X + {}^{0}_{-1}e \rightarrow {}^{A}_{Z-1}Y + \nu$$

Q-value $= (m_x - m_y)\ c^2$

β^- Decay

$${}^{A}_{Z}X \longrightarrow {}_{Z+1}^{A}Y + {}^{0}_{-1}e + \bar{\nu}$$

Q value $= (m_x - m_y)\ c^2$

$0 < KE_{electron} <$ Q-value

$0 \le E_{\bar{\nu}} <$ Q–value

β^+-decay

$$\overset{m_x}{{}^{A}_{Z}X} \rightarrow \overset{m_y}{{}^{A}_{Z-1}Y} + \overset{m_e}{{}^{0}_{+1}e} + \nu$$

Q-value $= (m_x - m_y - 2m_e)\ c^2$

Nuclear Fission

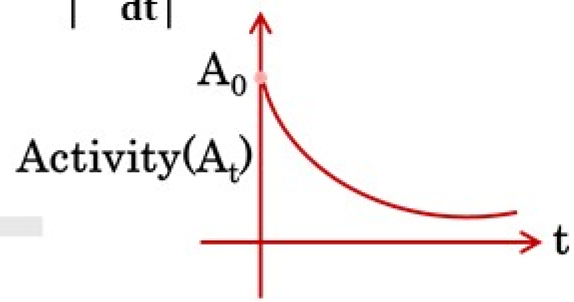
$${}^{235}U + {}^{0}_{1}n \rightarrow X + Y + \underset{\text{avg}=2.5\ \text{neutrons}}{\underline{(2\ \text{or}\ 3)\ \text{neutrons}}}$$

Nuclear Fusion

Light nuclei fuse (combine) together in nuclear fusion reaction. Energy released in fusion is much more than in fission per nucleon.

Activity

$$A = \left| -\frac{dN}{dt} \right| \qquad A = \lambda N$$



$$\Rightarrow A = \frac{A_0}{2^{t/t_{1/2}}} \qquad A = A_0 e^{-\lambda t}$$

SI unit is Becquerel (Bq)

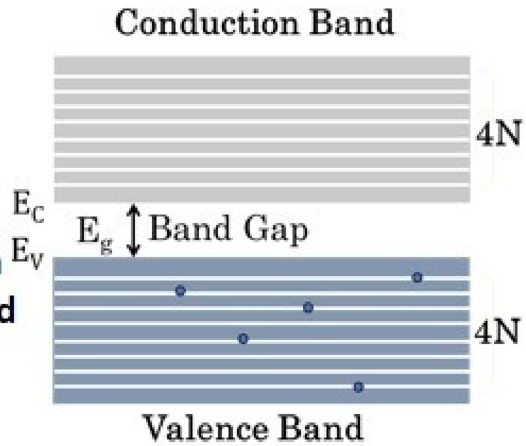
1 becquerel (1 Bq)=1 decay/sec

Other unit is curie

1 Ci $= 3.70 \times 10^{10}$ decays/sec

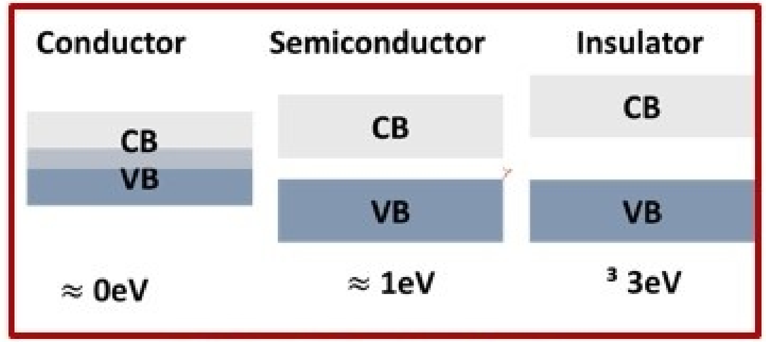
Semiconductor

Energy levels with continuous energy variation form Energy Band.

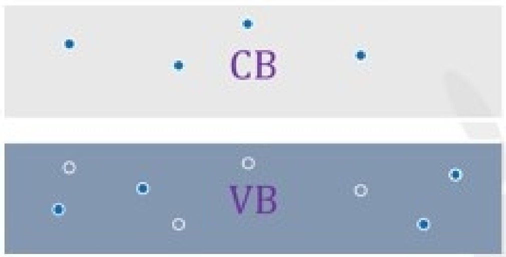


Energy gap between conduction band and valence band,

$\boxed{E_g = E_C - E_V}$



Intrinsic Semiconductor



In intrinsic semiconductor, the number of free electrons, n_e, is equal to the number of holes, n_h

$n_e = n_h = n_i$ (Intrinsic carrier concentration)

At thermal equilibrium $\boxed{\text{Generation rate = Recombination rate}}$

$$n_e \times n_h = n_i^2$$

- The deliberate addition of desirable impurity is called Doping.
- Added impurity atoms are called Dopants.

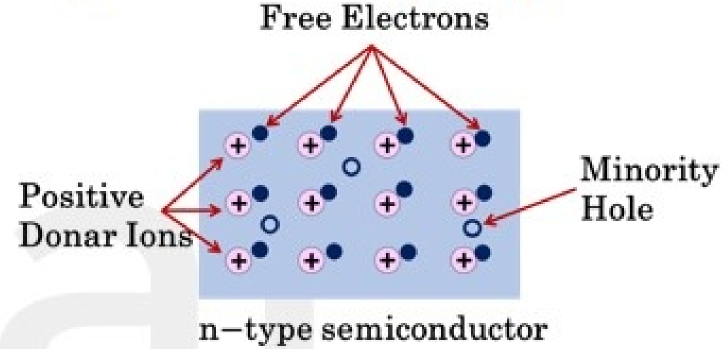
There are two types of dopants used in doping

(a) Pentavalent : Arsenic (As), Antimony (Sb), Phosphorous (P)

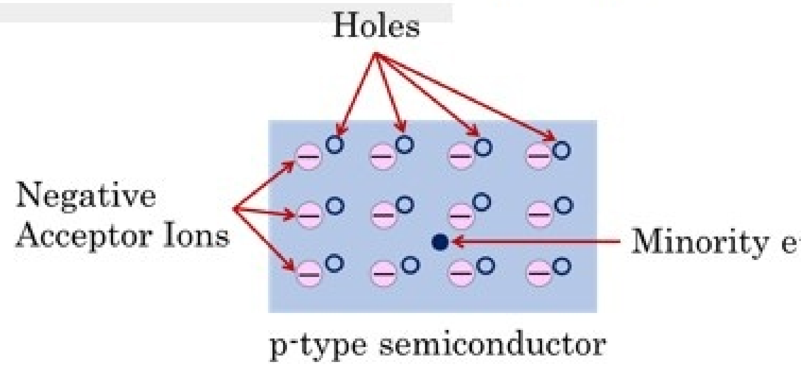
(b) Trivalent : Indium (In), Boron (B), Aluminium (Al)

Extrinsic Semiconductor

1. n-type semiconductor $n_e >> n_h$



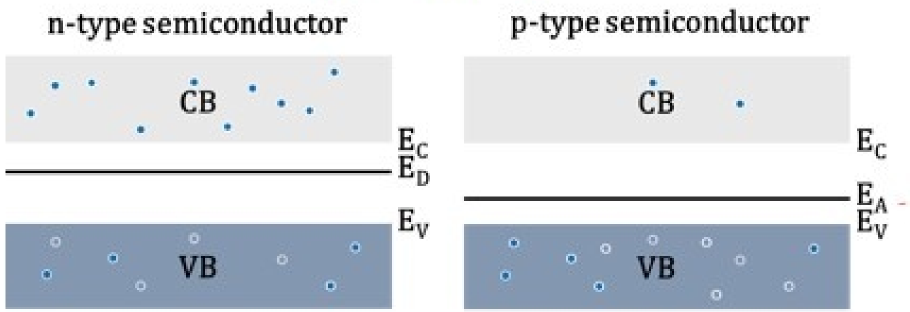
2. p-type semiconductor $n_h >> n_e$



Semiconductor

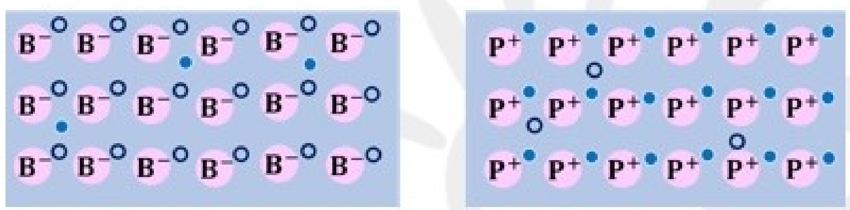
In extrinsic semiconductor the electron and hole concentration in a semiconductor in the thermal equilibrium is given by

Mass action law $n_e \times n_h = n_i^2$



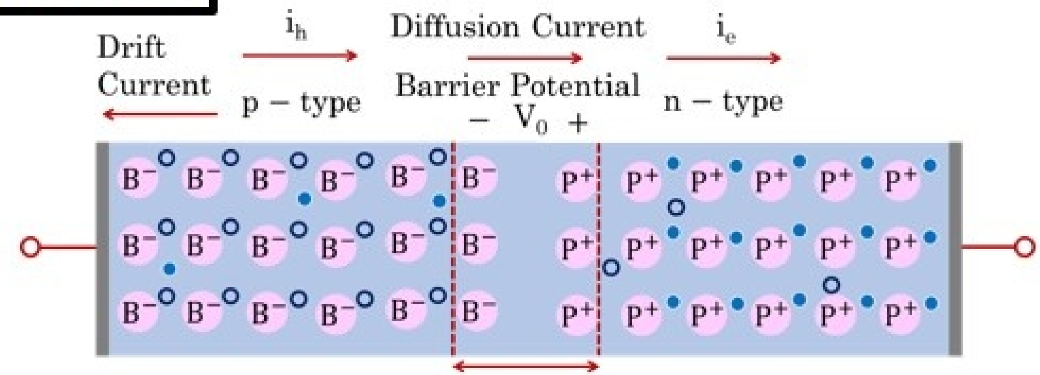
$n_h << n_e$ $n_h >> n_e$

p – type   n – type



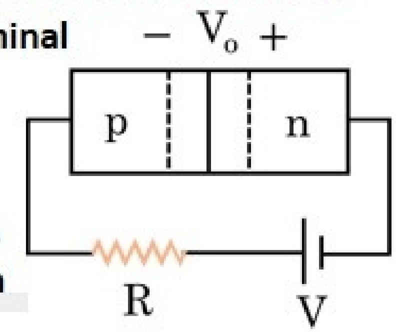
p–n Junction Diode

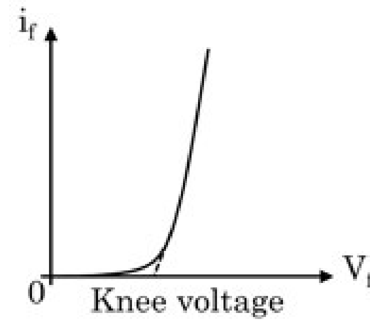
Si : V_0 = 0.7 eV   Ge : V_0 = 0.3 eV



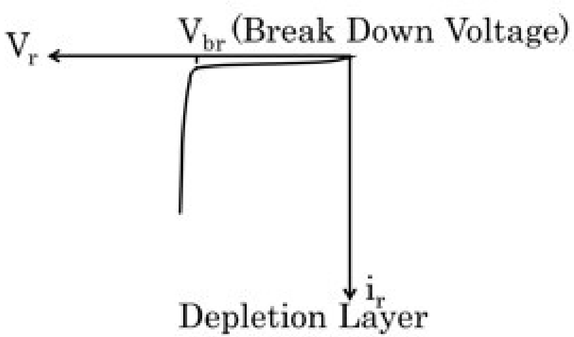
p–n Junction Under forward bias

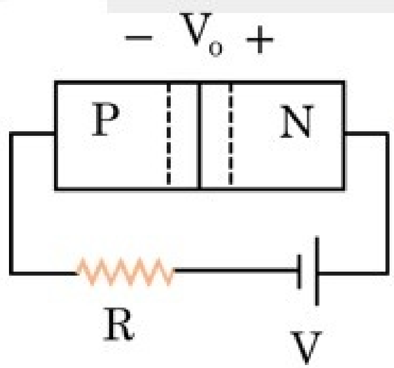
- When p side is connected to positive terminal and n side to the negative terminal, it is called forward bias.
- Due to applied voltage, electrons from n side cross the depletion region, and holes from p side cross the junction and reach n side. This process under forward bias is known as minority carrier injection.
- The total diode forward current is due to Diffusion.





p–n Junction Diode Under Reverse Bias





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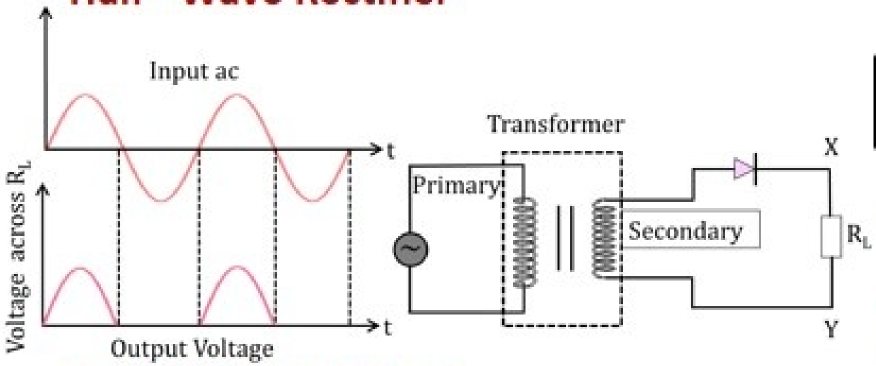
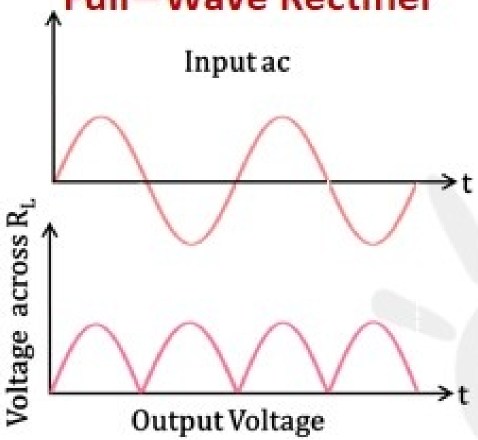
Semiconductor

Half–Wave Rectifier

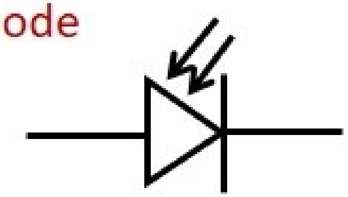


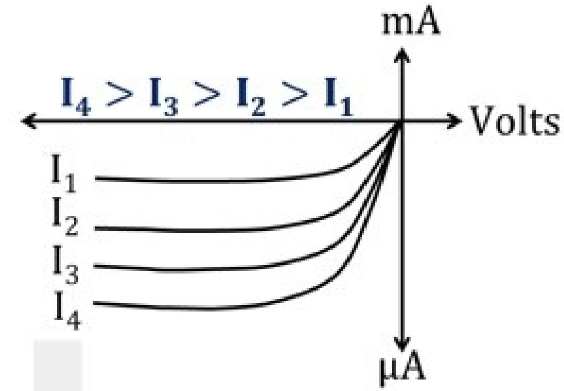
Full–Wave Rectifier



Photodiode

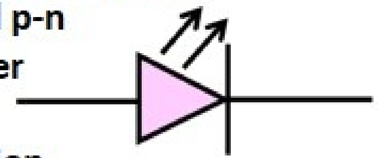
Photodiode operated under reverse bias.





Light Emitting Diode

It is a heavily doped p-n junction which under forward bias emits spontaneous radiation.

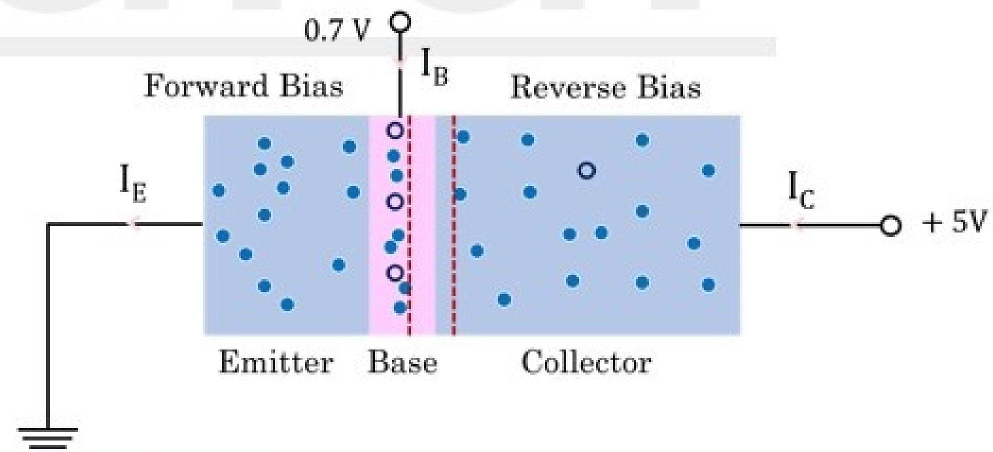


On recombination, the energy is released in the from of photons.

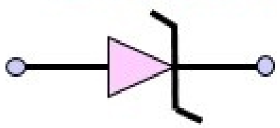
Solar cell

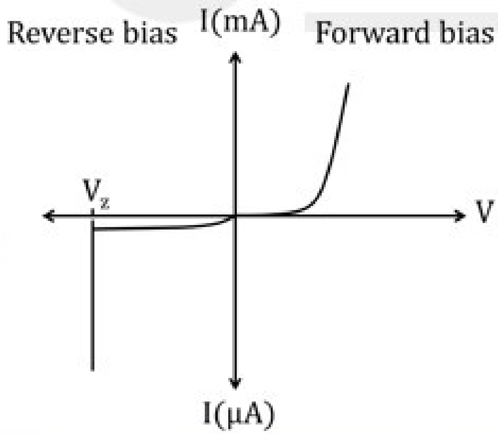
Electron – hole pair generated due to light with $h\nu > E_g$.

Transistor



Zener Diode

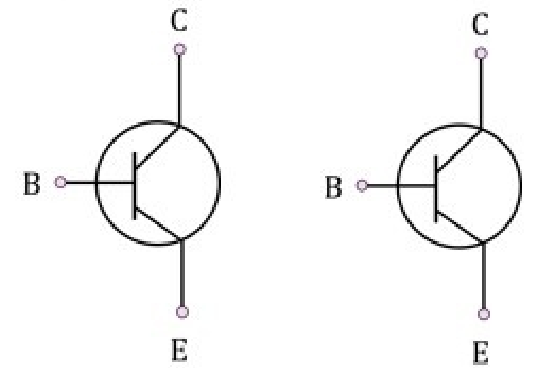




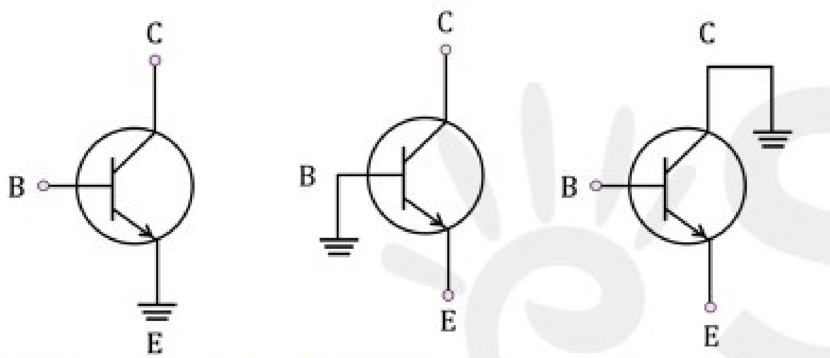
Zener diode acts as a Voltage Regulator.

Semiconductor

npn Transistor **pnp Transistor**



Common Emitter Common Base Common Collector

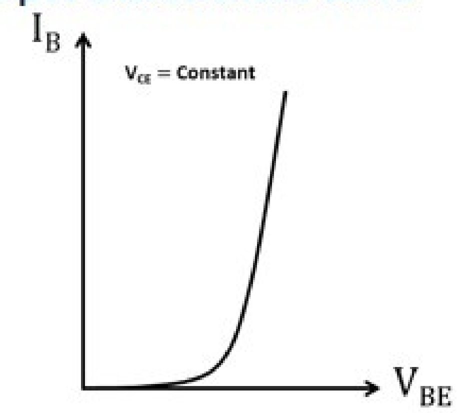


For common emitter current amplification factor:

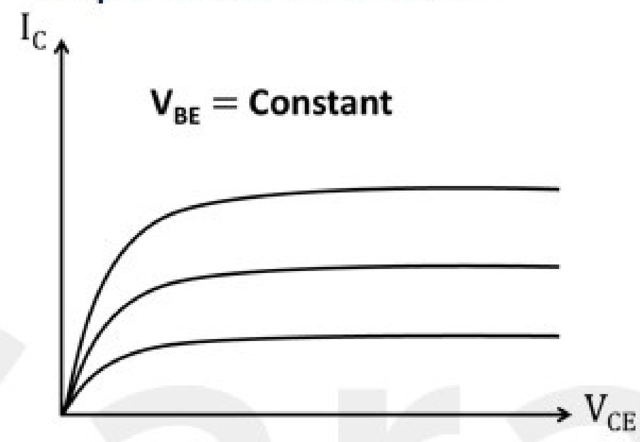
$$\beta_{dc} = \frac{I_C}{I_B}$$

Common Emitter Configuration

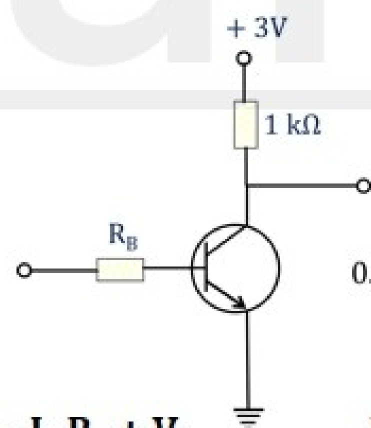
Input Characteristic Curve



Output Characteristic Curve

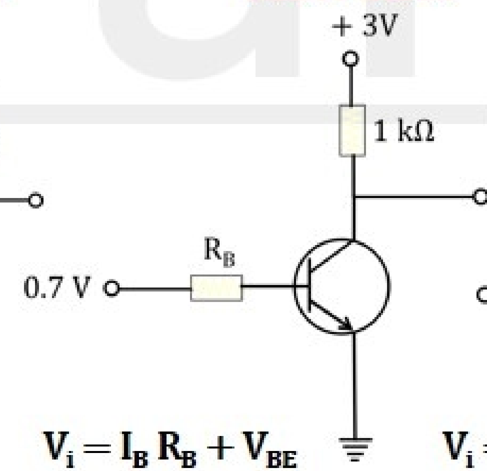


Cutoff Mode



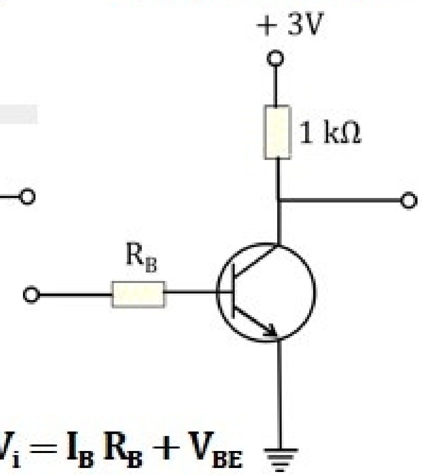
$$V_i = I_B R_B + V_{BE}$$
$$V_0 = V_{CC} - I_C R_C$$

Active Mode



$$V_i = I_B R_B + V_{BE}$$
$$V_0 = V_{CC} - I_C R_C$$

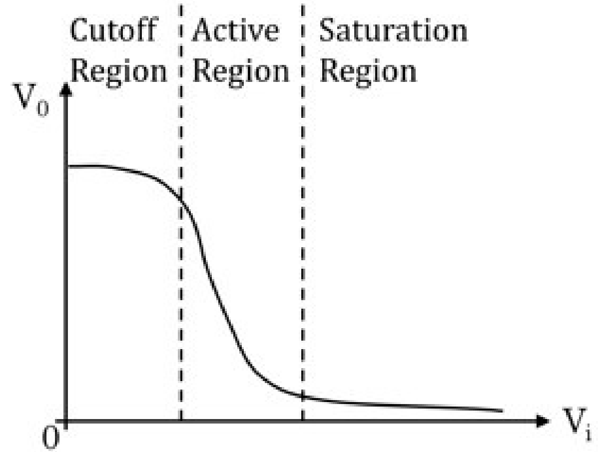
Saturation Mode



$$V_i = I_B R_B + V_{BE}$$
$$V_0 = V_{CC} - I_C R_C$$

Semiconductor

Logic Gates



- Transistor works as a Switch in Cutoff mode and Saturation mode.
- For using the transistor as an amplifier we use active region of the V_0 and V_i curve.

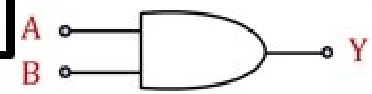
Voltage Gain

$$A_V = \frac{\Delta V_0}{\Delta V_i} = \frac{-\Delta I_C R_C}{\Delta I_B R_B}$$

Power Gain

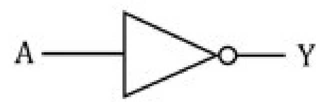
$$A_P = \beta_{ac} \times A_V$$

AND Gate



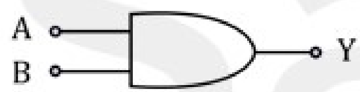
Input		Output
A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

NOT Gate



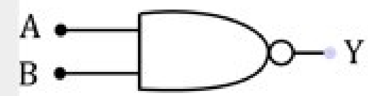
Input	Output
A	Y
0	1
1	0

AND Gate



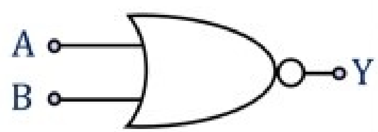
Input		Output
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

NAND Gate



Input		Output		
A	B	AND	NOT	NAND
0	0	0	1	1
0	1	0	1	1
1	0	0	1	1
1	1	1	0	0

NOR Gate



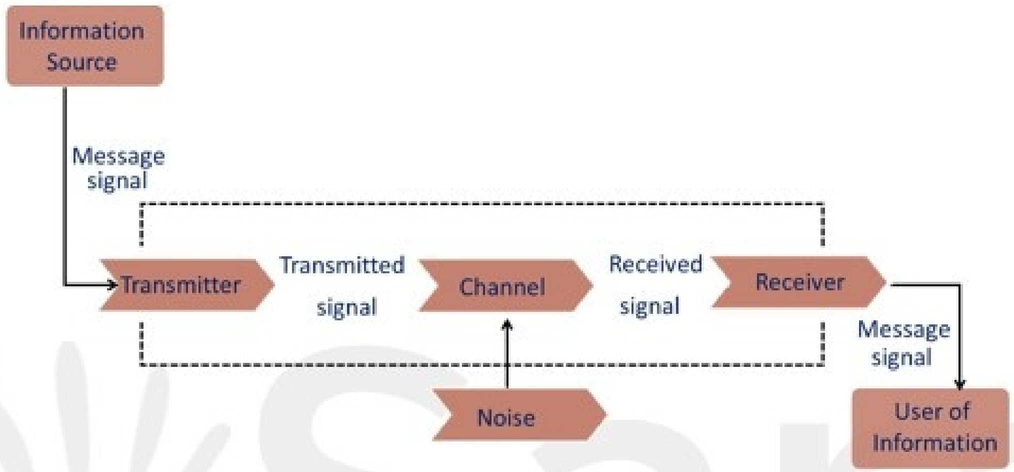
Input		Output		
A	B	OR	NOT	NOR
0	0	0	1	1
0	1	1	0	0
1	0	1	0	0
1	1	1	0	0

Communication System

Communication is the act of transmission of information.

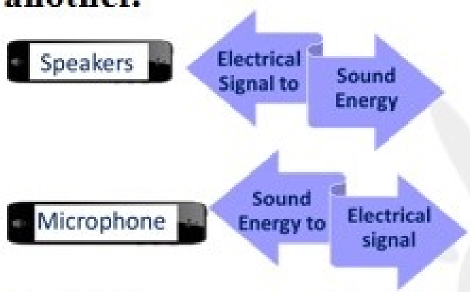
Elements of a Communication System

1. Transmitter
2. Medium/Channel
3. Receiver



Basic Terminology

1. **Transducer** is the device that converts one form of energy into another.



2. **Signal** is the information converted in electrical form.
Signals can be analog or digital.

3. **Noise**-There are unwanted signals that tend to disturb the transmission and processing of message signals.

4. **A transmitter** processes the incoming message signal to make it suitable for transmission through a channel and subsequent reception.

5. **A receiver** extracts the desired message signals from the received signals at the channel output.

6. **Attenuation**-It is the loss of strength of a signals while propagating through a medium.

7. **Amplification**-It is the process of increasing the amplitude (and therefore the strength) of a signal using an electronic circuit called the amplifier.

Point to point communication - takes place over a link between a single transmitter and a receiver as in telephony.

Broadcast - there are a large number of receivers corresponding to a single transmitter.

8. **Range**- It is the largest distance between the source and the destination up to which the signal is received with sufficient strength.

9. **Bandwidth** - It is the frequency range over which an equipment operates or the portion of the spectrum occupied by the signal.

10. **Modulation** - The information contained in the low frequency message signal is superimposed on a high frequency wave, which acts as a carrier of the information..

11. **Demodulation** - The process of retrieval of original information from the carrier wave at the receiver end is termed as demodulation.

12. **Repeaters** - A repeater is a combination of a receiver and a transmitter.

Communication System

Bandwidth of Transmission Medium

Wire (Coaxial Cable)	750 MHz
Optical communication using fibers	1THz−1000 THz (microwaves− ultra violet)
Standard AM broadcast	540kHz −1600 kHz
FM broadcast	88−108 MHz

Television

VHF (very high frequencies) TV	54-72 MHz 76-88 MHz
UHF (Ultra high frequency) TV	174-216 MHz 420-890 MHz
Mobile to base Station	896-901 MHz
Base station to mobile	840-935 MHz

Satellite Communication

Up linking	5.925−6.425 GHz
Downlinking	3.7 − 4.2 GHz

Bandwidth for Analog Signals

Signal	Frequency range	Bandwidth required
Speech	300−3100 Hz	3100−300 = 2800 Hz
Music	High frequencies produced by musical instrument	20 kHz
Picture	–	4.2 MHz
TV	Contains both voice and picture	6 MHz

Different Layers of Atmosphere and their Interaction with the Propagating Electromagnetic Waves

Atmospheric stratum (layer)	Height over earth's surface (approx)	Exists during	Frequencies most likely affected
1. Troposphere	10 km	Day and night	VHF (upto several GHz)
2. Ionosphere			
(i) D (part of stratosphere)	65-75 km	Day only	Reflects LF, absorbs MF & HF to some degree
(ii) E (part of stratosphere)	100 km	Day only	Helps surface waves, reflects HF
(iii) F_1 (Part of Mesosphere)	170-190 km	Daytime, merges with F_2 at night	Partially absorbs HF waves yet allowing them to reach F_2
(iv) F_2 (Thermosphere)	300 km at night, 250-400 km during daytime	Day and night	Efficiently reflects HF waves particularly at night

Ground Wave Propagation

This propagation is suitable for low and medium frequency i.e. upto 2 or 3 MHz only.

(a) The radio waves which travel through atmosphere following the surface of earth are known as ground waves or surface waves and their propagation is called ground wave propagation or surface wave propagation.

(b) The ground wave transmission becomes weaker with increase in frequency because more absorption of ground waves takes place at higher frequency during propagation through atmosphere.

(c) The maximum range of ground or surface wave propagation depends on two factors :

(i) The frequency of the radio waves and

(ii) Power of the transmitter

(d) The ground wave propagation is generally used for local band broadcasting and is commonly called medium wave.

Communication System

Sky Wave Propagation

(a) The sky waves are the radio waves of frequency between 2 MHz to 30 MHz.

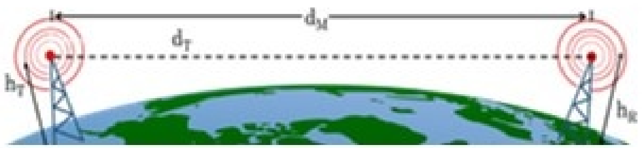
(b) The ionospheric layer acts as a reflector for a certain range of frequencies (3 to 30 MHz). Electromagnetic waves of frequencies higher than 30 MHz penetrate the ionosphere and escape.

(c) The highest frequency of radio waves which when sent straight towards the layer of ionosphere gets reflected from ionosphere and returns to the earth is called critical frequency. It is given by

$f_c = 9\,(N_{max})^{1/2}$, where N is the number density of electron/m^3.

Space Wave Propagation

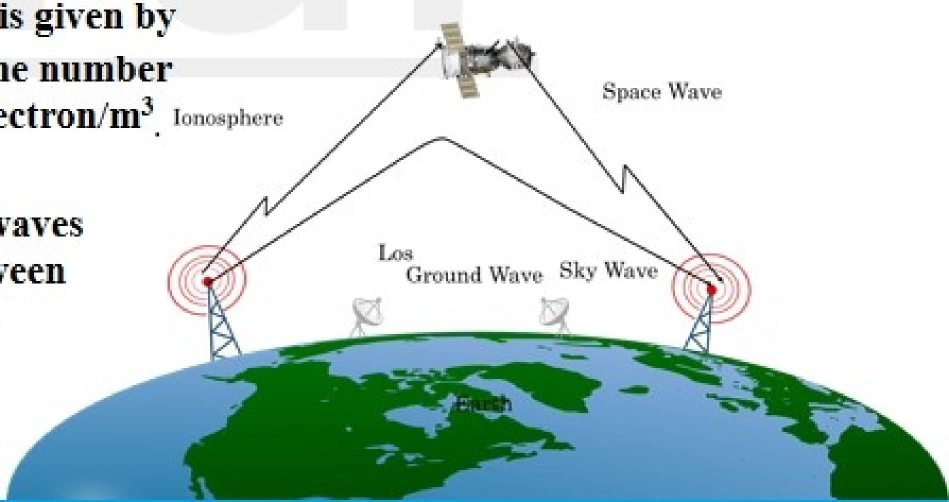
The space waves are the radio waves of very high frequency (i.e. between 30 MHz to 300 MHz or more).h



d_M is the distance between the two antennas having heights h_T and h_R above the earth is given by:

$$d_M = \sqrt{2Rh_T} + \sqrt{2Rh_R}$$

where h_R is the height of receiving antenna.



Communication System

Modulation

The signal which results from this process is known as modulated signal.

Need For Modulation

(i) To avoid interference

(ii) To design antennas of practicable size

The minimum height of antenna should be $\lambda/4$

where

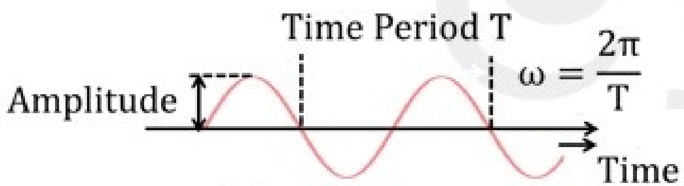
λ is wavelength of modulating signal.

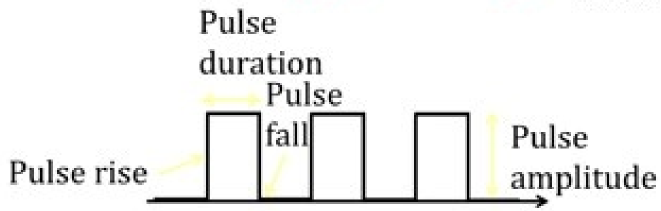
(iii) Effective Power Radiated by an Antenna

$$P \propto \left(\frac{1}{\lambda}\right)^2$$

For eff. transmission, $P \uparrow \Rightarrow \lambda \downarrow \Rightarrow \nu \uparrow$

Hence, high frequency signals are desirable





Carrier Wave : Sinusoidal

A sinusoidal carrier wave can be represented as $c(t) = A_c \sin(\omega_c t + \phi)$

Difference Between AM,FM and PM

	AM	FM	PM
Definition	•The amplitude of the carrier wave is varied in accordance with the information signal	•The frequency of the carrier wave is varied in accordance with the information signal	•The phase of the carrier wave is varied in accordance with the information signal
Variation	Frequency and phase remains same.	Amplitude and phase remains same.	Amplitude and frequency remains same.
Frequency bands	540-1600 kHz	88-108 MHz	
Noise	More susceptible to noise	Less susceptible to noise	
Advantage	Cheap ,can be transmitted to very long distances	Less prone to interference, sound quality is good	

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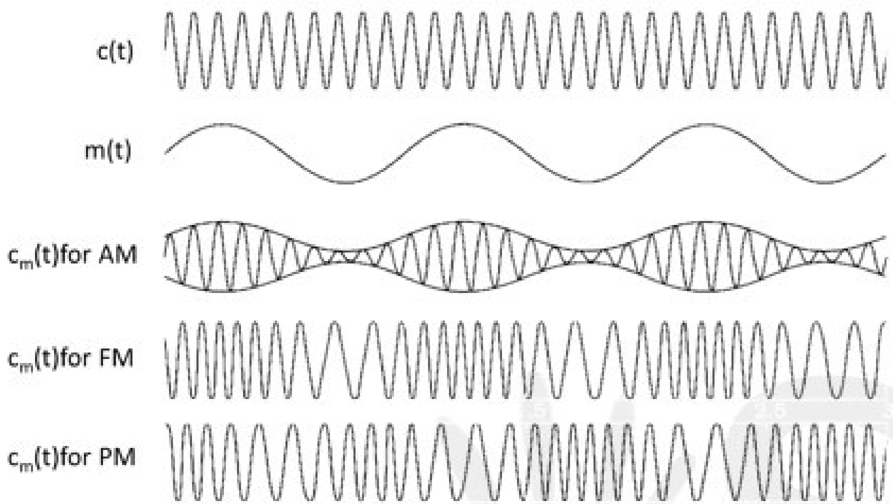
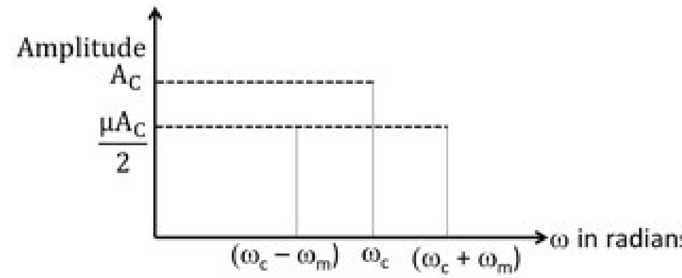
Communication System

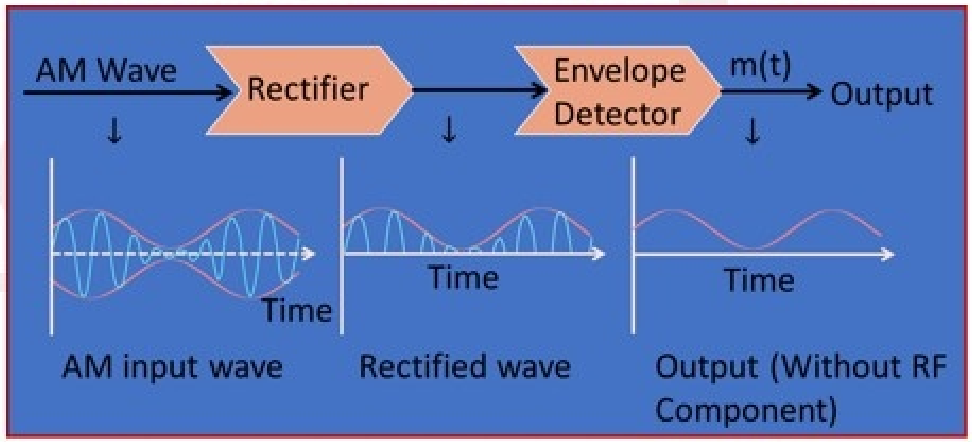


Frequency Spectrum of the Modulated Signal

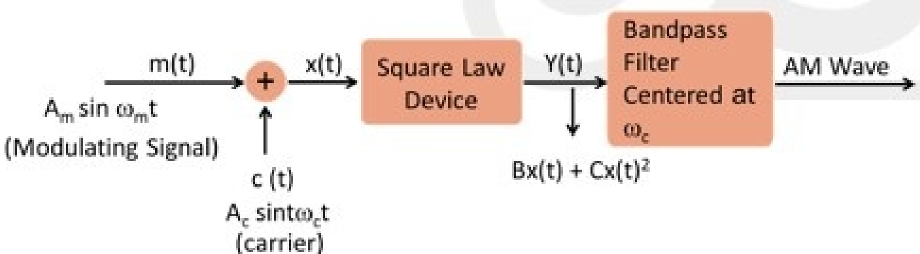
$$c_m(t) = A_c \sin\omega_c t + \frac{\mu A_c}{2}\cos(\omega_c - \omega_m)t - \frac{\mu A_c}{2}\cos(\omega_c + \omega_m)t$$

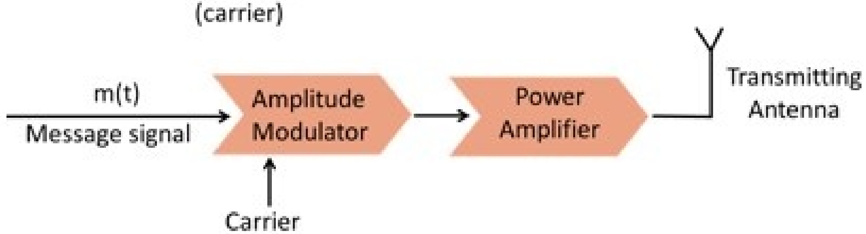


Block Diagram of a Detector for AM Signal

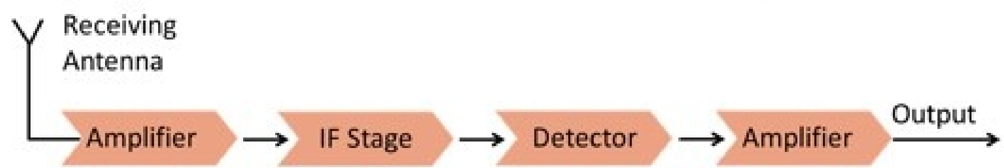


Production of Amplitude Modulated Wave





Detection of Amplitude Modulated Wavet



ERROR IN MEASUREMENT- Significant Digits

Scientific Notation

$a \times 10^b$

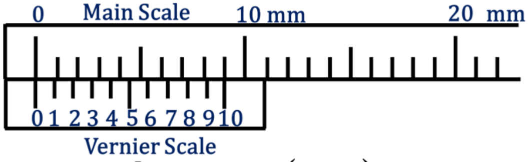
'a' is a number between 1 and 10

Arithmetic Operations (Rules for Rounding off)	
Multiplication or Division	Addition or Subtraction
The final result should retain as many significant digits as are there in the original numbers with the least significant digits.	The final result should retain as many decimal places as are there in the original numbers with the least decimal places.

Rules for Determining the Number of Significant Digits	
For numbers with indicated decimal	For numbers with no indicated decimal
1. All nonzero digits (1-9) are to be counted as significant	1. All nonzero digits (1-9) are to be counted as significant.
2. Zeros that have any nonzero digits anywhere to the left of them are considered significant zeros.	2. The terminal or trailing zero(s) are not significant.
3. All other zeros not covered in rule (2) above are NOT considered significant zeros.	--

ERROR IN MEASUREMENT- Vernier Caliper & Screw Gauge

Vernier Caliper



1 Main Scale Division (MSD) = 1mm

1 Vernier Scale Division (VSD) = 0.9 mm

Length = MS Reading + Least Count of Vernier × VS division Coinciding with MS

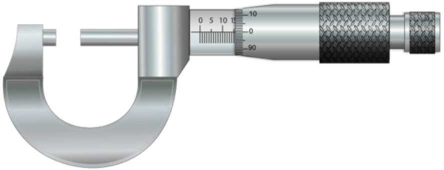
Least Count of Vernier Calipers = 1 MSD – 1 VSD = 0.1 mm

Zero Error is subtracted from the reading to get the corrected value.

Positive Zero Error: If nothing is placed and zero of VS is to the RIGHT of zero of MS , then the zero error is positive.

Negative Zero Error: If nothing is placed and zero of circular scale is ABOVE the line of MS , then the zero error is negative.

Screw Gauge



PITCH: Distance between two consecutive threads of the screw.

$$\text{Least Count} = \frac{\textbf{Pitch}}{\textbf{No. of division on circular Scale}}$$

Reading = Main Scale Reading + Circular Scale Count

Corrected Value = Reading – Zero Error

Positive Zero Error: If nothing is placed and zero of circular scale is BELOW the line of MS , then the zero error is positive.

Corrected Value = Reading – Zero Error

Negative Zero Error: If nothing is placed and zero of circular scale is ABOVE the line of MS , then the zero error is negative.

Corrected Value = Reading – Zero Error