Property - 6

6. The value of a determinant is not changed by adding to the elements of any row (or column) the same multiples of the corresponding elements of any other row (or column).

5. Special Determinants

1. $\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = (x-y)(y-z)(z-x)$

2. $\begin{vmatrix} 1 & x & x^3 \\ 1 & y & y^3 \\ 1 & z & z^3 \end{vmatrix} = (x-y)(y-z)(z-x)(x+y+z)$

3. $\begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$

4. $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 3abc - a^3 - b^3 - c^3$

$= -(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$= -\frac{1}{2}(a+b+c).\{(a-b)^2+(b-c)^2+(c-a)^2\}$

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Determinant

6. Factor Theorem

If by putting x = a the value of a determinant vanishes then, (x – a) will be a factor the determinant. This is known as FACTOR THEOREM.

7. Multiplication of Determinants

Introduction

$D_1 = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$ $D_2 = \begin{vmatrix} p & q \\ r & s \end{vmatrix}$

$D = D_1 \times D_2$

$$D = \begin{vmatrix} ap+br & aq+bs \\ cp+dr & cq+ds \end{vmatrix}$$

(Row by column multiplication)

Note

Multiplication can also be done

1) Row by Row
2) Column by Row
3) Column by Column

8. Cramer's Rule System of Linear Equations

Types of solutions
- Solution exist (At least one solution) (Consistent)
  - Unique/ Exactly one sol.
    - All variables zero is the only solution $x = 0, y = 0, z = 0$ → Trivial solution
    - At least one non – zero variable satisfy the system → Non - Trivial solution, Non - Zero solution
  - Infinite solutions
- No solution (Inconsistent)

9. Non – Homogeneous System

$a_1x + b_1y + c_1z = d_1$ .... (i)

$a_2x + b_2y + c_2z = d_2$ .... (ii)

$a_3x + b_3y + c_3z = d_3$ .... (iii)

$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ $D_x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$

$D_y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$ $D_z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$

Then, $x = \frac{D_x}{D}$ $y = \frac{D_y}{D}$ $z = \frac{D_z}{D}$

This is known as the CRAMER'S RULE