Key Point

2. Sum of product of elements of any row (column) with cofactors of corresponding elements of any other row (column) is ZERO.

$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$\Rightarrow a_{11}C_{21} + a_{12}C_{22} + a_{13}C_{23} = 0$$

$$D = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 6 \\ 5 & 7 & 8 \end{vmatrix} = 38$$

$C_{11} = -42 \quad C_{12} = -2 \quad C_{13} = 28$

$C_{21} = 5 \quad C_{22} = -7 \quad C_{23} = 3$

$C_{31} = 12 \quad C_{32} = 6 \quad C_{33} = -8$

$$D_c = \begin{vmatrix} -42 & -2 & 28 \\ 5 & -7 & 3 \\ 12 & 6 & -8 \end{vmatrix} = (38)^2$$

3. $D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ $D_C = \begin{vmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{vmatrix}$

Then, $D_C = D^2$

$\therefore$ In general, $D_C = D^{n-1}$

Where 'n' is the order of determinant.

Determinant

4. Properties of Determinants

Property - 1

1. The value of a determinant remains unchanged, if the rows & columns are interchanged.

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \quad \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = D^T$$

$C_1 \to R_1 \quad C_2 \to R_2 \quad C_3 \to R_3$

Property - 2

2. If any two rows (or columns) of a determinant are interchanged, the value of determinant is changed in sign only.

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \quad R_1 \leftrightarrow R_2 \quad \begin{vmatrix} a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix} = -D$$

Property - 3

3. If a determinant has any two rows (or columns) identical, then its value is ZERO.

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_2 & b_2 & c_2 \end{vmatrix} \quad \text{Here, } R_2 = R_3$$

$= a_1 \times (b_2c_2 - b_2c_2) - b1 \times (a_2c_2 - a_2c_2)$

$+ c1 \times (a_2b_2 - a2b2) = 0$

Property - 4

4. If all the elements of any row (or column) are multiplied by the same number, then value of the determinant is multiplied by that number.

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \quad \& \quad D' = \begin{vmatrix} Ma_1 & Mb_1 & Mc_1 \\ a_2 & b_2 & c_2 \\ Na_3 & Nb_3 & Nc_3 \end{vmatrix}$$

Then, $D' = M.N.D$

Property - 5

5. If each element of any row (or column) can be expressed as a sum of two or more terms, then the determinant can be expressed as the sum of two or more determinants.

$$D = \begin{vmatrix} a+x & b+y \\ c & d \end{vmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} + \begin{vmatrix} x & y \\ c & d \end{vmatrix}$$

$= (a+x)d - c(b+y) \quad (ad - bc) + (xd - yc)$

$= (ad - bc) + (xd - yc)$