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Determinant

1. Introduction

Value of the Determinant

$$D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \qquad D = \begin{vmatrix} 2 & 1 \\ 3 & 4 \end{vmatrix}$$

$D = a_1b_2 - a_2b_1 \quad D = 2 \times 4 - 3 \times 1$

$= 8 - 3 = 5$

Note

A determinant of order 1 is the number itself.

$\Rightarrow |x| = x$

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Expanding the Determinant

$$a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

This is know as row-wise expansion of the determinant.

2. General Representation of Determinant

A general third order determinant is represented by :

Note

$$D = \begin{vmatrix} a_{11} & \bar{a}_{12} & a_{13} \\ \bar{a}_{21} & a_{22} & \bar{a}_{23} \\ a_{31} & \bar{a}_{32} & a_{33} \end{vmatrix}$$

In the similar fashion we can expand a determinant in 6 different ways using elements of R_1, R_2, R_3, C_1, C_2 & C_3.

3. Minors and Cofactors

Minors

General third order determinant :

$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Minor of elements :

$$a_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} \quad a_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} \quad a_{13} = \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

Minor of element $a_{23} = \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}$

General third order determinant :

$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Hence, Minor of an element is defined as the minor determinant obtained by deleting a particular row and column in which that element lies.

Cofactors

General third order determinant :

$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Cofactor of any element a_{ij} of determinant D is given by :

$$C_{ij} = (-1)^{i+j}.M_{ij}$$

$\therefore$ Cofactor is nothing but minor with a proper sign.

$$C_{12} = (-1)^{1+2}.M_{12} = (-1)\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$$

$$C_{ij} = (-1)^{i+j}.M_{ij}$$

$$C_{22} = (-1)^{2+2}.M_{22} = (1)\begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix}$$

We know that,

$D = a_{11}M_{11} - a_{12}M_{12} + a_{13}M_{13}$

$D = a_{11}M_{11} + a_{12}(-M_{12}) + a_{13}M_{13}$

$\Rightarrow D = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$

Key Point

1. Sum of product of elements of any row (column) with their corresponding cofactors is equal to the value of DETERMINANT.

$$\Rightarrow \sum_{j=1}^{n} a_{ij}C_{ij} = D$$


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