## 1. Introduction

### Value of the Determinant

$$\mathbf{D} = \begin{vmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \end{vmatrix} \quad \mathbf{D} = \begin{vmatrix} \mathbf{a}_1 \\ \mathbf{b}_2 \end{vmatrix} \quad \mathbf{D} = \begin{vmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \end{vmatrix}$$

 $D = a_1b_2 - a_2b_1$   $D = 2 \times 4 - 3 \times 1$ = 8 - 3 = 5

#### Note

A determinant of order 1 is the number itself.  $\begin{vmatrix} a_1 & b_1 & c_1 \end{vmatrix}$ 

$$\Rightarrow |\mathbf{x}| = \mathbf{x} \begin{vmatrix} \mathbf{a}_2 & \mathbf{b}_2 & \mathbf{c}_2 \\ \mathbf{a}_3 & \mathbf{b}_3 & \mathbf{c}_3 \end{vmatrix}$$

## **Expanding the Determinant**

$$a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

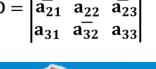
This is know as row-wise expansion of the determinant.

#### 2. General Representation of Determinant

A general third order determinant is represented by :

Note  

$$D = \begin{vmatrix} a_{11} & \overline{a_{12}} & a_{13} \\ \overline{a_{21}} & a_{22} & \overline{a_{23}} \\ \overline{a_{21}} & \overline{a_{22}} & \overline{a_{23}} \end{vmatrix}$$







In the similar fashion we can expand a determinant in 6 different ways using elements of R<sub>1</sub>, R<sub>2</sub>, R<sub>3</sub>, C<sub>1</sub>, C<sub>2</sub> & C<sub>3</sub>. **3. Minors and Cofactors**

#### Minors

General third order determinant :

$$\mathbf{D} = \begin{vmatrix} \mathbf{a}_{11} & \mathbf{a}_{12} & \mathbf{a}_{13} \\ \mathbf{a}_{21} & \mathbf{a}_{22} & \mathbf{a}_{23} \\ \mathbf{a}_{31} & \mathbf{a}_{32} & \mathbf{a}_{33} \end{vmatrix}$$

Minor of elements :

$$\mathbf{a}_{11} = \begin{vmatrix} \mathbf{a}_{22} & \mathbf{a}_{23} \\ \mathbf{a}_{32} & \mathbf{a}_{33} \end{vmatrix} \quad \mathbf{a}_{12} = \begin{vmatrix} \mathbf{a}_{21} & \mathbf{a}_{23} \\ \mathbf{a}_{31} & \mathbf{a}_{33} \end{vmatrix} \quad \mathbf{a}_{13} = \begin{vmatrix} \mathbf{a}_{21} & \mathbf{a}_{22} \\ \mathbf{a}_{31} & \mathbf{a}_{32} \end{vmatrix}$$

Minor of element  $a_{23} = \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}$ 

General third order determinant :

$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Hence, Minor of an element is defined as the minor determinant obtained by deleting a particular row and column in which that element lies.

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#### Cofactors

General third order determinant :

 $D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ 

Cofactor of any element  $a_{ij}$  of determinant D is given by :

$$\mathbf{C}_{\mathbf{ij}} = (-\mathbf{1})^{\mathbf{i}+\mathbf{j}} \cdot \mathbf{M}_{\mathbf{ij}}$$

... Cofactor is nothing but minor with a proper sign.

$$C_{12} = (-1)^{1+2} \cdot M_{12} = (-1) \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$$
$$C_{ij} = (-1)^{i+j} \cdot M_{ij}$$
$$C_{22} = (-1)^{2+2} \cdot M_{22} = (1) \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix}$$
$$We \text{ know that,}$$
$$D = a_{11}M_{11} - a_{12}M_{12} + a_{13}M_{13}$$
$$D = a_{11}M_{11} + a_{12}(-M_{12}) + a_{13}M_{13}$$
$$\Rightarrow D = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$$

## **Key Point**

1. Sum of product of elements of any row (column) with their corresponding cofactors is equal to the value of DETERMINANT.

$$\Rightarrow \sum_{j=1}^n a_{ij}C_{ij} = D$$

