Binomial Theorem

6. Properties of Binomial Coefficients & Summation of Series

- $B_{/}B_{0} * B_{0}B_{1} * B_{1}B_{2} * — *Bm_{╫0}B_{m} < {}^{2n}C_{n} ╞0$

$$= \frac{(2n)!}{(n+1)!\,(n-1)!}$$

- $B_{/}B_{1} * B_{0}B_{2} * B_{1}B_{3} * — *Bm_{╫1}B_{m} < {}^{2n}C_{n} ╞1$

$$= \frac{(2n)!}{(n+2)!\,(n-2)!}$$

- $B_{/}^{1} ├B_{0}^{1} * B_{1}^{1} * —— * ' ├0(^{m}B_{m}^{1} <$

$$\begin{cases} 0 & \text{if } n \text{ is odd} \\ (-1)^{n/2}\,{}^{n}C_{n/2} & \text{if } n \text{ is even} \end{cases}$$

7. Sum of Coefficients in an Expansion

- Sum of all the coefficients in an expansion can be obtained by putting variables = 1.
- Sum of coefficients of EVEN power of x can be obtained by

$$\frac{(\text{Put variables} = 1) + (\text{Put variables} = -1)}{2}$$

- Sum of coefficients of ODD power of x can be obtained by

$$\frac{(\text{Put variables} = 1) - (\text{Put variables} = -1)}{2}$$

8. Application of Binomial Theorem

Divisibility Problems

$x^n - y^n$ is always divisible by $(x - y)$

9. Multinomial Theorem

$$(x+y)^n = \sum_{r+s=n}^{n} \frac{n!}{s!\,r!} x^s y^r \quad \text{Where, } s + r = n$$

$$(x+y+z)^n = \sum_{r+s+t=n}^{n} \frac{n!}{s!\,r!\,t!} x^s y^r z^t \quad \text{Where, } s + r + t = n$$

This result can be generalized in the following form

$$(x_1 + x_2 + \dots + x_k)^n = \sum_{r_1+r_2+\dots+r_k=n}^{n} \frac{n!}{r_1!\,r_2!\dots r_k!} x_1^{r_1} x_2^{r_2} \dots x_k^{r_k}$$

where,

(i) $r_1 + r_2 + r_3 + \dots\dots + r_k = n$

(ii) $r_1, r_2, r_3, \dots\dots\ r_k$ Î w

(iii) n Î N

(iv) The general term in the above expansion

$$= \frac{n!}{r_1!\,r_2!\,r_3!\dots r_k!} \cdot x_1^{r_1} x_2^{r_2} x_3^{r_3} \dots\dots x_k^{r_k}$$

(v) Number of terms = ${}^{n+k-1}C_{k-1}$

10. Binomial Theorem for Negative & Fractional Index

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots + \infty$$

where,

(i) n is a negative integer or a fraction.

(ii) $|x| < 1$

General term in the expansion of $(1 + x)^n$ is :

$$T_{r+1} = \frac{n(n-1)(n-2)\dots\dots\dots\dots[n-(r-1)]}{r!}x^r$$

11. Important expansions for n ∈ Q & |x| < 1

'\` ($(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots\dots + \infty$

'a($(1+x)^{-n} = 1 + (-n)x + \frac{(-n)(-n-1)}{2!}x^2 + \frac{(-n)(-n-1)(-n-2)}{3!}x^3 + \dots + \infty$

'b($(1-x)^n = 1 + n(-x) + \frac{n(n-1)}{2!}(-x)^2 + \frac{n(n-1)(n-2)}{3!}(-x)^3 + \dots\dots + \infty$

'c($(1-x)^{-n} =$

$$1 + (-n)(-x) + \frac{(-n)(-n-1)}{2!}(-x)^2 + \frac{(-n)(-n-1)(-n-2)}{3!}(-x)^3 + \dots\dots \infty$$