Binomial Theorem

1. Binomial Theorem

General Expansion

$(x + y)^n = {}^nC_0\, x^n y^0 + {}^nC_1\, x^{n-1} y^1 + {}^nC_2\, x^{n-2} y^2 + \ldots + {}^nC_n\, x^0 y^n$

General Term :

$T_{r+1} = {}^nC_r\, x^{n-r} y^r$ where, $0 \le r \le n$

1) The number of terms in the expansion of $(x + y)^n$ is $(n + 1)$ i.e. one more than the index .
2) The sum of the indices of x & y in each term is n.
3) Power of first variable (x) decreases while of second variable (y) increases.
4) Binomial coefficients of the terms equidistant from the beginning and from the end are equal.
5) Binomial coefficients of the middle term is greatest.
6) m^{th} - Term from the END

T_m [from the end] ⟷ T_{n-m+2} [from the beginning]

2. Middle Term

Middle term in the expansion of $(I + II)^n$ is

$T_{\frac{n}{2}+1}$ when n is even

$T_{\frac{n+1}{2}}$ & $T_{\frac{n+3}{2}}$ when n is odd

In binomial expansion, middle term has greatest binomial coefficient and if there are 2 middle terms, their coefficients will be equal.

$\Rightarrow$ ${}^nC_r$ will be max

- where $r = \frac{n}{2}$, if n is even
- where $r = \frac{n-1}{2}$ or $\frac{n+1}{2}$, if n is odd

3. Number of Terms in Expansion

(a) If n is ODD, then number of terms in $(x + a)^n \pm (x - a)^n$ is $\frac{n+1}{2}$

(b) If n is EVEN, then number of terms in

(i) $(x + a)^n + (x - a)^n$ is $\frac{n}{2} + 1$

(ii) $(x + a)^n - (x - a)^n$ is $\frac{n}{2}$

4. Numerically Greatest Term in the expansion of $(a + bx)^n$

$$\left(\frac{n+1}{1+\left|\frac{I}{II}\right|}\right) - 1 \le r \le \left(\frac{n+1}{1+\left|\frac{I}{II}\right|}\right)$$

5. Standard Binomial Expansion

$$(1 + x)^n = C_0x^0 + C_1x^1 + C_2x^2 + \ldots\ldots + C_nx^n$$

Note : Binomial coefficient & Coefficient of x^r are equal

6. Properties of Binomial Coefficients & Summation of Series

$$\sum_{r=0}^{n} {}^nC_r = 1^m$$

$$\sum_{r=0}^{n} '\ |0(q\ {}^nC_r = 0$$

$$C_0 + C_2 + C_4 + \ldots = C_1 + C_3 + C_5 + \ldots = 2^{n-1}$$

- ${}^nC_1 + 2.{}^nC_2 + 3.{}^nC_3 + \ldots\ldots\ldots + (n-1).{}^nC_{n-1} + n.{}^nC_n = n.\, 2^{n-1}$
- $(1)^2.C_1 + (2)^2.C_2 + (3)^2.C_3 + \ldots\ldots\ldots + (n)^2.C_n = n\,(1 + n)\, 2^{n-2}$
- $C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \frac{C_3}{4} + \ldots\ldots + \frac{C_n}{n+1} = \frac{2^{n+1} - 1}{n+1}$
- $C_0 - \frac{C_1}{2} + \frac{C_2}{3} - \ldots + (-1)^n \frac{C_n}{n+1} = \frac{1}{n+1}$
- $C_0^2 + C_1^2 + C_2^2 + C_3^2 + \ldots\ldots + C_n^2 = {}^{2n}C_n = \frac{(2n)!}{n!\,n!}$
- $B_/ B_q * B_0 B_{q*0\,*}$ —— $* \ Bm_{|qBm} < {}^{2n}C_{n\,|q} = \frac{(2n)!}{(n+r)!\,(n-r)!}$