Class X
Mathematics –Standard
Question Paper

Max. Marks: 80 **Duration: 3 hrs**

General Instructions:

All the questions are compulsory.

1. *The question paper consists of 40 questions divided into 4 sections A, B, C, and D.*

2. *Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises of 6 questions of 4 marks each.*

3. *There is no overall choice. However, an internal choice has been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each, and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.*

4. *Use of calculators is not permitted.*

SECTION A

Question numbers 1 to 20 carry 1 mark each.

Q 1- Q 10 are multiple choice questions. Select the most appropriate answer from the given options.

Q1. π is:

(A) A rational number (B) Not a real number

(C) An irrational number (D) Terminating decimal

Q2. Which of the following is/are correct?

(i) Every integer is a rational number.

(ii) The sum of a rational number and an irrational number is an irrational number.

(iii) Every real number is rational

(iv) Every point on a number line is associated with a real number.

(A) (i), (ii) and (iii) (B) (i), (ii), (iii) and (iv)

(C) (i), (ii) and (iv) (D) (ii), (iii) and (iv)

Q3. The product of a non-zero rational and an irrational number is :

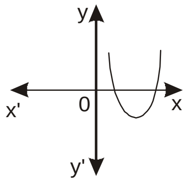
(A) always irrational (B) always rational

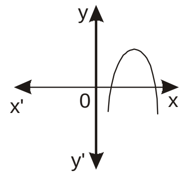
(C) rational or irrational (D) one

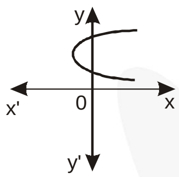
Q4. Degree of a zero polynomial is:

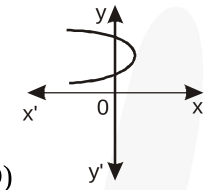
(A) 0 (B) 1 (C) not defined (D) none of these

Q5. If a < 0, then graph of $y = ax^2 + bx + c$ can be:

(A) 

(B) 

(C) 

(D) 

Q6. If two positive integers p and q can be expressed as $p = ab^2$ and $q = a^3b$; a, b being prime numbers, then LCM (p, q) is:

(A) ab (B) a^2b^2 (C) a^3b^2 (D) a^3b^3

Q7. Find the value of y if x = 3 in given equation: $-3x + 2y - 3 = 0$.

(A) 15 (B) 6 (C) 7 (D) –6

Q8. The pair of equations x = 0 and y = – 7 has

(A) One solution (B) Two solutions

(C) Infinitely many solutions (D) No solution

Q9. The equation $4p^2x^2 + 12pqx + 5q^2 = 0$ have $(p, q \neq 0)$

(A) real and equal roots (B) real and unequal roots

(C) no real roots (D) None of these

Q10. If the sum of first n terms of an AP be $3n^2 - n$ and its common difference is 6, then its first term is:

(A) 2 (B) 3 (C) 1 (D) 4

(Q 11 - Q15) Fill in the Blanks

Q11. Graph of a linear polynomial is ……………….

Q12. Every quadratic polynomial can have at most ……………

Q13. Value of D when root of $ax^2 + bx + c = 0$ are real and unequal will be……………….

Q14. The first and last term of an A.P. are 1 and 11. If the sum of its terms is 36, then the number of terms will be …………….

Q15. A polynomial of degree n has ………………

(Q16 - Q20) Answer the following

Q16. Find two irrational numbers lying between $\sqrt{2}$ and $\sqrt{3}$.

Q17. Find the degree of the polynomial:

(i) $2y^{12} + 3y^{10} - y^{15} + y + 3$

(ii) 8

Q18. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ & $\frac{c_1}{c_2}$ and without drawing them, find out whether the lines representing the following pair of linear equations intersect at a point, are parallel or coincide: $5x - 4y + 8 = 0$, $7x + 6y - 9 = 0$

OR

Find the value of m, if x = 2 is a zero of quadratic polynomial $3x^2 - mx + 4$.

Q19. Write the first three terms in each of the sequence defined by $a_n = 3n + 2$

Q20. From given rational number check whether it is terminating or non-terminating $\frac{13}{3125}$.

SECTION B

Question numbers 21 to 26 carry 2 mark each.

Q21. The n^{th} term of a sequence is $3n - 2$. Is the sequence an A.P.? If so, find its 10^{th} term.

Q22. Solve the following quadratic equations: $7x^2 = 8 - 10x$

Q23. Solve the following systems of equations by eliminating 'y': 3x – y = 3,7x + 2y = 20

Q24. Find the remainder when $4x^3 - 3x^2 + 2x - 4$ is divided by x – 1

Q25. Using Euclid's division algorithm, find the H.C.F. of 135 and 225

Q26. Find the value of the polynomial $5x - 4x^2 + 3$ at: $x = -1$

SECTION C

Question numbers 27 to 34 carry 3 mark each.

Q27. Find the prime factors of 21252 using factor tree method.

Q28. Solve 2x + 3y = 11 and 2x – 4y = – 24 and hence find the value of 'm' for which y = mx + 3.

OR

Find the roots of the following quadratic equations (if they exist) by the method of completing the square $2x^2 - 7x + 3 = 0$

Q29. For what value of m, roots of the equation (3m + 1) $x^2 + (11 + m)x + 9 = 0$ are equal?

Q30. Determine the general term of an A.P. whose 7^{th} term is –1 and 16^{th} term 17.

Q31. Explain why 7 × 11 × 13 + 13 and 7 × 6 × 5 × 4 × 3 + 5 are composite numbers.

OR

Find the zeroes of the quadratic polynomial $9x^2 - 5$ and verify the relation between the zeroes and its coefficients.

Q32. Which term of the sequence 4, 9, 14, 19, ... is 124?

Q33. Solve the following system of equations 43x + 35y = 207; 35x + 43y = 183

Q34. Find a cubic polynomial with the sum of its zeroes, sum of the products of its zeroes taken two at a time, and product of its zeroes as 2, – 7 and –14, respectively.

SECTION D

Question numbers 35 to 40 carry 4 mark each.

Q35. Show that any positive integer which is of the form 6q + 1 or 6q + 3 or 6q + 5 is odd, where q is some integer.

Q36. A motor boat, whose speed is 15 km/hr in still water, goes 30 km downstream and comes back in a total of 4 hours 30 minutes. Determine the speed of the stream.

OR

Obtain all the zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

Q37. If m times m^{th} term of an A.P. is equal to n times its n^{th} term, show that the (m + n) term of the A.P. is zero.

Q38. The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the number. Find the number.

Q39. Solve, $\frac{1}{x+y} + \frac{2}{x-y} = 2$ $\frac{2}{x+y} - \frac{1}{x-y} = 3$ where $x + y \neq 0$ and $x - y \neq 0$

OR

If α and β are the roots of the quadratic equation $ax^2 + bx + c = 0, (a \neq 0)$ then find the values of:

(i) $\alpha^2 + \beta^2$ (ii) $\alpha^3 + \beta^3$

Q40. The sum of n, 2n, 3n terms of an A.P. are S_1, S_2, S_3 respectively.

Prove that $S_3 = 3(S_2 - S_1)$.

Class X

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Answer Paper

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SECTION A

1. C

2. C

3. A

4. C

5. B

6. C

7. B

8. A

9. B

10. A

11. Straight line

12. Two zero

13. D > 0

14. 6

15. Exactly n zeroes

16. We know that, if a and b are two distinct positive irrational numbers, then $\sqrt{ab}$ is an irrational number lying between a and b.

$\therefore$ Irrational number between $\sqrt{2}$ and $\sqrt{3}$ is $\sqrt{\sqrt{2}\times\sqrt{3}} = \sqrt{\sqrt{6}} = 6^{1/4}$

Irrational number between $\sqrt{2}$ and $6^{1/4}$ is $\sqrt{\sqrt{2}\times 6^{1/4}} = 2^{1/4} \times 6^{1/8}$.

Hence required irrational number are $6^{1/4}$ and $2^{1/4} \times 6^{1/8}$.

17. (i) Since the term with highest exponent (power)

The highest power of the variable is 15 $\Rightarrow$ degree = 15.

(ii) $8 = 8x^0 \Rightarrow$ degree = 0

18. Comparing the given equations with standard forms of equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ we have,

$a_1 = 5, b_1 = -4, c_1 = 8;$

$a_2 = 7, b_2 = 6, c_2 = -9$

$\therefore \frac{a_1}{a_2} = \frac{5}{7}, \frac{b_1}{b_2} = \frac{-4}{6}$

$\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Thus, the lines representing the pair of linear equations are intersecting.

OR

18. Since, x = 2 is a zero of $3x^2 - mx + 4$

$\Rightarrow 3(2)^2 - m\times 2 + 4 = 0$

$\Rightarrow 12 - 2m + 4 = 0$, i.e., m = 8.

19. We have, $a_n = 3n + 2$

Putting n = 1, 2 and 3, we get

$a_1 = 3 \times 1 + 2 = 3 + 2 = 5,$

$a_2 = 3 \times 2 + 2 = 6 + 2 = 8,$

$a_3 = 3 \times 3 + 2 = 9 + 2 = 11$

Thus, the required first three terms of the sequence defined by $a_n = 3n + 2$ are 5, 8, and 11.

20. $\frac{13}{3125} = \frac{13}{(5)^5} = \frac{13 \times 2^5}{2^5 \times 5^5} = \frac{(13 \times 32)}{(10)^5}$ = terminating

SECTION B

21. We have $a_n = 3n - 2$

Clearly a_n is a linear expression in n. So, the given sequence is an A.P. with common difference 3.

Putting n = 10, we get

$a_{10} = 3 \times 10 - 2 = 28$

22. $7x^2 = 8 - 10x$

$\Rightarrow 7x^2 + 10x - 8 = 0$

$\Rightarrow 7x^2 + 14x - 4x - 8 = 0$

$\Rightarrow 7x(x + 2) - 4(x + 2) = 0$

$\Rightarrow (x + 2)(7x - 4) = 0$

$\Rightarrow x + 2 = 0$ or $7x - 4 = 0$

$\Rightarrow x = -2$ or $x = \frac{4}{7}$

23. We have;

$3x - y = 3$ .... (1)

$7x + 2y = 20$ .... (2)

From equation (1), we get;

$3x - y = 3$

$\Rightarrow y = 3x - 3$

Substituting the value of 'y' in equation (2), we get;

$\Rightarrow 7x + 2 \times (3x - 3) = 20$

$\Rightarrow 7x + 6x - 6 = 20$

$\Rightarrow 13x = 26$

$\Rightarrow x = 2$

Now, substituting x = 2 in equation (1), we get;

$3 \times 2 - y = 3$

$\Rightarrow y = 3$

Hence, x = 2, y = 3.

24. Let $p(x) = 4x^3 - 3x^2 + 2x - 4$

When p(x) is divided by (x – 1), then by remainder theorem, the required remainder will be p (1)

$p(1) = 4(1)^3 - 3(1)^2 + 2(1) - 4$

$= 4 \times 1 - 3 \times 1 + 2 \times 1 - 4$

$= 4 - 3 + 2 - 4 = -1$

25. Starting with the larger number i.e., 225,

we get, $225 = 135 \times 1 + 90$

Now taking divisor 135 and remainder 90,

we get, $135 = 90 \times 1 + 45$

Further taking divisor 90 and remainder 45,

we get, $90 = 45 \times 2 + 0$

$\therefore$ Required H.C.F. = 45

26. Let $p(x) = 5x - 4x^2 + 3$

At $x = -1, p(-1) = 5(-1) - 4(-1)^2 + 3 = -5 - 4 + 3 = -6$

SECTION C

27.

(2) 21252
(2) 10626
(3) 5313
(7) 1771
(11) 253
23

$\therefore 21252 = 2 \times 2 \times 3 \times 7 \times 11 \times 23 = 2^2 \times 3 \times 11 \times 7 \times 23.$

28. We have,

$2x + 3y = 11$ .... (1)

$2x - 4y = -24$ .... (2)

From (1), we have $2x = 11 - 3y$

Substituting $2x = 11 - 3y$ in (2),

we get $11 - 3y - 4y = -24$

$-7y = -24 - 11$

$\Rightarrow -7y = -35$

$\Rightarrow y = 5$

Putting $y = 5$ in (1),

we get, $2x + 3 \times 5 = 11$

$2x = 11 - 15$

$\Rightarrow x = -\frac{4}{2} = -2$

Hence, $x = -2$ and $y = 5$

Again, putting $x = -2$ and $y = 5$ in $y = mx + 3$,

we get, $5x = m(-2) + 3$

$\Rightarrow -2m = 5 - 3$

$\Rightarrow m = \frac{2}{-2} = -1$

OR

28. $2x^2 - 7x + 3 = 0 \Rightarrow x^2 - \frac{7}{2}x + \frac{3}{2} = 0$

$\Rightarrow \frac{7}{4}\ \frac{3}{2}\ x^2 - 2 \times x \times \frac{7}{4} + \frac{3}{2} = 0$

$\Rightarrow x^2 - 2 \times x \times \frac{7}{4} + \left(\frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2 + \frac{3}{2} = 0$

$\Rightarrow \left(x - \frac{7}{4}\right)^2 - \frac{49}{16} + \frac{3}{2} = 0$

$\Rightarrow \left(x - \frac{7}{4}\right)^2 - \left(\frac{49 - 24}{16}\right) = 0$

$\Rightarrow \left(x - \frac{7}{4}\right)^2 - \frac{25}{16} = 0$

i.e., $\left(x - \frac{7}{4}\right)^2 = \frac{25}{16}$

$\Rightarrow x - \frac{7}{4} = \pm \frac{5}{4}$

i.e., $x - \frac{7}{4} = \frac{5}{4}$ = or $x - \frac{7}{4} = -\frac{5}{4}$

$\Rightarrow x = \frac{7}{4} + \frac{5}{4}$ or $x = \frac{7}{4} - \frac{5}{4}$

$\Rightarrow x = 3$ or $x = \frac{1}{2}$

29. Comparing the given equation with $ax^2+bx+c=0$;

we get: a = 3m + 1, b = 11 + m and c = 9

$\therefore$ Discriminant, $D=b^2-4ac$

$=(11+m)^2-4(3m+1)\times 9$

$=121+22m+m^2-108m-36$

$=m^2-86m+85$

$=m^2-85m-m+85$

= m (m – 85) – 1 (m – 85)

= (m – 85) (m – 1)

Since the roots are equal, D = 0

$\Rightarrow$ (m – 85) (m – 1) = 0

$\Rightarrow$ m – 85 = 0 or m – 1 = 0

$\Rightarrow$ m = 85 or m = 1

30. Let a be the first term and d be the common difference of the given A.P. Let the A.P. be $a_1, a_2, a_3, \ldots\ldots a_n, \ldots\ldots$

It is given that $a_7=-1$ and $a_{16}=17$

a + (7 – 1) d = – 1 and, a + (16 – 1) d = 17

$\Rightarrow$ a + 6d = – 1 .... (i)

and, a + 15d = 17 .... (ii)

Subtracting equation (i) from equation (ii),

we get, 9d = 18

$\Rightarrow$ d = 2

Putting d = 2 in equation (i),

we get, a + 12 = – 1

$\Rightarrow$ a = – 13

Now, General term = a_n

= a + (n – 1) d = – 13 + (n – 1) × 2 = 2n – 15

31. Since,

7 × 11 × 13 + 13 = 13 × (7 × 11 + 1) = 13 × 78 = 13 × 13 × 3 × 2;

that is, the given number has more than two factors and it is a composite number.

Similarly, 7 × 6 × 5 × 4 × 3 + 5 = 5 × (7 × 6 × 4 × 3 + 1)

= 5 × 505 = 5 × 5 × 101 $\Rightarrow$ The given no. is a composite number.

OR

31. We have, $9x^2 - 5 = (3x)^2 - (\sqrt{5})^2 = (3x - \sqrt{5})(3x + \sqrt{5})$

So, the value of $9x^2 - 5$ is 0,

when $3x - \sqrt{5} = 0$ or $3x + \sqrt{5} = 0$

i.e., when $x = \frac{\sqrt{5}}{3}$ or $x = \frac{-\sqrt{5}}{3}$

Sum of the zeroes $= \frac{\sqrt{5}}{3} - \frac{\sqrt{5}}{3} = 0 = \frac{-(0)}{9} = \frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$

Product of the zeroes $= \left(\frac{\sqrt{5}}{3}\right)\left(\frac{-\sqrt{5}}{3}\right) = \frac{-5}{9} = \frac{\text{constant term}}{\text{coefficient of } x^2}$

32. Clearly, the given sequence is an A.P. with first term a = 4 and common difference d = 5.

Let 124 be the n^{th} term of the given sequence.

Then, $a_n = 124$

$a + (n - 1) d = 124$

$\Rightarrow 4 + (n - 1) \times 5 = 124$

$\Rightarrow n = 25$

Hence, 25^{th} term of the given sequence is 124.

33. The given system of equations is;

$43x + 35y = 207$ .... (1)

$35x + 43y = 183$ .... (2)

Adding equation (1) and (2),

we get; $78x + 78y = 390$

$\Rightarrow 78(x + y) = 390$

$\Rightarrow x + y = 5$ .... (3)

Subtracting equation (2) from the equation (1),

we get, $8x - 8y = 24$

$\Rightarrow x - y = 3$ .... (4)

Adding equation (3) and (4),

we get, $2x = 8$

$\Rightarrow x = 4$

Putting x = 4 in equation (3),

we get, $4 + y = 5$

$\Rightarrow y = 1$

Hence, the solution of the system of equation is; $x = 4$, $y = 1$.

34. Let the cubic polynomial be

$ax^3 + bx^2 + cx + d$

$\Rightarrow x^3 + \frac{b}{a}x^2 + \frac{c}{a}x + \frac{d}{a}$ .... (1)

and its zeroes are α, β and γ, then

$\alpha + \beta + \gamma = 2 = -\frac{b}{a}$

$\alpha\beta + \beta\gamma + \alpha\gamma = -7 = \frac{c}{a}$

$\alpha\beta\gamma = -14 = -\frac{d}{a}$

Putting the values of $\frac{b}{a}$, $\frac{c}{a}$ and $\frac{d}{a}$ in (1),

we get, $x^3 + (-2)x^2 + (-7)x + 14$

$\Rightarrow x^3 - 2x^2 - 7x + 14$

SECTION D

35. If a and b are two positive integers such that a is greater than b; then according to Euclid's division algorithm; we have

$a = bq + r$; where q and r are positive integers and $0 \le r < b$.

Let $b = 6$, then $a = bq + r$

$\Rightarrow a = 6q + r$; where $0 \le r < 6$.

When $r = 0 \Rightarrow a = 6q + 0 = 6q$;

which is even integer

When $r = 1 \Rightarrow a = 6q + 1$ which is odd integer

When $r = 2 \Rightarrow a = 6q + 2$ which is even.

When $r = 3 \Rightarrow a = 6q + 3$ which is odd.

When $r = 4 \Rightarrow a = 6q + 4$ which is even.

When $r = 5 \Rightarrow a = 6q + 5$ which is odd.

This verifies that when $r = 1$ or 3 or 5; the integer obtained is

$6q + 1$ or $6q + 3$ or $6q + 5$ and each of these integers is a positive odd number.

Hence the required result.

36. Let the speed of the stream = x km/hr

$\Rightarrow$ The speed of the boat downstream = (15 + x) km/hr.

and, the speed of the boat upstream = (15 – x) km/hr

Now, time taken to go 30 km downstream = $\frac{30}{15+x}$ hrs.

and, time take to come back 30 km upstream = $\frac{30}{15-x}$ hrs.

Given: the time taken for both the journeys = 4 hours 30 min. = $4\frac{1}{2}$ hrs = $\frac{9}{2}$ hrs

$$\therefore \frac{30}{15+x} + \frac{30}{15-x} = \frac{9}{2}$$

$$\Rightarrow \frac{30(15-x)+30(15+x)}{(15+x)(15-x)} = \frac{9}{2}$$

i.e., $\frac{450-30x+450+30x}{225-x^2} = \frac{9}{2}$

$\Rightarrow 2 \times 900 = 9\,(225 - x^2)$

On dividing both the sides by 9,

we get, $2 \times 100 = 225 - x^2$

i.e., $x^2 = 225 - 200$

$\Rightarrow x^2 = 25$ and $x = \pm 5$

Rejecting the negative value of x,

we get: x = 5

i.e., the speed of the steam = 5 km/hr

OR

36. Since two zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$, $x = \sqrt{\frac{5}{3}}$, $x = -\sqrt{\frac{5}{3}}$

$\Rightarrow \left(x-\sqrt{\frac{5}{3}}\right)\left(x+\sqrt{\frac{5}{3}}\right) = x^2 - \frac{5}{3}$ or $3x^2 - 5$ is a factor of the given polynomial.

Now, we apply the division algorithm to the given polynomial and $3x^2 - 5$.

$$
\begin{array}{r|l}
 & x^2 + 2x + 1 \\ \hline
3x^2 - 5 & 3x^4 + 6x^3 - 2x^2 - 10x - 5 \\
 & 3x^4 \qquad\quad - 5x^2 \\
 & - \qquad\qquad + \\ \hline
 & \quad 6x^3 + 3x^2 - 10x - 5 \\
 & \quad 6x^3 \qquad\quad - 10x \\
 & \quad - \qquad\qquad + \\ \hline
 & \qquad\quad 3x^2 \qquad - 5 \\
 & \qquad\quad 3x^2 \qquad - 5 \\
 & \qquad\quad - \qquad\quad + \\ \hline
 & \qquad\qquad 0
\end{array}
$$

So, $3x^4 + 6x^3 - 2x^2 - 10x - 5 = (3x^2 - 5)(x^2 + 2x + 1) + 0$

Quotient = $x^2 + 2x + 1 = (x + 1)^2$

Zeroes of $(x + 1)^2$ are –1, –1.

Hence, all its zeroes are $\sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}, -1, -1$.

37. Let a be the first term and d be the common difference of the given A.P.

Then, m times m^{th} term = n times n^{th} term

$\Rightarrow ma_m = na_n$

$\Rightarrow m\{a + (m - 1)d\} = n\{a + (n - 1)d\}$

$\Rightarrow m\{a + (m - 1)d\} - n\{a + (n - 1)d\} = 0$

$\Rightarrow a(m - n) + \{m(m - 1) - n(n - 1)\}d = 0$

$\Rightarrow a(m - n) + (m - n)(m + n - 1)d = 0$

$\Rightarrow (m - n)\{a + (m + n - 1)d\} = 0$

$\Rightarrow a + (m + n - 1)d = 0$

$\Rightarrow a_{m+n} = 0$

Hence, the $(m + n)^{th}$ term of the given A.P. is zero.

38. Let the ten's and the unit's digits in the number be x and y, respectively. So, the number may be written as 10x + y.

When the digits are reversed, x becomes the unit's digit and y becomes the ten's digit.

The number can be written as 10y + x.

According to the given condition, $x + y = 9$ .... (1)

We are also given that nine times the number i.e., 9(10x + y) is twice the numbers obtained by reversing the order of the number i.e. 2(10y + x).

$\therefore 9(10x + y) = 2(10y + x)$

$\Rightarrow 90x + 9y = 20y + 2x$

$\Rightarrow 90x - 2x + 9y - 20y = 0$

$\Rightarrow 88x - 11y = 0$

$\Rightarrow 8x - y = 0$ .... (2)

Adding (1) and (2), we get, $9x = 9$

$\Rightarrow x = 1$

Putting $x = 1$ in (1),

we get $y = 9 - 1 = 8$

Thus, the number is

$10 \times 1 + 8 = 10 + 8 = 18$

39. Taking $\frac{1}{x+y} = u$ and $\frac{1}{x-y} = v$ the above system of equations becomes

$u + 2v = 2$ .... (1)

$2u - v = 3$ .... (2)

Multiplying equation (1) by 2, and (2) by 1, we get;

$2u + 4v = 4$ .... (3)

$2u - v = 3$ .... (4)

Subtracting equation (4) from (3), we get;

$5v = 1 \Rightarrow v = \frac{1}{5}$

Putting $v = 1/5$ in equation (1), we get;

$u + 2 \times \frac{1}{5} = 2 \Rightarrow u = 2 - \frac{2}{5} = \frac{8}{5}$

Here, $u = \frac{8}{5} = \frac{1}{x+y} \Rightarrow 8x + 8y = 5$ .... (5)

And, $v = \frac{1}{5} = \frac{1}{x-y} \Rightarrow x - y = 5$ .... (6)

Multiplying equation (5) with 1, and (6) with 8, we get;

$8x + 8y = 5$ .... (7)

$8x - 8y = 40$ .... (8)

Adding equation (7) and (8), we get;

$16x = 45$

$\Rightarrow x = \frac{45}{16}$

Now, putting the above value of x in equation (6), we get;

$\frac{45}{16} - y = 5$

$\Rightarrow \; y = \frac{45}{16} - 5 = \frac{-35}{16}$

Hence, solution of the system of the given equations is;

$x = \frac{45}{16}, \; y = \frac{-35}{16}$

OR

39. We know, sum of roots $(\alpha + \beta) = -\frac{b}{a}$

And, product of roots $(\alpha\,\beta) = \frac{c}{a}$; therefore :

(i) $(\alpha+\beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$

$\Rightarrow \alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= \left(-\frac{b}{a}\right)^2 - 2\frac{c}{a} = \frac{b^2}{a^2} - 2\,\frac{c}{a} = \frac{b^2 - 2ac}{a^2}$

(ii) $\alpha^3 + \beta^3 = (\alpha+\beta)\left(\alpha^2 + \beta^2 - \alpha\beta\right)$

$= \left(-\frac{b}{a}\right)\left(\frac{b^2 - 2ac}{a^2} - \frac{c}{a}\right)$

$= \left(-\frac{b}{a}\right)\left(\frac{b^2 - 2ac - ac}{a^2}\right) = -\frac{b(b^2 - 3ac)}{a^3}$

40. Let a be the first term and d be the common difference of the given A.P. Then,

S_1 = Sum of n terms

$\Rightarrow S_1 = \frac{n}{2}\{2a + (n-1)d\}$ ....(i)

$S_2 =$ Sum of 2 n terms

$\Rightarrow S_2 = \frac{2n}{2}\,[2a + (2n - 1)\,d]$ ....(ii)

and, S_3 = Sum of 3n terms

$\Rightarrow S_3 = \frac{3n}{2}\,[2a + (3n - 1)\,d]$ ....(iii)

Now, $S_2\, S_1 = \frac{2n}{2}\,[2a + (2n - 1)\,d] - \frac{n}{2}\,[2a + (n - 1)\,d]$

$S_2 - S_1 = \frac{n}{2}[2\,\{2a + (2n - 1)\,d\} - \{2a + (n - 1)d\}]$

$= \frac{n}{2}\,[2a + (3n - 1)\,d]$

$\therefore\ 3(S_2 - S_1) = \frac{3n}{2}\ [2a + (3n - 1)\ d] = S_3$

[Using (iii)]

Hence, $S_3 = 3(S_2 - S_1)$