eSaral

Physics Revision Series

SHM
JEE Main 2019
Jan + Apr All Shifts Qs
Analysis & Solutions

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5. A rod of mass 'M' and length '2L' is suspended at its middle by a wire. It exhibits torsional oscillations; If two masses each of 'm' are attached at distance 'L/2' from its centre on both sides, it reduces the oscillation frequency by 20%. The value of ratio m/M is close to :

(1) 0.17 (2) 0.37 (3) 0.57 (4) 0.77

Ans. (2)

Sol. Frequency of torsonal oscillations is given by

$$f = \frac{k}{\sqrt{I}}$$

$$f_1 = \frac{k}{\sqrt{\frac{M(2L)^2}{12}}}$$

$$f_2 = \frac{k}{\sqrt{\frac{M(2L)^2}{12} + 2m\left(\frac{L}{2}\right)^2}}$$

$$f_2 = 0.8\ f_1$$

$$\frac{m}{M} = 0.375$$

7. A particle is executing simple harmonic motion (SHM) of amplitude A, along the x-axis, about x = 0. When its potential Energy (PE) equals kinetic energy (KE), the position of the particle will be :

(1) $\frac{A}{2}$ (2) $\frac{A}{2\sqrt{2}}$ (3) $\frac{A}{\sqrt{2}}$ (4) A

Ans. (3)

Sol. Potential energy (U) = $\frac{1}{2}kx^2$

Kinetic energy (K) = $\frac{1}{2}kA^2 - \frac{1}{2}kx^2$

According to the question, U = k

$\therefore \frac{1}{2}kx^2 = \frac{1}{2}kA^2 - \frac{1}{2}kx^2$

$x = \pm\frac{A}{\sqrt{2}}$

$\therefore$ Correct answer is (3)

14. A particle executes simple harmonic motion with an amplitude of 5 cm. When the particle is at 4 cm from the mean position, the magnitude of its velocity in SI units is equal to that of its acceleration. Then, its periodic time in seconds is :

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(1) $\frac{7}{3}\pi$ (2) $\frac{3}{8}\pi$

(3) $\frac{4\pi}{3}$ (4) $\frac{8\pi}{3}$

Ans. (4)

$v = \omega\sqrt{A^2 - x^2}$ ____(1)

$a = -\omega^2 x$ ____(2)

$|v| = |a|$ ____(3)

$\omega\sqrt{A^2 - x^2} = \omega^2 x$

$A^2 - x^2 = \omega^2 x^2$

$5^2 - 4^2 = \omega^2(4^2)$

$\Rightarrow 3 = \omega \times 4$

$T = 2\pi/\omega$

20. A particle undergoing simple harmonic motion has time dependent displacement given by $x(t) = A\sin\frac{\pi t}{90}$. The ratio of kinetic to potential energy of this particle at t = 210 s will be :

(1) 2 (2) $\frac{1}{9}$ (3) 3 (4) 1

Ans. **(3)**

Sol. $k = \frac{1}{2}m\omega^2A^2\cos^2\omega t$

$U = \frac{1}{2}m\omega^2A^2\sin^2\omega t$

$\frac{k}{U} = \cot^2\omega t = \cot^2\frac{\pi}{90}(210) = \frac{1}{3}$

Hence ratio is 3 (most appropriate)

29. A body of mass 1 kg falls freely from a height of 100 m on a platform of mass 3 kg which is mounted on a spring having spring constant $k = 1.25 \times 10^6$ N/m. The body sticks to the platform and the spring's maximum compression is found to be x. Given that $g = 10\ ms^{-2}$, the value of x will be close to :

(1) 4 cm

(2) 8 cm

(3) 80 cm

(4) 40 cm

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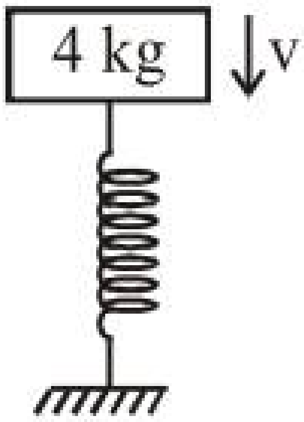
Ans. (1)

Sol. Velocity of 1 kg block just before it collides with 3kg block = $\sqrt{2gh} = \sqrt{2000}\ m/s$

Applying momentum conversation just before and just after collision.

$$1 \times \sqrt{2000} = 4v \Rightarrow v = \frac{\sqrt{2000}}{4}\ m/s$$



Initial compression of spring

$1.25 \times 10^6\ x_0 = 30 \Rightarrow x_0 \approx 0$

applying work energy theorem,

$W_g + W_{sp} = \Delta KE$

$$\Rightarrow 40 \times x + \frac{1}{2} \times 1.25 \times 10^6 (0^2 - x^2)$$

$$= 0 - \frac{1}{2} \times 4 \times v^2$$

solving $x \approx 4$ cm

5. A simple pendulum of length 1 m is oscillating with an angular frequency 10 rad/s. The support of the pendulum starts oscillating up and down with a small angular frequency of 1 rad/s and an amplitude of 10^{-2} m. The relative change in the angular frequency of the pendulum is best given by :-

(1) 10^{-3} rad/s

(2) 10^{-1} rad/s

(3) 1 rad/s

(4) 10^{-5} rad/s

Ans. (1)

Sol. Angular frequency of pendulum

$$\omega = \sqrt{\frac{g_{eff}}{\ell}}$$

$$\therefore \frac{\Delta\omega}{\omega} = \frac{1}{2}\frac{\Delta g_{eff}}{g_{eff}}$$

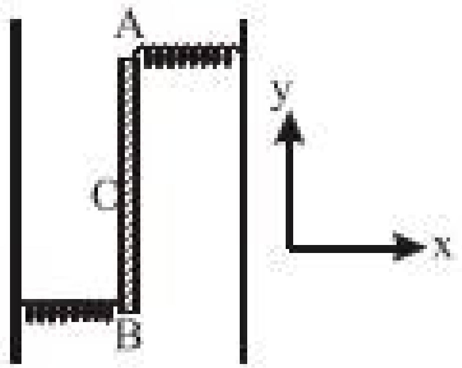
$$\Delta\omega = \frac{1}{2}\frac{\Delta g}{g} \times \omega$$

[ω_s = angular frequency of support]

$$\Delta\omega = \frac{1}{2} \times \frac{2A\omega_s^2}{100} \times 100$$

$$\Delta\omega = 10^{-3} \text{ rad/sec.}$$

1. Two light identical springs of spring constant k are attached horizontally at the two ends of a uniform horizontal rod AB of length ℓ and mass m. The rod is pivoted at its centre 'O' and can rotate freely in horizontal plane. The other ends of the two springs are fixed to rigid supports as shown in figure. The rod is gently pushed through a small angle and released. The frequency of resulting oscillation is:



(1) $\frac{1}{2\pi}\sqrt{\frac{6k}{m}}$

(2) $\frac{1}{2\pi}\sqrt{\frac{2k}{m}}$

(3) $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$

(4) $\frac{1}{2\pi}\sqrt{\frac{3k}{m}}$

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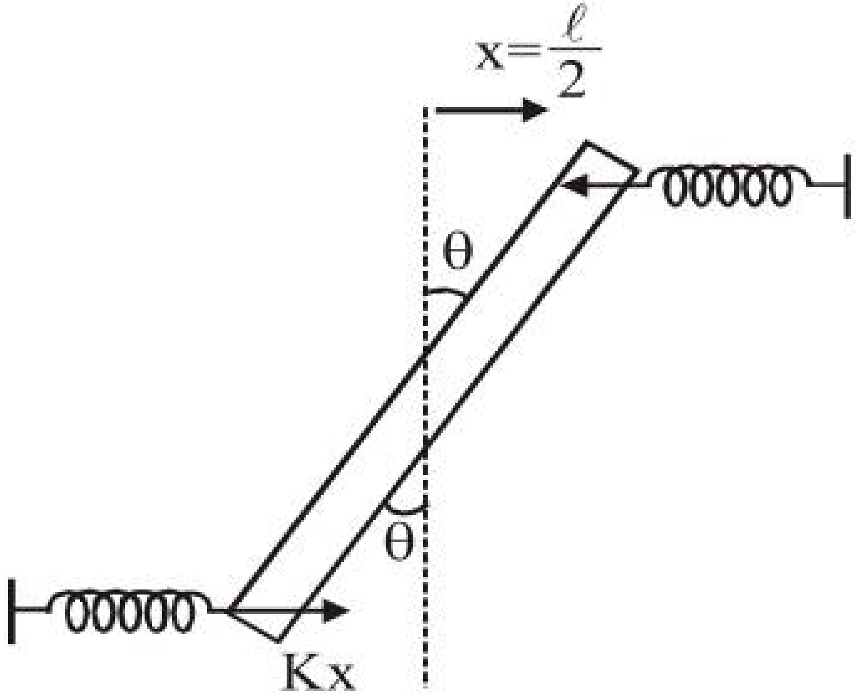
Ans. (1)

Sol.



$$\tau = -2Kx\frac{\ell}{2}\cos\theta$$

$$\Rightarrow \tau = \left(\frac{K\ell^2}{2}\right)\theta = -C\theta$$

$$\Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{C}{I}} = \frac{1}{2\pi}\sqrt{\frac{\frac{K\ell^2}{2}}{\frac{M\ell^2}{12}}} \Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{6K}{M}}$$

4. A simple harmonic motion is represented by: **JEE Main 12-Jan-2019-Evening**

$y = 5(\sin 3\pi t + \sqrt{3}\cos 3\pi t)$ cm

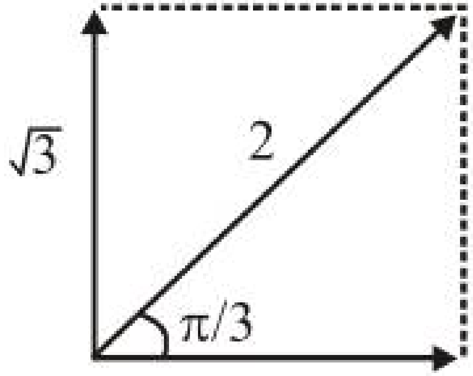
The amplitude and time period of the motion are:

(1) 5cm, $\frac{3}{2}$s (2) 5cm, $\frac{2}{3}$s

(3) 10cm, $\frac{3}{2}$s (4) 10cm, $\frac{2}{3}$s

Ans. **(4)**

Sol.



$$y = 5\left[\sin(3\pi t) + \sqrt{3}\cos(3\pi t)\right]$$

$$= 10\sin\left(3\pi t + \frac{\pi}{3}\right)$$

Amplitude = 10 cm

$$T = \frac{2\pi}{w} = \frac{2\pi}{3\pi} = \frac{2}{3}\text{sec}$$

17. A damped harmonic oscillator has a frequency of 5 oscillations per second. The amplitude drops to half its value for every 10 oscillations. The time it will take to drop to $\frac{1}{1000}$ of the original amplitude is close to :-

(1) 100 s (2) 20 s (3) 10 s (4) 50 s

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Sol. $A = A_0e^{-\gamma t}$

$A = \frac{A_0}{2}$ after 10 oscillations

$\because$ After 2 seconds

$$\frac{A_0}{2} = A_0e^{-\gamma(2)}$$

$2 = e^{2\gamma}$

$\ell n2 = 2\gamma$

$$\gamma = \frac{\ell n2}{2}$$

$\because A = A_0e^{-\gamma t}$

$$\ell n\frac{A_0}{A} = \gamma t$$

$$\ell n1000 = \frac{\ell n2}{2}t$$

$$2\left(\frac{3\ell n10}{\ell n2}\right) = t$$

$$\frac{6\ell n10}{\ell n2} = t$$

$t = 19.931$ sec

$t \approx 20$ sec

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6. A simple pendulum oscillating in air has period T. The bob of the pendulum is completely immersed in a non-viscous liquid. The density of the liquid is $\frac{1}{16}$ th of the material of the bob. If the bob is inside liquid all the time, its period of oscillation in this liquid is :

(1) $4T\sqrt{\frac{1}{15}}$ (2) $2T\sqrt{\frac{1}{10}}$

(3) $4T\sqrt{\frac{1}{14}}$ (4) $2T\sqrt{\frac{1}{14}}$

Sol. For a simple pendulum $T = 2\pi\sqrt{\frac{L}{g_{eff}}}$

situation 1 : when pendulum is in air $\rightarrow g_{eff} = g$

situation 2 : when pendulum is in liquid

$$\rightarrow g_{eff} = g\left(1-\frac{\rho_{liquid}}{\rho_{body}}\right) = g\left(1-\frac{1}{16}\right) = \frac{15g}{16}$$

So, $$\frac{T'}{T} = \frac{2\pi\sqrt{\frac{L}{15g/16}}}{2\pi\sqrt{\frac{L}{g}}} \Rightarrow T' = \frac{4T}{\sqrt{15}}$$

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18. The displacement of a damped harmonic oscillator is given by
$x(t) = e^{-01.1t} \cos(10\pi t + \phi)$. Here t is in seconds. The time taken for its amplitude of vibration to drop to half of its initial value is close to :

(1) 13 s (2) 7 s (3) 27 s (4) 4 s

Sol. $A = A_0 e^{-0.1t} = \dfrac{A_0}{2}$

$\ln 2 = 0.1t$

$t = 10 \ln 2 = 6.93 \approx 7$ sec

14. A spring whose unstretched length is l has a force constant k. The spring is cut into two pieces of unstretched lengths l_1 and l_2 where, $l_1 = nl_2$ and n is an integer. The ratio k_1/k_2 of the corresponding force constants, k_1 and k_2 will be :

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(1) $\frac{1}{n^2}$ (2) n^2 (3) $\frac{1}{n}$ (4) n

Sol. $k_1 = \frac{C}{\ell_1}$

$k_2 = \frac{C}{\ell_2}$

$\frac{k_1}{k_2} = \frac{C\ell_2}{\ell_1 C}\ell_2 = \frac{\ell_2}{n\ell_2} = \frac{1}{n}$